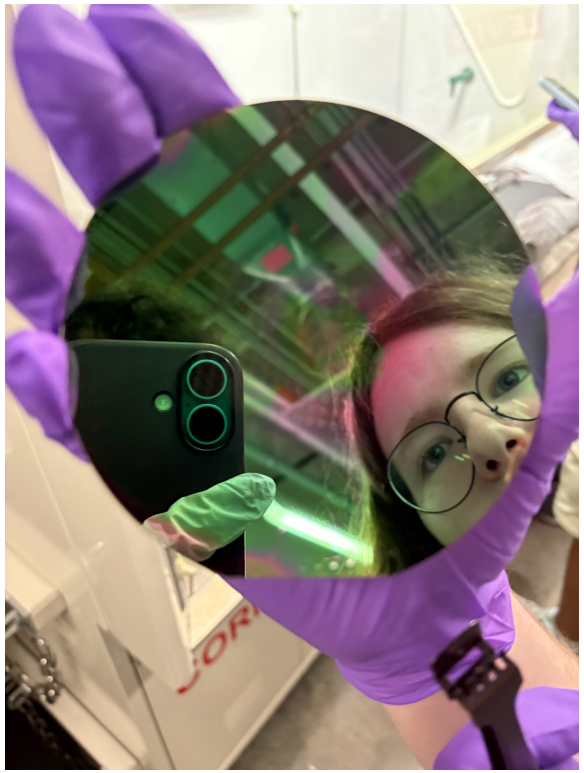
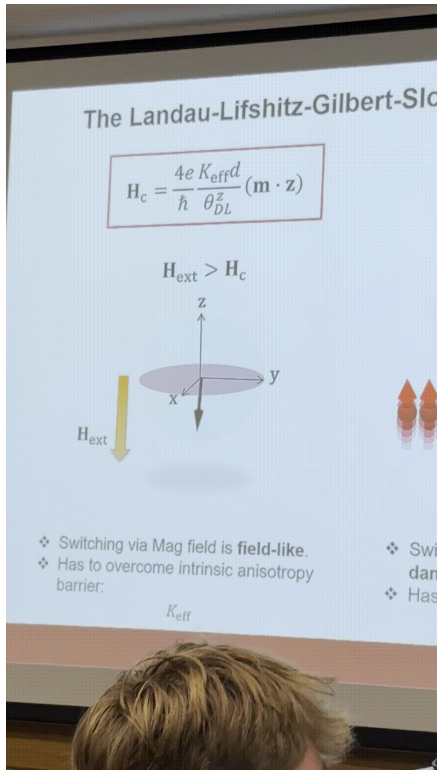
SHRIKAR DULAM

10.13.25

HOFSTADTER'S BUTTERFLY

QUANTUM + CHIPS !!!



