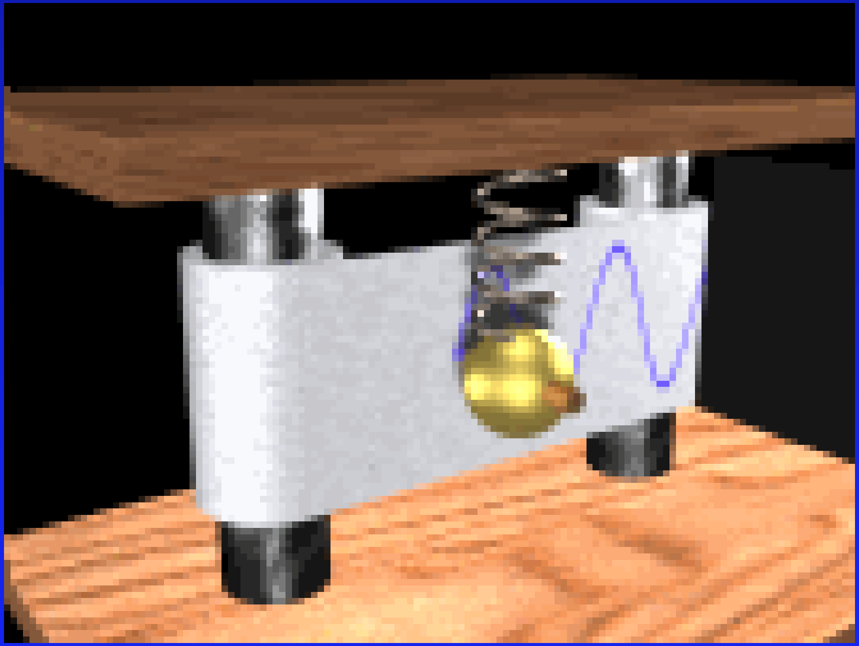
Exam III

Physics 101: Lecture 19
Elasticity and Oscillations



Overview

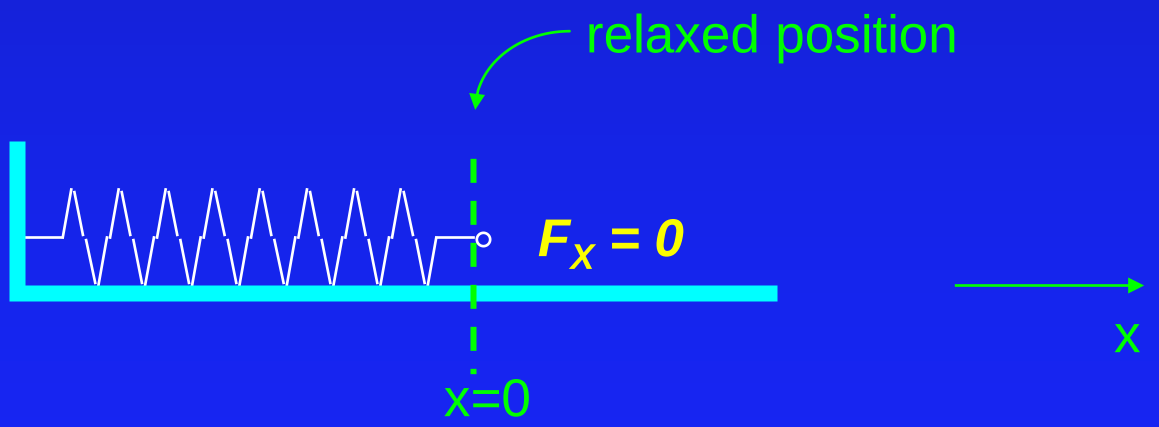
- Springs (review)
  - ➔ Restoring force proportional to displacement
  - ➔ $F = -k\ x$ (often a good approximation)
  - ➔ $U = \frac{1}{2}\ k\ x^2$
- Today
  - ➔ Simple Harmonic Motion
  - ➔ Springs Revisited
  - ➔ Young's Modulus (where does k come from?)

Springs

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.

➔ $F_X = -k\,x$

Where x is the displacement from the relaxed position and k is the constant of proportionality.

