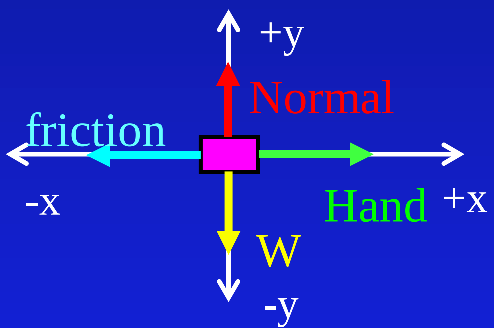
Physics 101: Lecture 06
Two Dimensional Dynamics

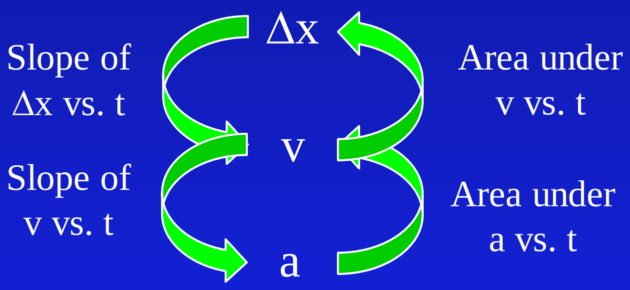
Today's lecture will cover Chapter 4

Overview

Draw a FBD to determine F_{Net} ⇄ Apply Newton's 2nd Law to determine acceleration ⇄ Use Kinematics to determine/describe motion of the object



$$\vec{F}_{Net} = m\vec{a}$$



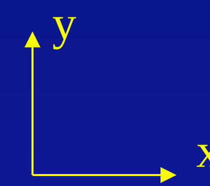
$x = x_0 + v_0 t + 1/2\, at^2$

$v = v_0 + at$

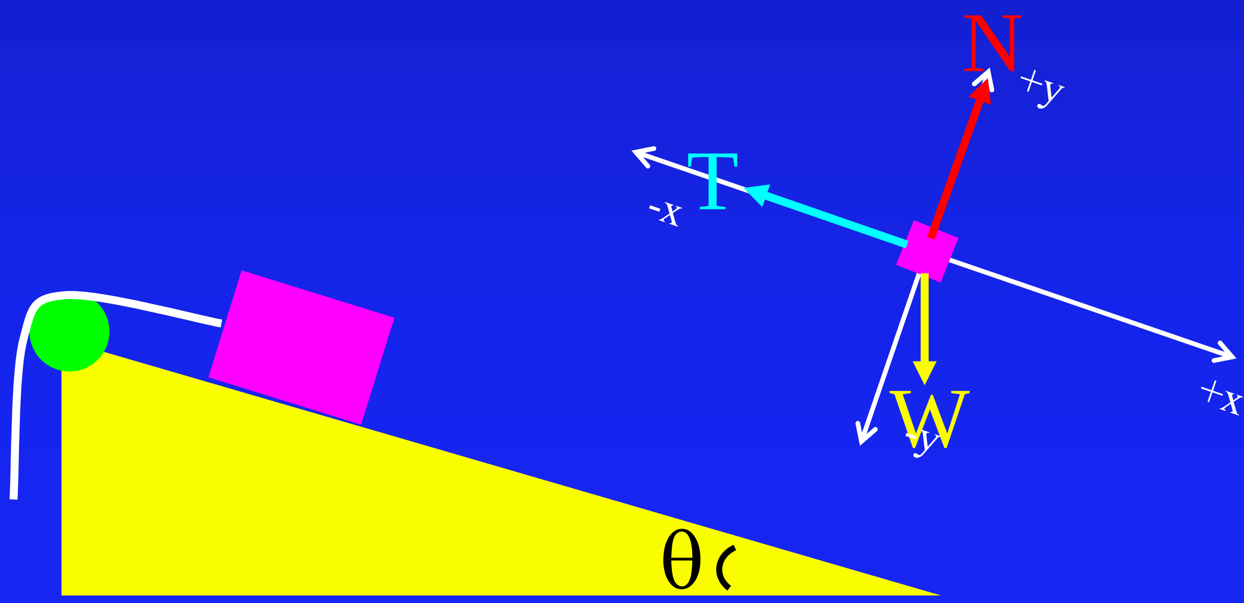
$v^2 = v_0^2 + 2a(x-x_0)$

Today: 2D Kinematics!