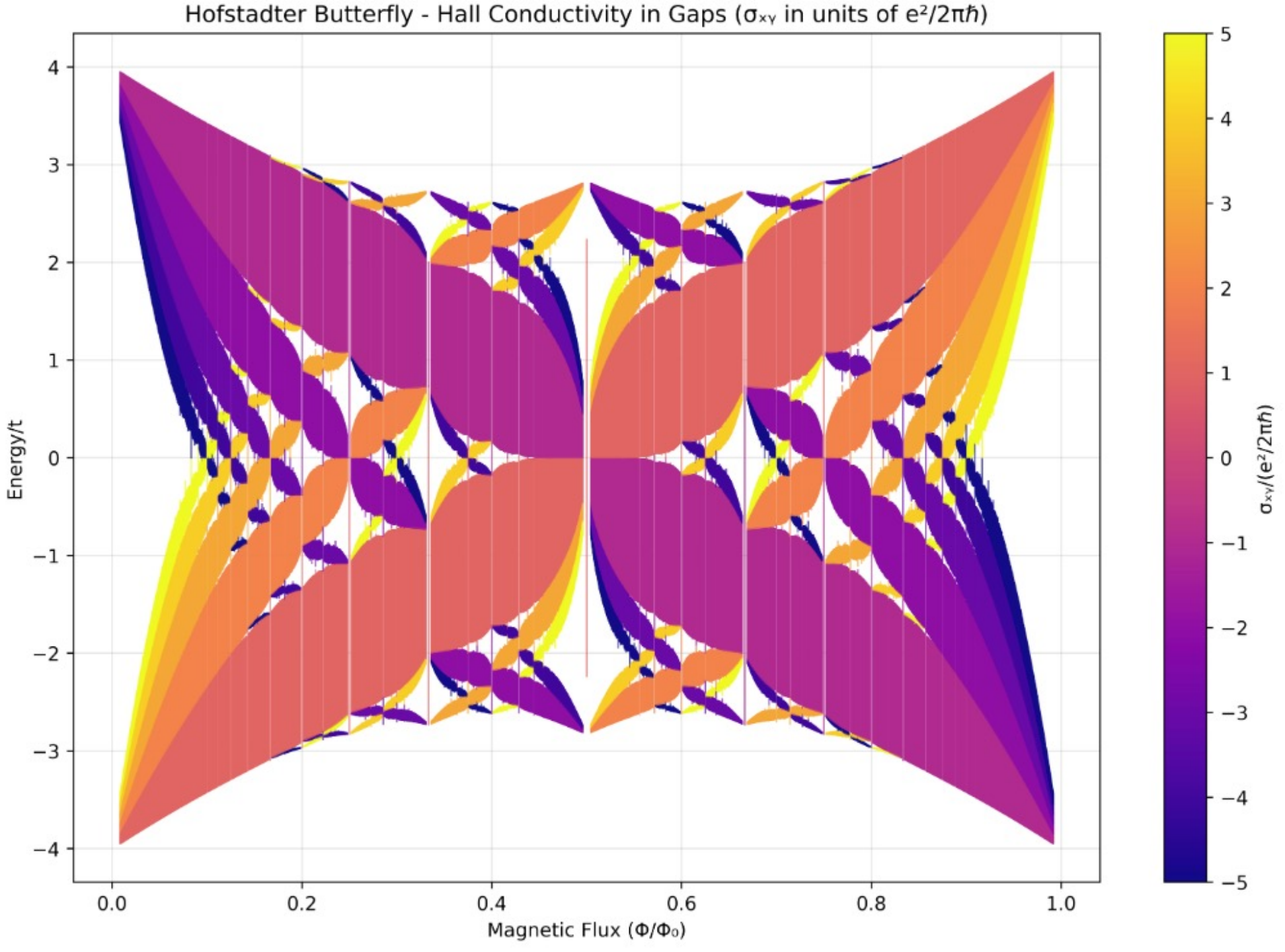


- Theory was developed by Rudolf Peierls and R.G. Harper in 1950s

- Discovered by Douglas Hofstadter in his 1976 PhD thesis

- Also called “Gplot” as a recursive structure and discussed in *Gödel, Escher, Bach*