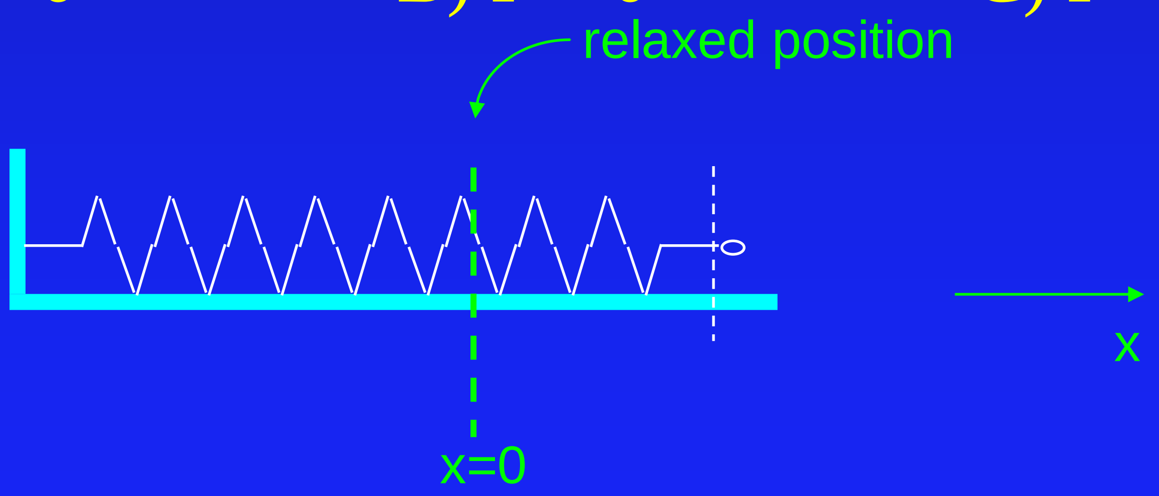


Springs ACT

- **Hooke's Law:** The force exerted by a spring is proportional to the distance the spring is stretched or compressed from its relaxed position.
  - ➔ $F_X = -k\,x$ Where x is the displacement from the relaxed position and k is the constant of proportionality.

What is force of spring when it is stretched as shown below.

A) F > 0   B) F = 0   C) F < 0



Springs

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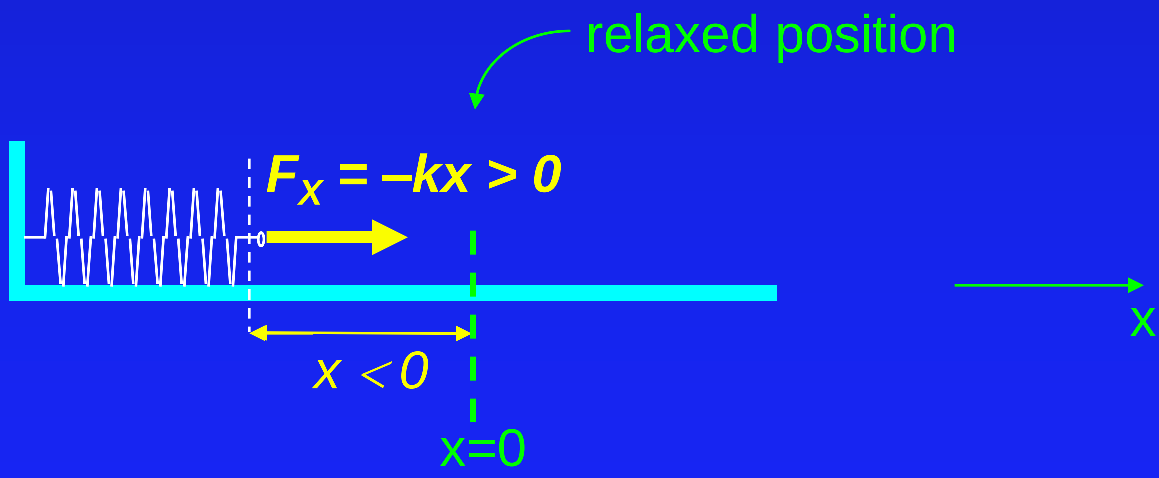
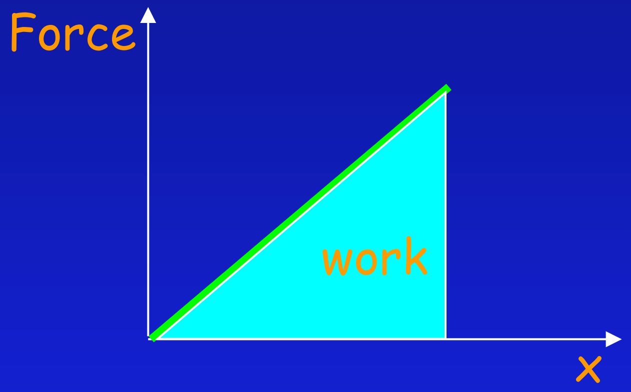