2-Dimensional Kinematics



- X and Y are INDEPENDENT!
- Break 2-D problem into two 1-D problems



$$F_{net,\,x} = ma_x$$

$$F_{net,\,y} = ma_y$$

Position, Velocity and Acceleration

- Position, Velocity and Acceleration are Vectors!

	x direction	y direction	
$\vec{v}_{av} = \dfrac{\vec{r}_f - \vec{r}_0}{t_f - t_0}$	$v_x = \dfrac{x_f - x_0}{t_f - t_0}$	$v_y = \dfrac{y_f - y_0}{t_f - t_0}$	$\lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2}$
$\vec{a}_{av} = \dfrac{\vec{v}_f - \vec{v}_0}{t_f - t_0}$	$a_x = \dfrac{v_{xf} - v_{x0}}{t_f - t_0}$	$a_y = \dfrac{v_{yf} - v_{y0}}{t_f - t_0}$	$\lvert\vec{a}\rvert = \sqrt{a_x^2 + a_y^2}$

- x and y directions are INDEPENDENT!

Velocity in Two Dimensions

A ball is rolling on a horizontal surface at 5 m/s. It then rolls up a ramp at a 25 degree angle. After 0.5 seconds, the ball has slowed to 3 m/s.

What is the magnitude of the change in velocity?

A) 0 m/s B) 2 m/s C) 2.6 m/s D) 3 m/s E) 5 m/s

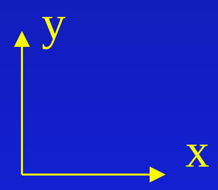
x-direction

v_{0x} = 5 m/s

v_{fx} = 3 m/s cos(25)

Δv_x = 3cos(25)–5 =-2.28m/s

y-direction

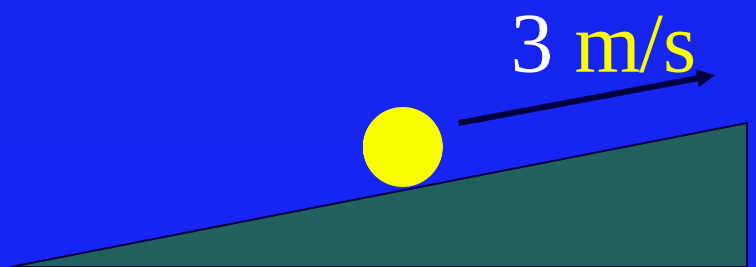


v_{0y} = 0 m/s

v_{fy} = 3 m/s sin(25)

Δv_y = 3sin(25)=+1.27 m/s

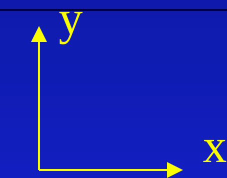
$$|\Delta v| = \sqrt{\Delta v_x^2 + \Delta v_y^2} = 2.6\ \text{m/s}$$



Acceleration in Two Dimensions

A ball is rolling on a horizontal surface at 5 m/s. It then rolls up a ramp at a 25 degree angle. After 0.5 seconds, the ball has slowed to 3 m/s. What is the average acceleration?

x-direction	y-direction
$a_x = \frac{-2.28\,\text{m/s}}{0.5\,\text{s}} = -4.56\,\text{m/s}^2$	$a_y = \frac{1.27\,\text{m/s}}{0.5\,\text{s}} = 2.54\,\text{m/s}^2$



$$|a| = \sqrt{a_x^2 + a_y^2} = 5.21\,\text{m/s}^2$$

Kinematics in Two Dimensions

- $x = x_0 + v_{0x}t + 1/2\, a_x t^2$
- $v_x = v_{0x} + a_x t$
- $v_x^2 = v_{0x}^2 + 2a_x\, \Delta x$