- David J. Thouless and his team characterized the colorful phases in 1982

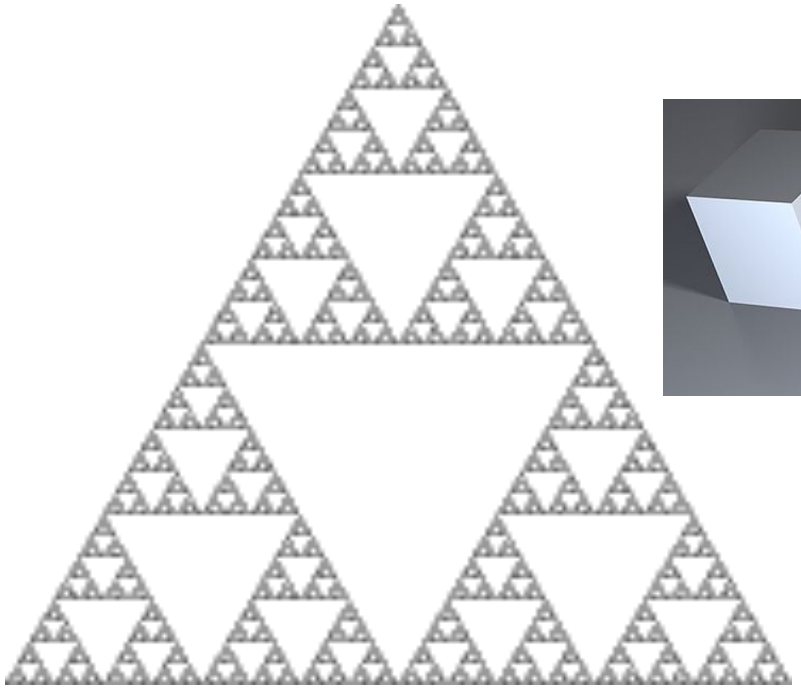


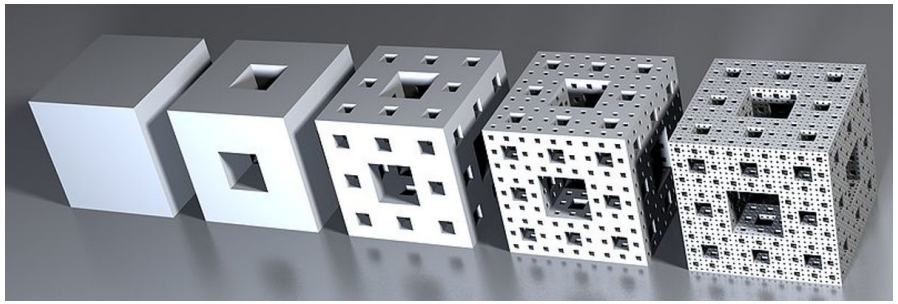
FRACTALS

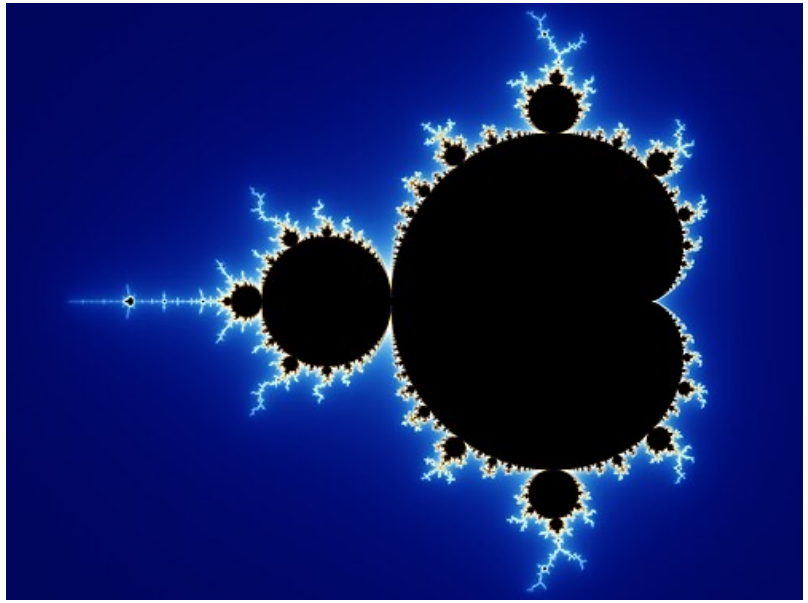
- A pattern that is self-similar at different scales

