# Physics 101: Lecture 13 Rotational Kinetic Energy and Rotational Inertia

# Overview of Semester

- Newton's Laws
  - ➔ $F_{Net} = m\, a$
- Work-Energy
  - ➔ $F_{Net} = m\, a$ multiply both sides by d
  - ➔ $W_{Net} = \Delta KE$ Energy is "conserved"
  - ➔ Useful when know Work done by forces
- Impulse-Momentum
  - ➔ $F_{Net} = m\, a = \Delta p/\Delta t$
  - ➔ Impulse = $\Delta p$
  - ➔ Momentum is conserved
  - ➔ Works in each direction independently

# Linear and Angular Motion

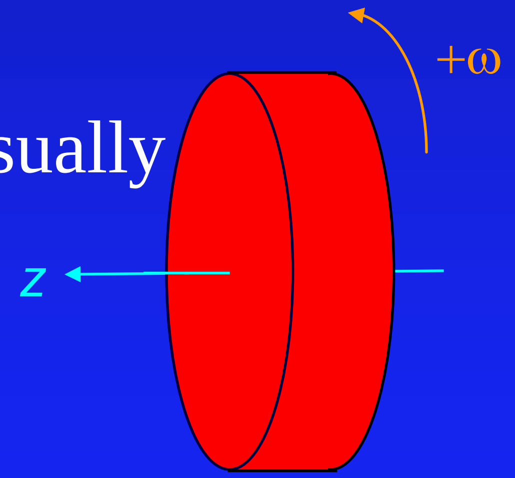
| | Linear | Angular |
|---|---|---|
| Displacement | $x$ | $\theta$ |
| Velocity | $v$ | $\omega$ |
| Acceleration | $a$ | $\alpha$ |
| Inertia | $m$ | $I$ |
| KE | $\frac{1}{2} m v^2$ | Today! |
| Newton's $2^{nd}$ | $F=ma$ | |
| Momentum | $p = mv$ | |

# Comment on axes and sign
## (i.e. what is positive and negative)

Whenever we talk about rotation, it is implied that there is a rotation "axis".

This is usually called the "z" axis (we usually omit the z subscript for simplicity).

Counter-clockwise (increasing $\theta$) is usually called positive.




Clockwise (decreasing $\theta$) is usually called negative. [demo]

# Energy ACT/demo

- When the bucket reaches the bottom, its potential energy has decreased by an amount mgh. Where has this energy gone?

A) Kinetic Energy of bucket

B) Kinetic Energy of flywheel

C) Both 1 and 2.

# Rotational Kinetic Energy

- Consider a mass $M$ on the end of a string being spun around in a circle with radius $r$ and angular frequency $\omega$ [demo]

  ➔ Mass has speed $v = \omega r$

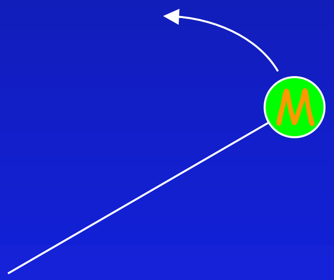
  ➔ Mass has kinetic energy

  » $K = \frac{1}{2} M v^2$

  » $\quad = \frac{1}{2} M \omega^2 r^2$

  » $\quad = \frac{1}{2} (M r^2) \omega^2$

  » $\quad \boxed{= \frac{1}{2} I \omega^2}$




- Rotational Kinetic Energy is energy due to circular motion of object.

# Rotational Inertia I

- Tells how much "work" is required to get object spinning. Just like mass tells you how much "work" is required to get object moving.
  - ➔ $K_{tran} = ½\, m\, v^2$   Linear Motion
  - ➔ $K_{rot} = ½\, I\, \omega^2$   Rotational Motion

- $I = \Sigma\, m_i r_i^2$   (units kg m$^2$)

- Note! Rotational Inertia (or "Moment of Inertia") depends on what you are spinning about (basically the $r_i$ in the equation).