- $y = y_0 + v_{0y}t + 1/2\, a_y t^2$
- $v_y = v_{0y} + a_y t$
- $v_y^2 = v_{0y}^2 + 2a_y\, \Delta y$

Must be able to identify variables in these equations!

x and y motions are independent!
They share a common time t

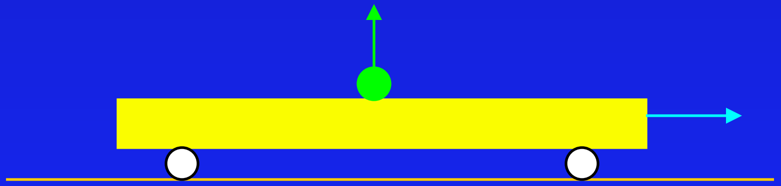
Train Act/Demo

A flatbed railroad car is moving along a track at constant velocity. A passenger at the center of the car throws a ball straight up. Neglecting air resistance, where will the ball land?

A. Forward of the center of the car

B. At the center of the car ← correct

C. Backward of the center of the car



- x- direction ball and car start with same position and velocity, a=0, so always have same position

Demo - train

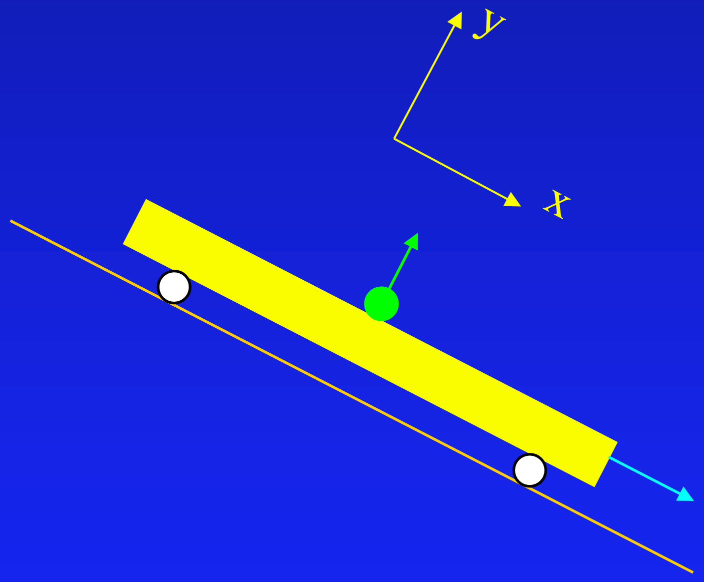
ACT

A flatbed railroad car is accelerating down a track due to gravity. The ball is shot perpendicular to the track. Where will it land?

A. Forward of the center of the car

B. At the center of the car

C. Backward of the center of the car



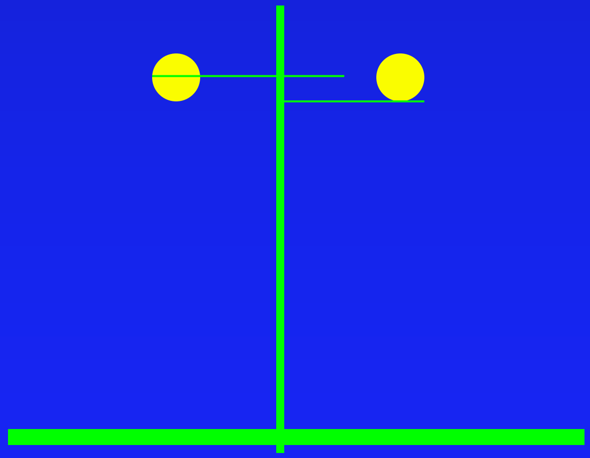
Projectile Motion ACT

One marble is given an initial horizontal velocity, the other simply dropped. Which marble hits the ground first?