Potential Energy in Spring

- Hooke's Law force is Conservative
  - ➔ $F = -k\,x$
  - ➔ $W = -1/2\; k\, x^2$

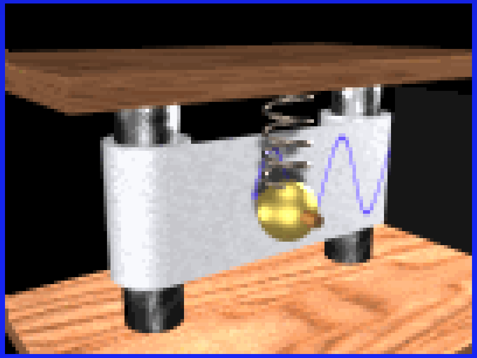


  - ➔ Work done only depends on initial and final position

  - ➔ Define Potential Energy $U_{spring} = \frac{1}{2}\, k\, x^2$

Simple Harmonic Motion

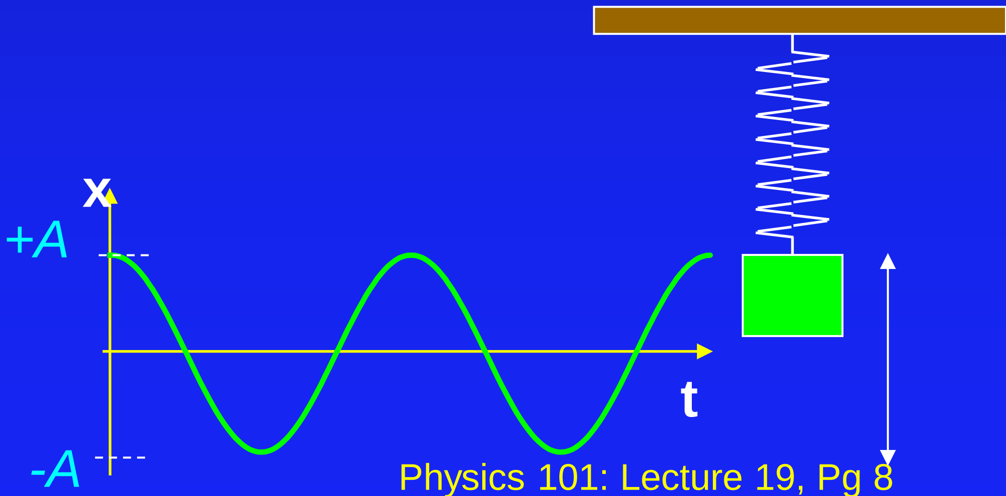
- Vibrations
  - ➔ Vocal cords when singing/speaking
  - ➔ String/rubber band
- Simple Harmonic Motion
  - ➔ Restoring force proportional to displacement
  - ➔ Springs $F = -kx$



