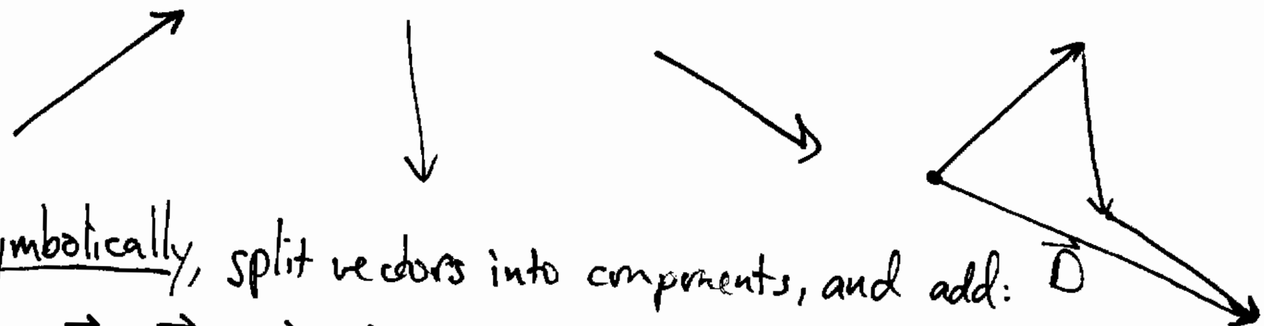
MATH

Vector Addition

Graphically, place vectors head to tail. The resultant is drawn from the tail of the first to the head of the last:

$$\vec{A} \quad + \quad \vec{B} \quad + \quad \vec{C} \quad = \quad \vec{D}$$



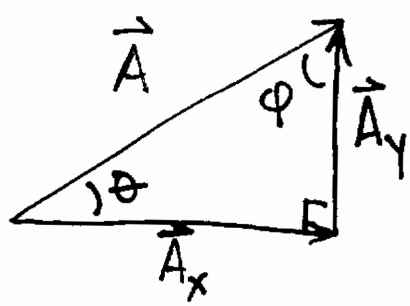
Symbolically, split vectors into components, and add:

$$\vec{A} + \vec{B} + \vec{C} = \vec{D}$$

$$\Rightarrow D_x = A_x + B_x + C_x$$

$$D_y = A_y + B_y + C_y$$

Where the x and y components are found using trigonometry, e.g.



$$A_x = A\cos\theta = A\sin\varphi$$

$$A_y = A\sin\theta = A\cos\varphi$$

and

$$A = \sqrt{A_x^2 + A_y^2}$$

Remember SOH CAH TOA

- SOH: Sine, Opposite, Hypotenuse
- CAH: Cosine, Adjacent, Hypotenuse
- TOA: Tangent, Opposite, Adjacent

Example type of Problem: Relative Velocities!

Variables for description of Motion

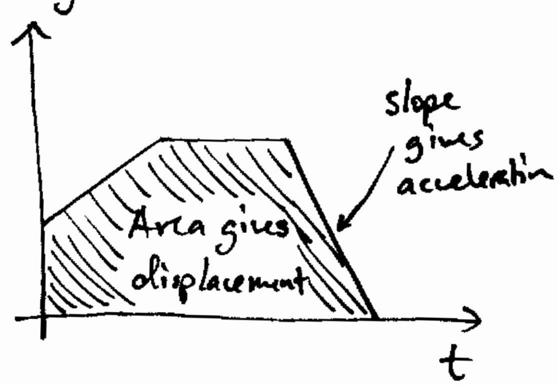
	Translational	Rotational
Displacement	Δx	$\Delta\theta$
velocity	$v = \frac{\Delta x}{\Delta t}$	$\omega = \frac{\Delta\theta}{\Delta t}$ (angular velocity)
acceleration	$a = \frac{\Delta v}{\Delta t}$	$\alpha = \frac{\Delta\omega}{\Delta t}$ (angular acceleration)

These variables are related to one another! By examining their plots versus time, you can extract the related variables:

$$\Delta x \xrightarrow{\text{Slope of } \Delta x \text{ vs } t} v \xrightarrow{\text{Slope of } v \text{ vs. } t} a$$

$$a \xrightarrow{\text{Area under } a \text{ vs. } t} v \xrightarrow{\text{Area under } v \text{ vs. } t} \Delta x$$

e.g.:



Because of these relationships, you can derive the Kinematic Equations:

Translational	Rotational
$\Delta x = v_0 t + \frac{1}{2}at^2$	$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$v_f = v_0 + at$	$\omega_f = \omega_0 + \alpha t$
$v_f^2 = v_0^2 + 2a\Delta x$	$\omega_f^2 = \omega_0^2 + 2\alpha\Delta\theta$

When working in 2D, each dimension is described by a set of kinematic equations and you work them independently. Example: Projectile motion

Forces and Newton's Laws

Newton's Laws:

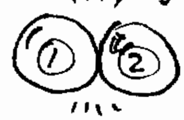
1. An object at rest will stay at rest, and an object in motion will remain in motion in a straight line unless acted upon by an external force.

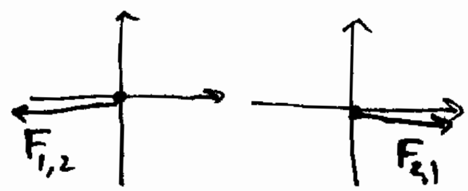
2. If an object experiences a net force, it will accelerate in the same direction as the force:

$$\vec{F}_{net} = m\vec{a}$$

3. For every action (force) there is an equal and opposite force

e.g. Colliding balls





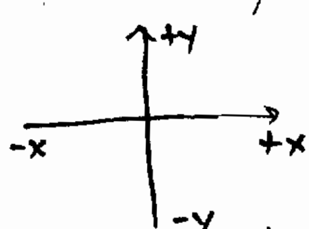
Force on ball 1 from ball 2

Force on ball 2 from ball 1

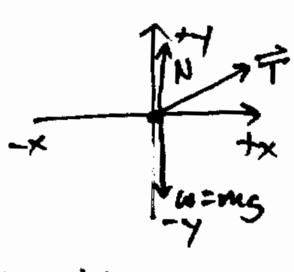
$$\vec{F}_{21} = -\vec{F}_{1,2}$$

The law we use the most is Newton's 2^{nd}: $\vec{F}_{net} = m\vec{a}$. To use it:

1. Draw a picture (free body diagram), labelling coordinate axes:



2. Identify and draw force vectors, e.g.



Example Problem:
Block on inclined plane.

3. Use your drawing to split vectors into components and write Newton's 2^{nd} law in each dimension

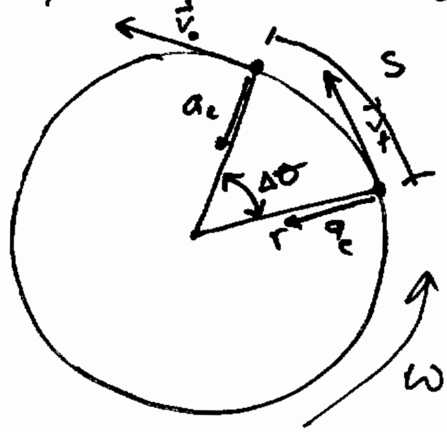
Rotation and Circular Motion

You should familiarize yourself with the angular variables:

$\Delta\theta = \frac{s}{r}$ (This is the definition of a radian!)

$\omega = \frac{\Delta\theta}{\Delta t}$

$\alpha = \frac{\Delta\omega}{\Delta t}$



Because these definitions are based on the definition of radians you MUST use radians for the rotational kinematic equation to work!

$$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega_f = \omega_0 + \alpha t$$
$$\omega_f^2 = \omega_0^2 + 2\alpha\Delta x$$

These are only true if your units are radians

The linear velocity ($\vec{v}$) of an object moving in a circle is everywhere tangent to its circular path, and is related to its angular velocity by

$$\vec{v} = r\omega$$

Similarly,

$$\Delta x = r\Delta\theta$$
$$a = r\alpha$$

Because the direction of its velocity vector is constantly changing, any object moving in a circle is constantly accelerated:

$$a_c = \frac{v^2}{r} = \omega^2 r$$ centripetal acceleration points radially inward

Example problems: A spinning disc, car driving in a circle

Friction

There are two kinds of friction:

static friction: friction between stationary objects

The maximum possible static frictional force is

$$f_{max} = \mu_s N$$

kinetic friction: friction between objects sliding relative to one another.

Has magnitude

$$f_k = \mu_k N$$

In general, static friction is stronger than kinetic friction; that is

$$\mu_s > \mu_k$$

It is harder to get something moving than to keep it moving!

Examples: Sliding block on inclined plane, car driving in a circle