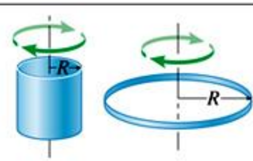
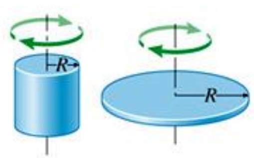
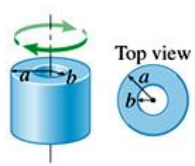
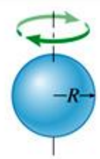
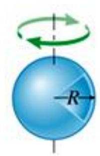
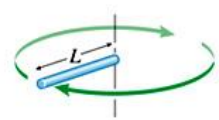
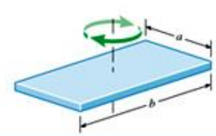
# Rotational Inertia Table

- For objects with finite number of masses, use $I = \Sigma\, m\, r^2$. For "continuous" objects, use table below (p. 263 of book).



**Table 8.1**

**Rotational Inertia for Uniform Objects with Various Geometrical Shapes**

| Shape | | Axis of Rotation | Rotational Inertia |
|---|---|---|---|
| Thin hollow cylindrical shell (or hoop) |  | Central axis of cylinder | $MR^2$ |
| Solid cylinder (or disk) |  | Central axis of cylinder | $\frac{1}{2}MR^2$ |
| Hollow cylindrical shell or disk |  | Central axis of cylinder | $\frac{1}{2}M(a^2+b^2)$ |
| Solid sphere |  | Through center | $\frac{2}{5}MR^2$ |
| Thin hollow spherical shell |  | Through center | $\frac{2}{3}MR^2$ |
| Thin rod |  | Perpendicular to rod through end | $\frac{1}{3}ML^2$ |
| Rectangular plate |  | Perpendicular to plate through center | $\frac{1}{12}M(a^2+b^2)$ |

# Merry Go Round

Four kids (mass m) are riding on a (light) merry-go-round rotating with angular velocity $\omega$=3 rad/s. In case A the kids are near the center (r=1.5 m), in case B they are near the edge (r=3 m). Compare the kinetic energy of the kids on the two rides.


A) $K_A > K_B$ B) $K_A = K_B$ C) $K_A < K_B$

# Inertia Rods

Two batons have equal mass and length.