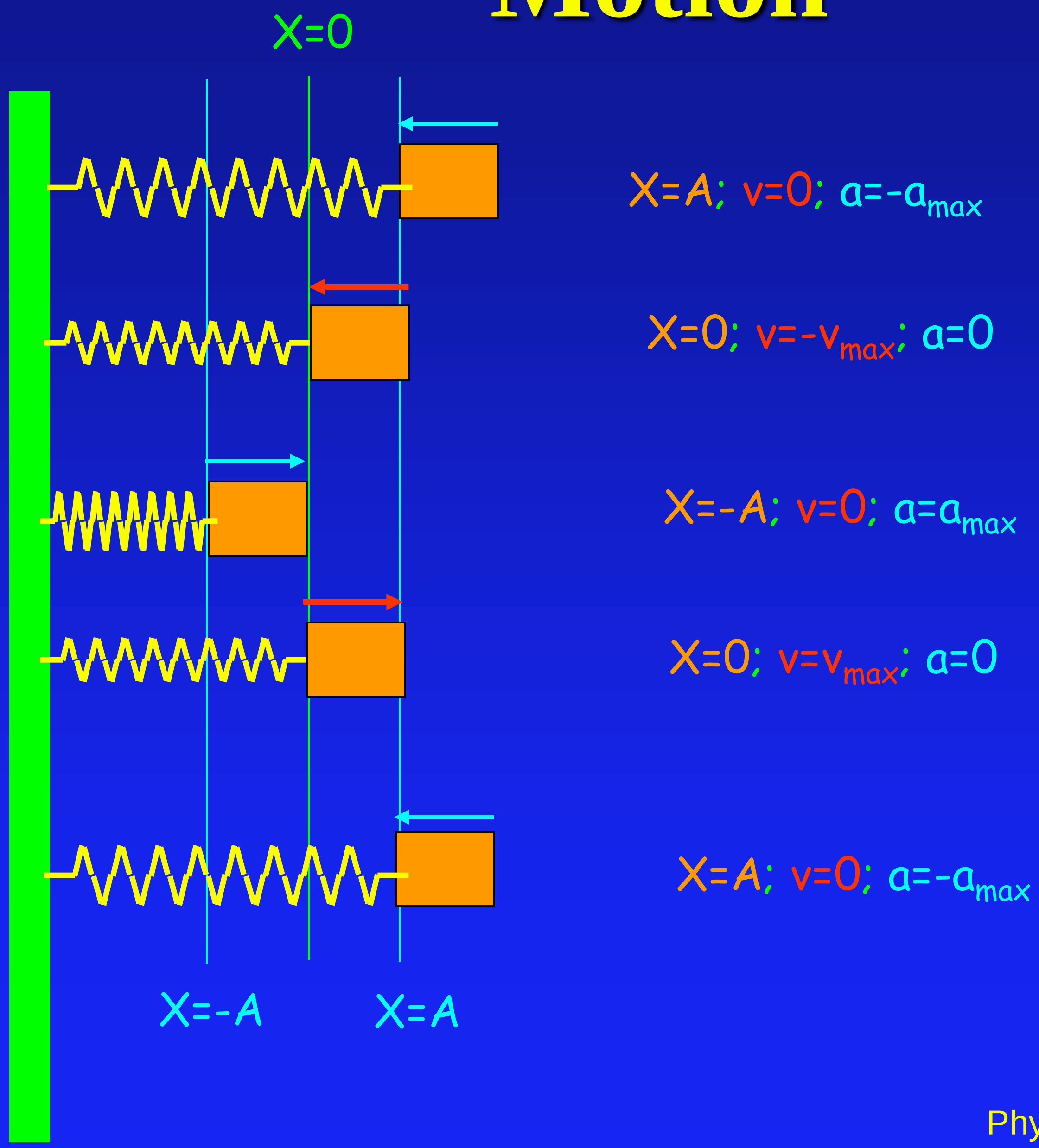
Spring ACT II

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the magnitude of the acceleration of the block biggest?

1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The acceleration of the mass is constant



Springs and Simple Harmonic Motion



Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$

$v(t) = -[A\omega]\sin(\omega t)$

$a(t) = -[A\omega^2]\cos(\omega t)$

OR

$x(t) = [A]\sin(\omega t)$

$v(t) = [A\omega]\cos(\omega t)$

$a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

$a_{max} = A\omega^2$

Period = T (seconds per cycle)

Frequency = f = 1/T (cycles per second)

Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$

***Energy ***

- A mass is attached to a spring and set to motion. The maximum displacement is $x = A$
  - ➔ $\Sigma W_{nc} = \Delta K + \Delta U$
  - ➔ $0 = \Delta K + \Delta U$ or Energy U+K is constant!

    Energy $= \frac{1}{2} k x^2 + \frac{1}{2} m v^2$
  - ➔ At maximum displacement $x = A$, $v = 0$

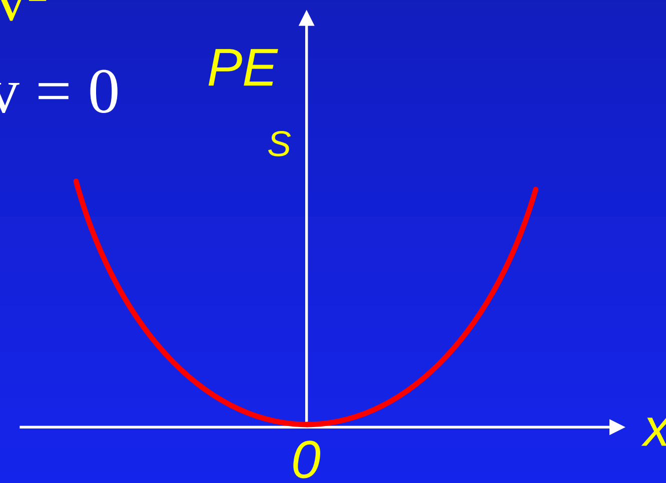
    Energy $= \frac{1}{2} k A^2 + 0$
  - ➔ At zero displacement $x = 0$