A) dropped

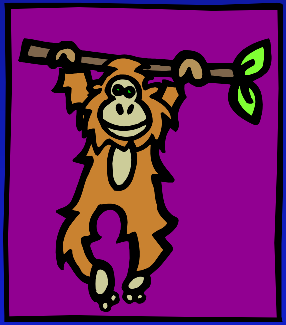
B) pushed

C) They both hit the ground at the same time



Monkey Checkpoint

You are a vet trying to shoot a tranquilizer dart into a monkey hanging from a branch in a distant tree. You know that the monkey is very nervous, and will let go of the branch and start to fall as soon as your gun goes off. In order to hit the monkey with the dart, where should you point the gun before shooting?



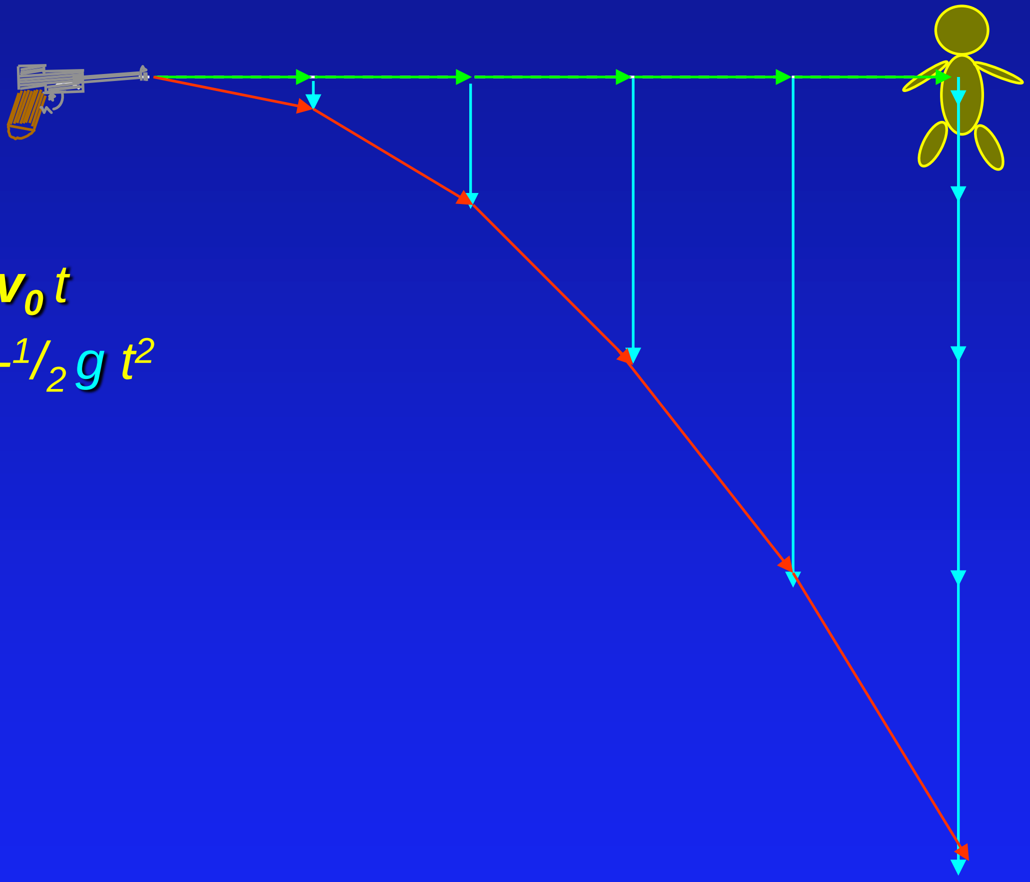
1 Right at the monkey

2 Below the monkey

3 Above the monkey

See text: 4-3

Shooting the Monkey...