Which will be "easier" to spin

A) Mass on ends

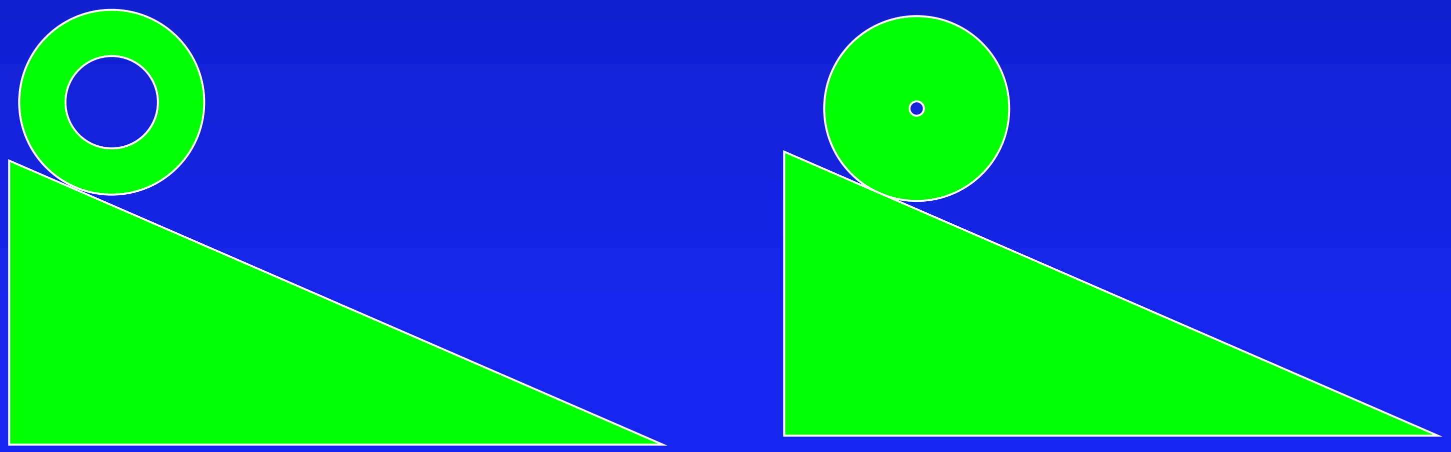
B) Same

C) Mass in center

# Checkpoint: Rolling Race (Hoop vs Cylinder)

A solid and hollow cylinder of equal mass roll down a ramp with height h. Which has greatest KE at bottom?

A) Solid B) Hollow C) Same



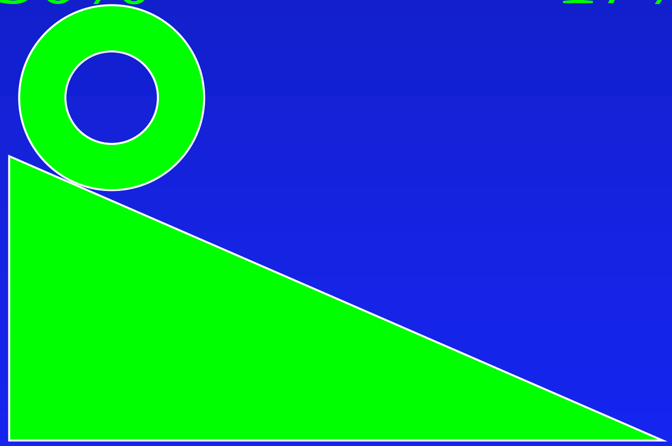
# Prelecture: Rolling Race (Hoop vs Cylinder)

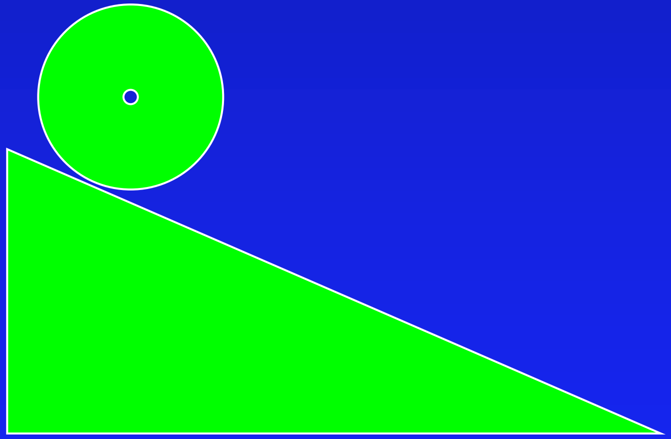
A solid and hollow cylinder of equal mass roll down a ramp with height h. Which has greatest speed at the bottom of the ramp?