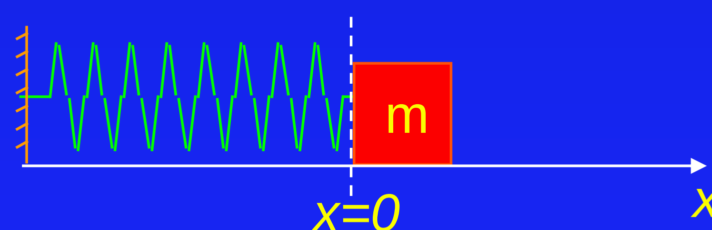
    Energy $= 0 + \frac{1}{2} m v_m^2$

  Since Total Energy is same

  $\frac{1}{2} k A^2 = \frac{1}{2} m v_m^2$

  $v_m = \text{sqrt}(k/m)\ A$

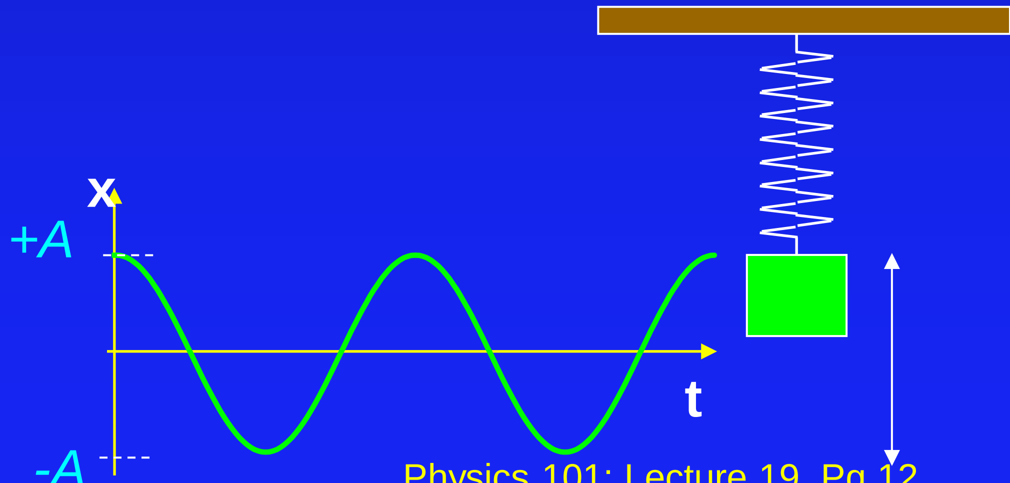




Prelecture 1+2

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude A. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the speed of the block biggest?

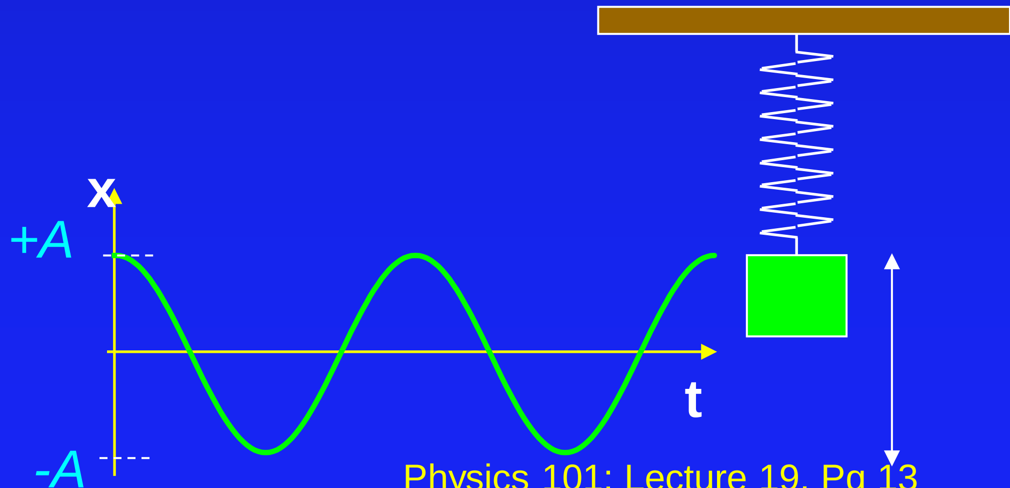
1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The speed of the mass is constant