$x = \mathbf{v_0}\, t$

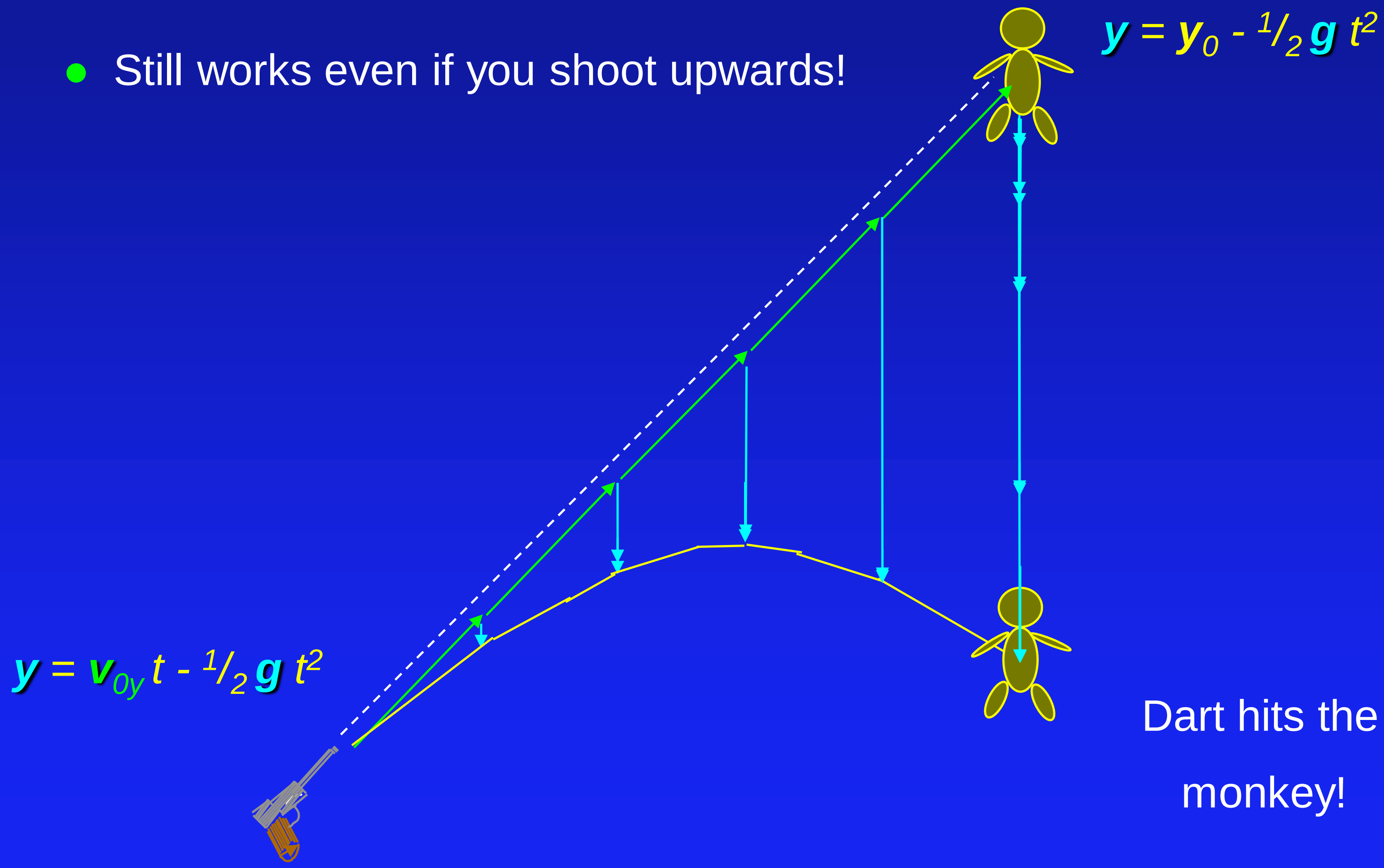
$y = -\frac{1}{2}\, g\, t^2$

$x = x_0$

$y = -\frac{1}{2}\, g\, t^2$

Shooting the Monkey...