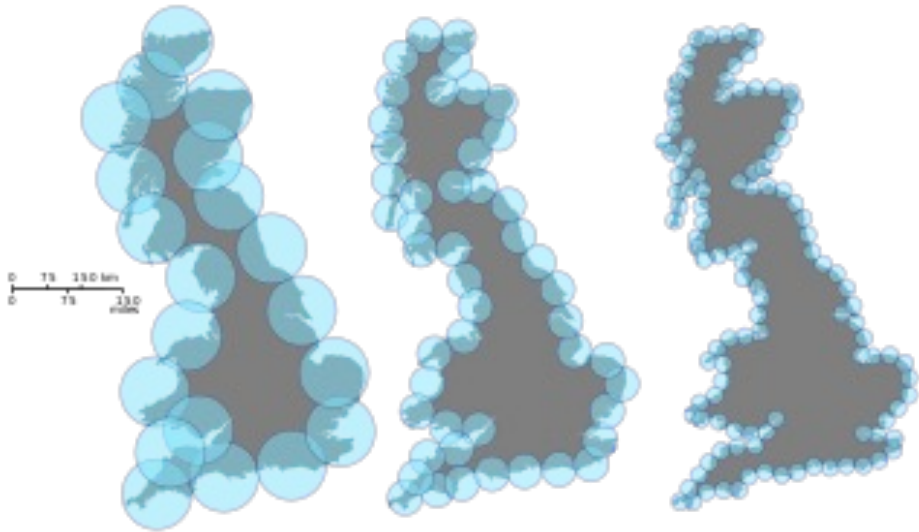
- Usually generated through iteration

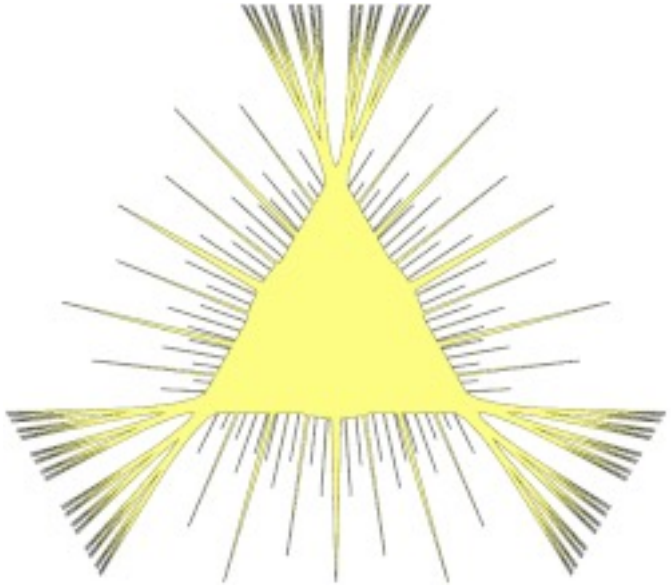
$$f(z) = z^2 + c$$











RATIONALITY

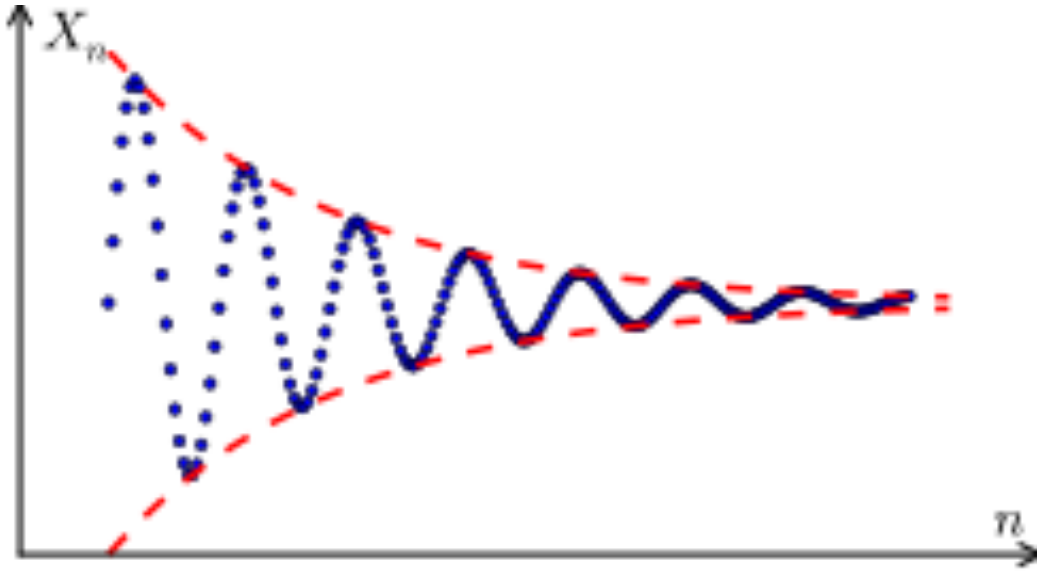
- If x = p/q for integers p,q, then x is **rational**