A) Solid 50%

B) Hollow 17%

C) Same 33%



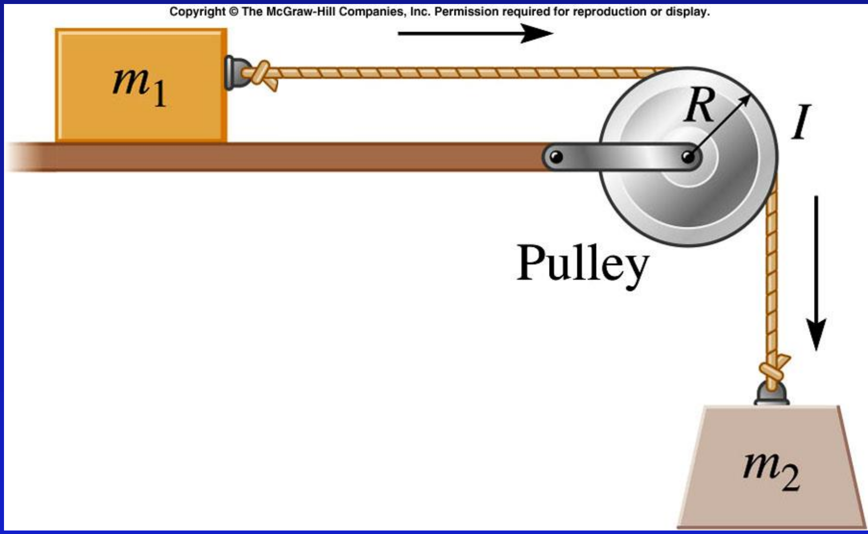


# Main Ideas

- Rotating objects have kinetic energy
  - ➔ $KE = \frac{1}{2} I \omega^2$
- Moment of Inertia $I = \Sigma mr^2$
  - ➔ Depends on Mass
  - ➔ Depends on axis of rotation
- Energy is conserved but need to include rotational energy too: $K_{rot} = \frac{1}{2} I \omega^2$

# Massless Pulley Example

Consider the two masses connected by a pulley as shown. Use conservation of energy to calculate the speed of the blocks after $m_2$ has dropped a distance h. Assume the pulley is massless.




$$E = K + U$$

$$E_0 = E_f$$

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$0 + 0 = -m_2 gh + \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2$$

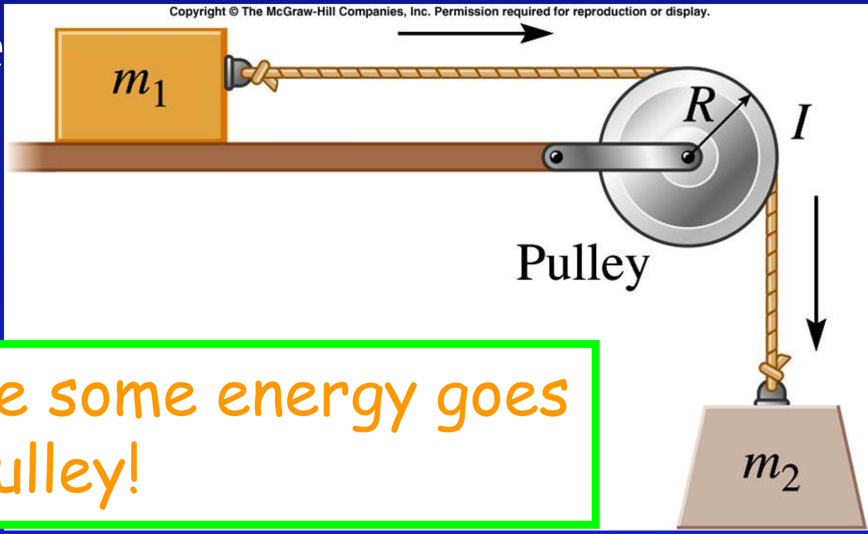
$$2 m_2 gh = m_1 v^2 + m_2 v^2$$

$$v = \sqrt{\frac{2 m_2 gh}{m_1 + m_2}}$$

Note: Tension does positive work on 1 and negative work on 2. Net work (on 1 and 2) by tension is ZERO.

# Massive Pulley Act

Consider the two masses connected by a pulley as shown. If the pulley is massive after m2 drops a distance h, the blocks will be moving




A) faster than

B) the same speed as

C) slower than

if it was a massless pulley

Slower because some energy goes into spinning pulley!

$$U_{initial} + K_{initial} = U_{final} + K_{final}$$

$$m_2 gh = +\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{4}Mv^2$$

$$0 = -m_2 gh + \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}I\omega^2$$

$$\boxed{v = \sqrt{\frac{2m_2 gh}{m_1 + m_2 + M/2}}}$$

$$m_2 gh = +\frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v}{R}\right)^2$$

# Summary

- Rotational Kinetic Energy $K_{rot} = \frac{1}{2} I \omega^2$
- Rotational Inertia $I = \Sigma m_i r_i^2$
- Energy Still Conserved!