- Still works even if you shoot upwards!



Dart hits the monkey!

Projectile Motion

$a_x = 0$ $a_y = -g$

- $x = x_0 + v_{0x}\, t$
- $v_x = v_{0x}$

- $y = y_0 + v_{0y}\, t - \frac{1}{2} g t^2$
- $v_y = v_{0y} - g\, t$
- $v_y^2 = v_{0y}^2 - 2\, g\, \Delta y$

- Choose direction where you "know" information
- Solve kinematics in that direction.
- Use *t* from that direction as *t* in other direction

Throw ball to Monkey

You throw a ball to a monkey who is on a platform 12 meters above and 5 meters to the right of you. Determine the speed and angle you should throw it such that it "just reaches" the monkey.

Y-direction (v_{0y})

$v_{fy}^2 - v_{0y}^2 = 2\ a\ \Delta y$

$v_{0y} = \text{sqrt}(2\ 9.8\ 12)$

$= 15.3$ m/s

$v_{fy} = v_{0y} + a\ t$

$t = v_{0y}/g = 1.56$ s.

X-direction (v_{0x})

$v_{0x} = d/t$

$= 5$ m / 1.56 s

$= 3.2$ m/s

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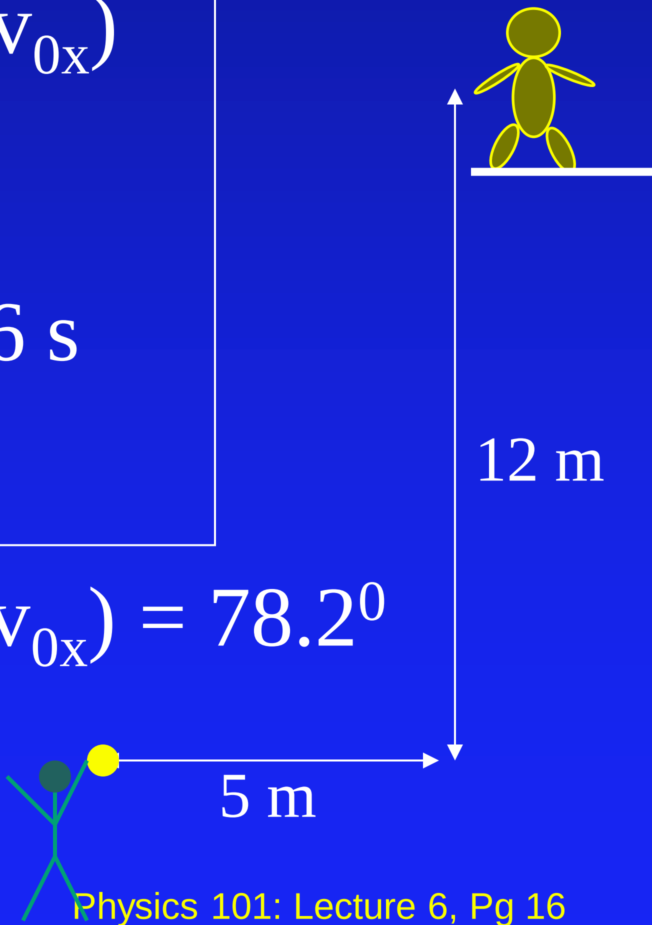
$\alpha = \arctan(v_{0y}/v_{0x}) = 78.2^0$

$|v| = (v_{0x}^2 + v_{0y}^2)^{1/2} = 15.6$ m/s



Summary of Concepts

- X and Y directions are Independent!
  - ➔ Position, velocity and acceleration are vectors
  - ➔ "Share" t
- F = m a applies in both x and y direction
- Projectile Motion
  - ➔ $a_x = 0$ in horizontal direction
  - ➔ $a_y = g$ in vertical direction