- To get the real numbers, we need to "complete" the number line with **irrationals**

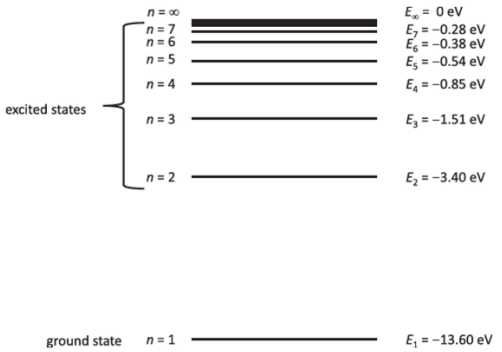
- Approximation reveals hidden structure

$$\log_2(3/2) = \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{3 + \cdots}}}}}$$

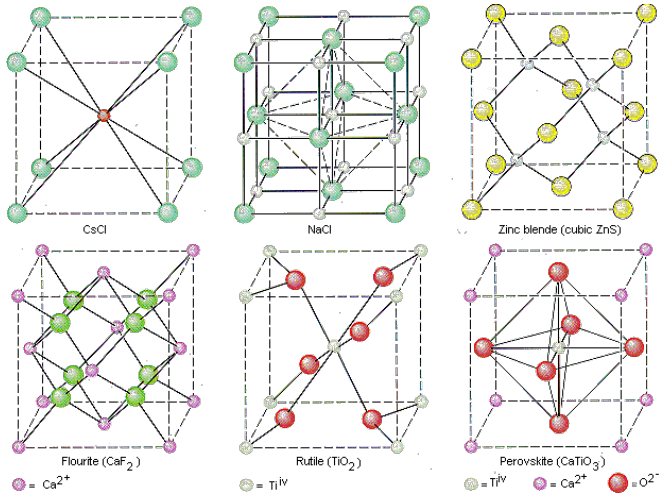
$$\frac{1}{2} < \frac{7}{12} < \frac{31}{53} < \ldots < \log_2(3/2) < \ldots < \frac{24}{41} < \frac{3}{5} < 1$$



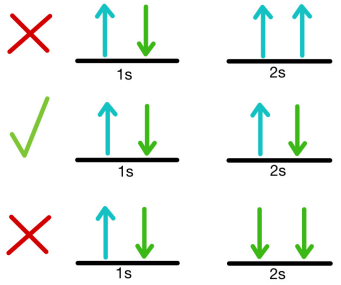
BAND STRUCTURES



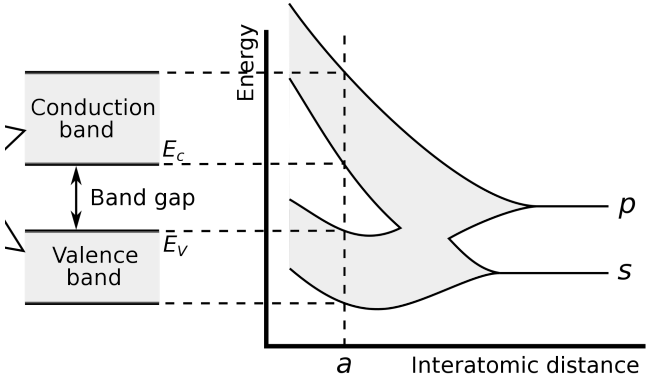
Electron energy states are quantized!