Prelecture 3+4

A mass on a spring oscillates back & forth with simple harmonic motion of amplitude *A*. A plot of displacement (x) versus time (t) is shown below. At what points during its oscillation is the total energy (K+U) of the mass and spring a maximum? (Ignore gravity).

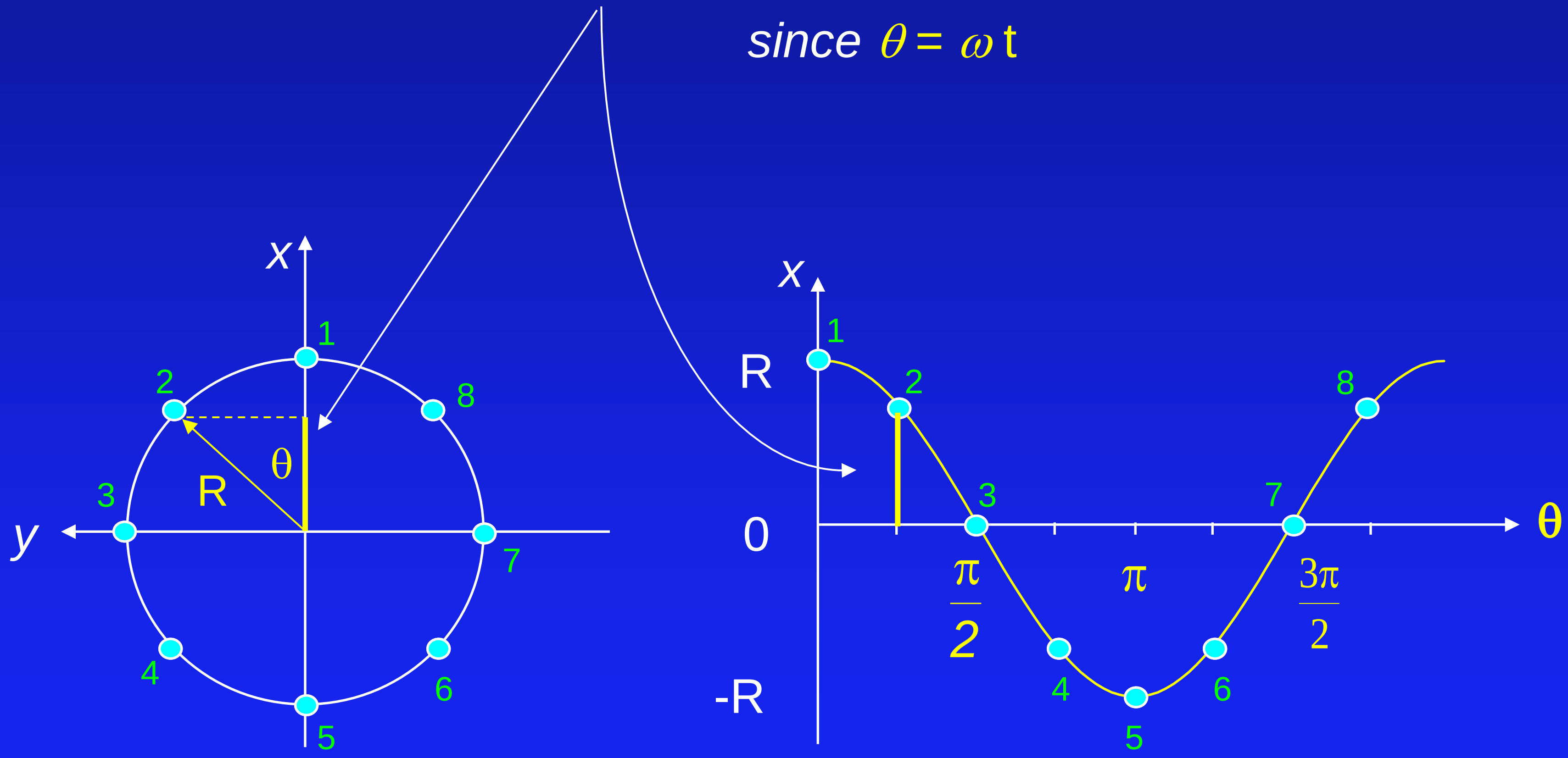
1. When x = +A or -A (i.e. maximum displacement)
2. When x = 0 (i.e. zero displacement)
3. The energy of the system is constant.



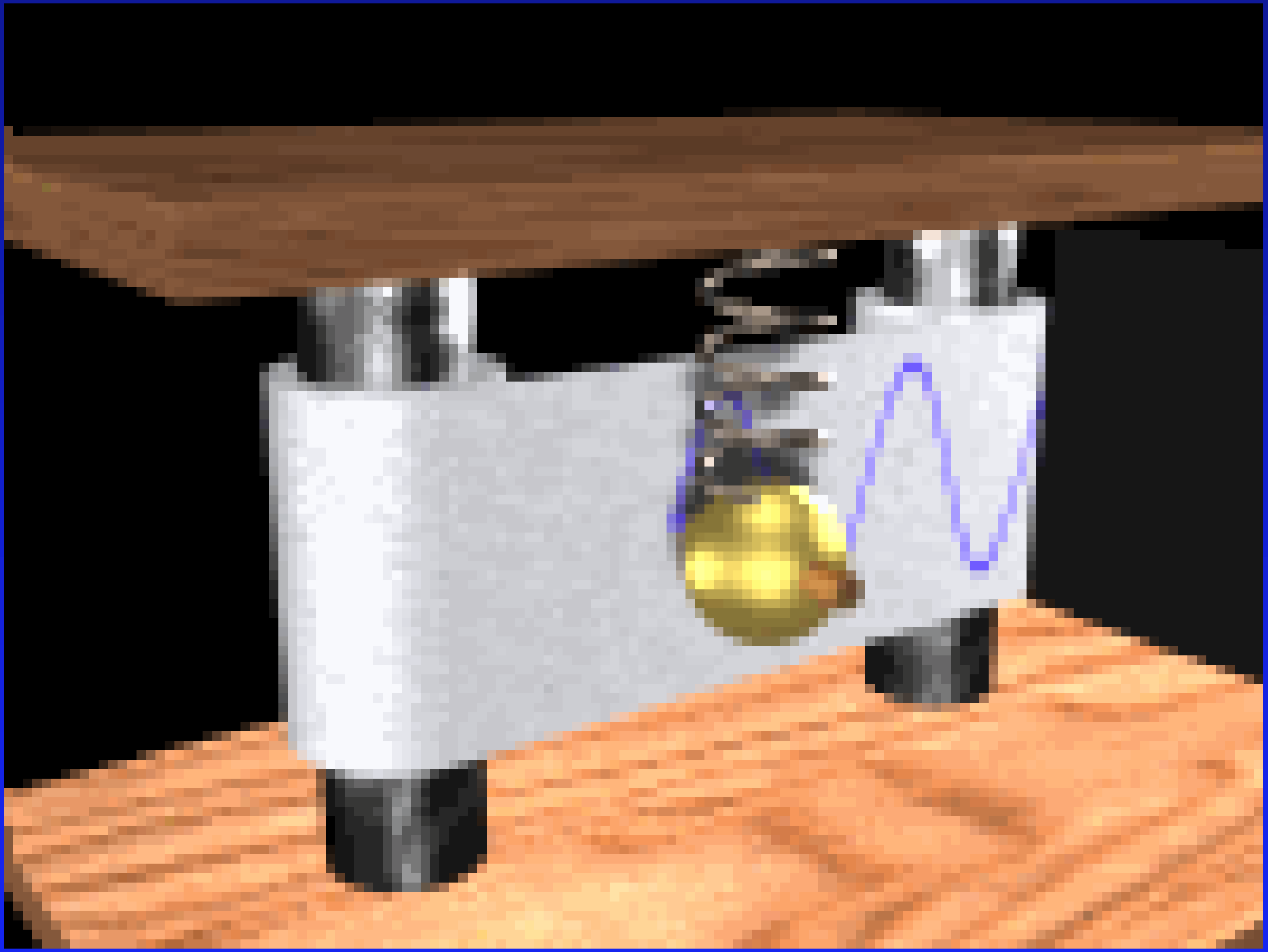
What does *moving in a circle* have to do with moving back & forth *in a straight line* ??