Scaling up quantum system into a solid crystalline lattice

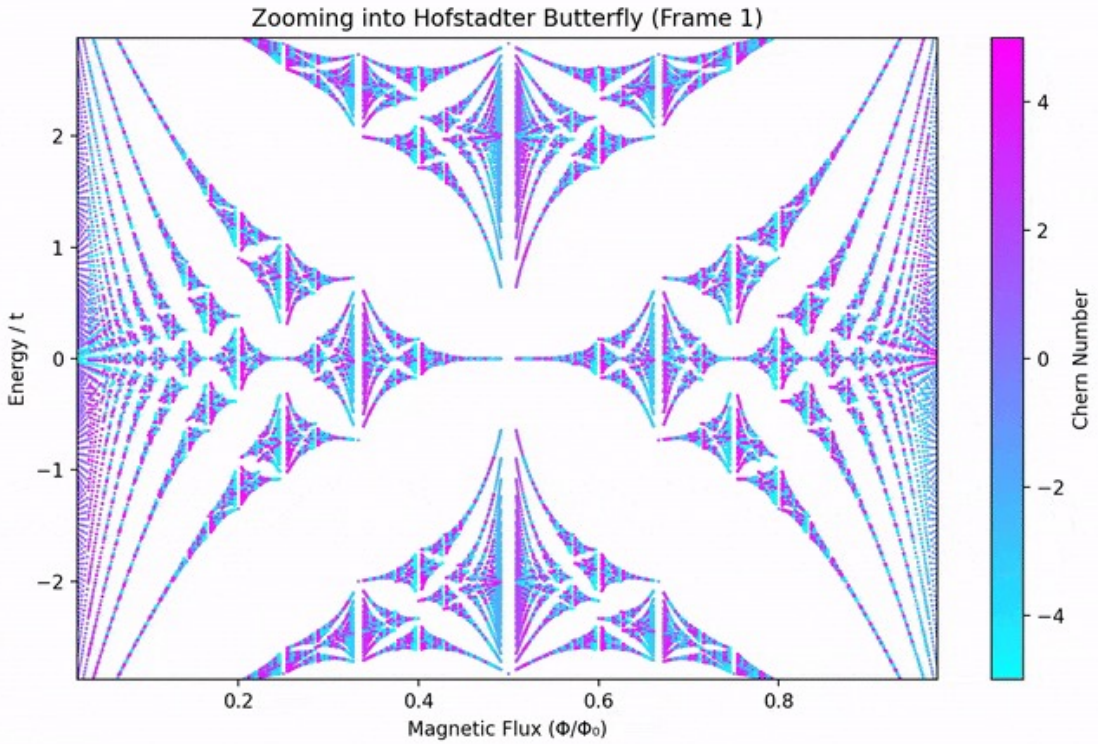


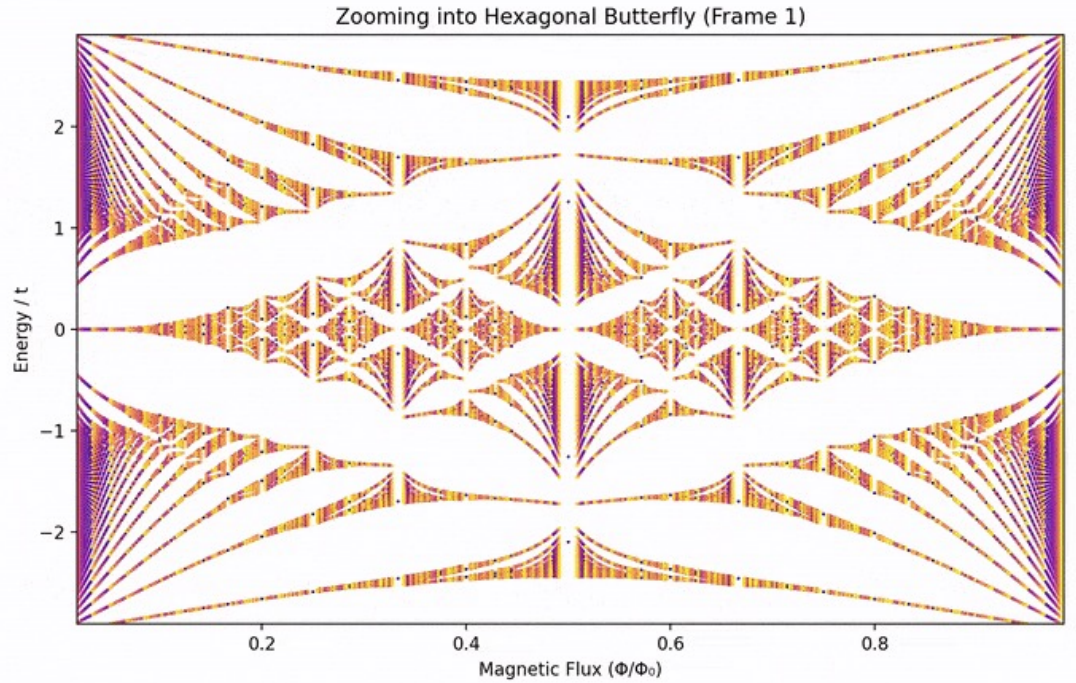
Pauli Exclusion prevents degeneracy



Get “continuous” bands of allowed energies

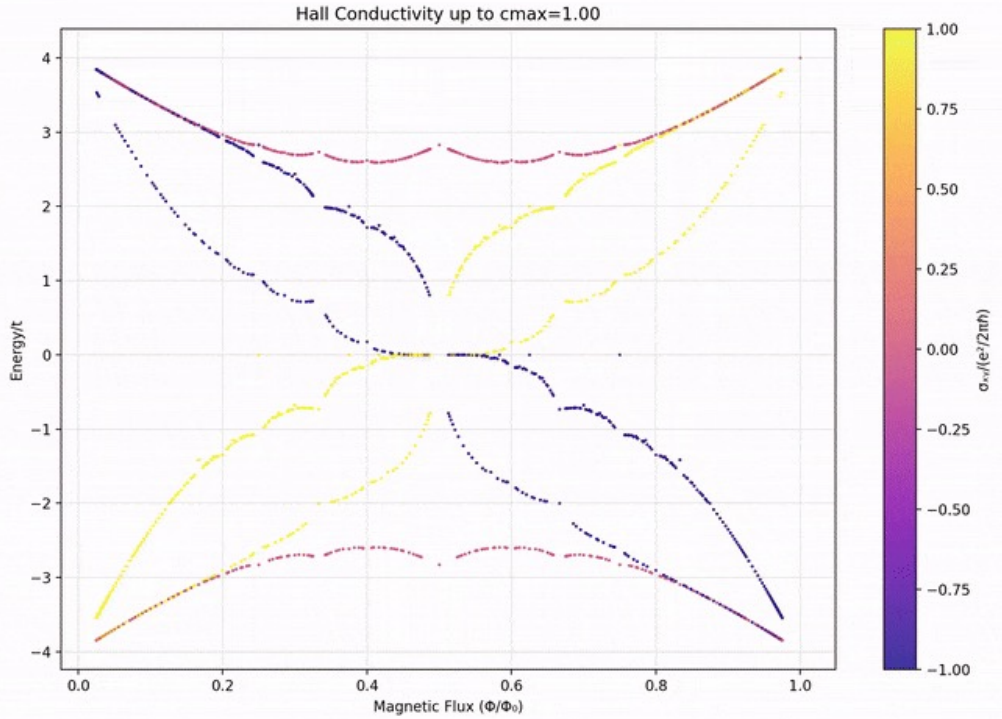
MIX IT ALL TOGETHER…





MORE TO UNCOVER!

Tons of topology introduced with magnetic field interaction (Quantum Hall Effect)



Butterfly can be observed in real settings like superconducting qubits and twisted bilayer graphene