$$x = R\cos\theta = R\cos(\omega t)$$

since $\theta = \omega t$



SHM and Circles



Simple Harmonic Motion:

$x(t) = [A]\cos(\omega t)$		$x(t) = [A]\sin(\omega t)$
$v(t) = -[A\omega]\sin(\omega t)$	OR	$v(t) = [A\omega]\cos(\omega t)$
$a(t) = -[A\omega^2]\cos(\omega t)$		$a(t) = -[A\omega^2]\sin(\omega t)$

$x_{max} = A$

$v_{max} = A\omega$

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Period = T (seconds per cycle)

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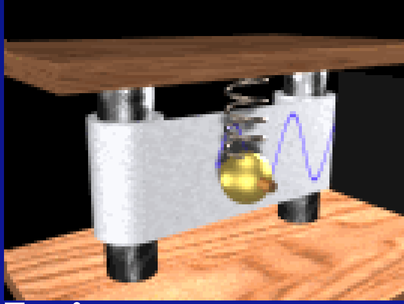
Angular frequency = $\omega = 2\pi f = 2\pi/T$

For spring: $\omega^2 = k/m$

Example



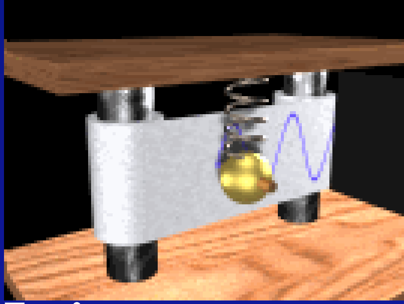
A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

Which equation describes the position as a function of time x(t) =

A) $5 \sin(\omega t)$ B) $5 \cos(\omega t)$ C) $24 \sin(\omega t)$

D) $24 \cos(\omega t)$ E) $-24 \cos(\omega t)$

Example

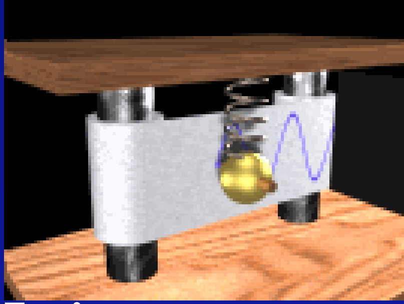


A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the total energy of the block spring system?

A) 0.03 J B) .05 J C) .08 J

Example

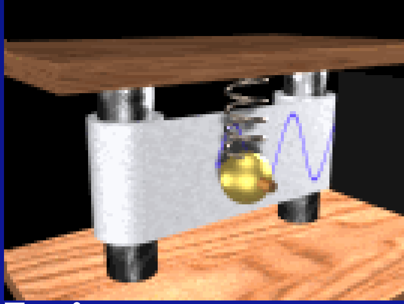


A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

What is the maximum speed of the block?

A) .45 m/s B) .23 m/s C) .14 m/s

Example



A 3 kg mass is attached to a spring (k=24 N/m). It is stretched 5 cm. At time t=0 it is released and oscillates.

How long does it take for the block to return to x=+5cm?

A) 1.4 s B) 2.2 s C) 3.5 s

Summary

- Springs
  - ➔ $F = -kx$
  - ➔ $U = \frac{1}{2} k x^2$
  - ➔ $\omega = \text{sqrt}(k/m)$

- Simple Harmonic Motion
  - ➔ Occurs when have linear restoring force $F = -kx$
  - ➔ $x(t) = [A] \cos(\omega t)$ or $[A] \sin(\omega t)$
  - ➔ $v(t) = -[A\omega] \sin(\omega t)$ or $[A\omega] \cos(\omega t)$
  - ➔ $a(t) = -[A\omega^2] \cos(\omega t)$ or $-[A\omega^2] \sin(\omega t)$

Young's Modulus

- Spring F = -k x [demo]
  - ➔What happens to "k" if cut spring in half?
  - ➔A) decreases B) same C) increases

- k is inversely proportional to length!
- Define
  - ➔Strain = $\Delta L / L$
  - ➔Stress = F/A
- Now
  - ➔Stress = Y Strain
  - ➔$F/A = Y \ \Delta L/L$
  - ➔$k = Y\, A/L$ from F = k x
- Y (Young's Modules) independent of L