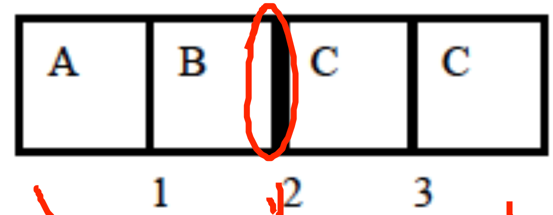
The next two questions pertain to the following situation

Consider a box having 4 bins, divided into two parts by a moveable partition. The left part has 2 distinguishable particles, A and B, and the right part has 2 indistinguishable particles, each called C. A microstate of the system is specified by identifying which particles are in which bins. Allow multiple occupancy in each side. One particular microstate is illustrated, and particles are free to move between bins on their respective sides of the partition.



1. With the partition as shown, what is the total entropy σ_T?

a. $\sigma_T = 1.52$
b. $\sigma_T = 1.94$
c. $\sigma_T = 2.48$

$\Omega_T = \Omega_1 * \Omega_2$

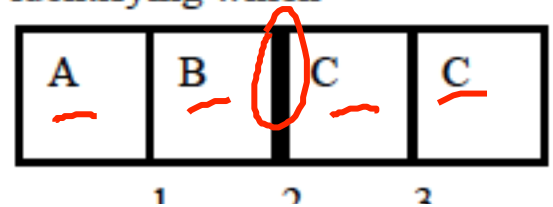
$\Omega_1 =$ [AB|], [A|B], [B|A], [|AB] $= 4$

$\Omega_2 =$ [cc|], [c|c], [|cc] $= 3$

$\Omega_T = 3 \cdot 4 = 12 \Rightarrow \sigma_T = \ln 12 = 2.48$

The next two questions pertain to the following situation

Consider a box having 4 bins, divided into two parts by a moveable partition. The left part has 2 distinguishable particles, A and B, and the right part has 2 indistinguishable particles, each called C. A microstate of the system is specified by identifying which particles are in which bins. Allow multiple occupancy in each side. One particular microstate is illustrated, and particles are free to move between bins on their respective sides of the partition.



2. If we now remove the partition and allow the particles to move freely in all four bins, what is the new total entropy? (Hint: The total number of microstates is the product of the microstates for the different type of particles, e.g., A, B, C.)

a. $\sigma_T = 2.97$
b. $\sigma_T = 3.56$
c. $\sigma_T = 4.16$
d. $\sigma_T = 5.08$
e. $\sigma_T = 5.54$

A,B 2 dist. in 4 multiple occ. bins

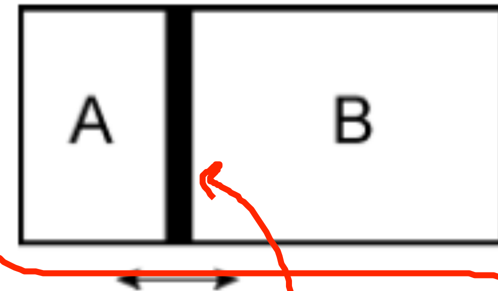
C,C 2 ind. in 4 " "

$$\Omega_{AB} \times \Omega_{CC} = \Omega_T = 16 \cdot 10 = 160 \Rightarrow \sigma_T = \ln 160 = 5.08$$

$$\Omega_{AB} = 4^2 = 16$$

$$\Omega_{CC} = \frac{(2+4-1)!}{2!\,3!} = \frac{5!}{3!\,2!} = \frac{5 \cdot 4}{2} = 10$$

3. Let two different (and not necessarily ideal) gases (A and B) at fixed equal temperature be confined to opposite sides of a fixed-volume cylinder with a sliding piston in between. In thermal equilibrium, which of these equations holds (with the usual definitions of the symbols):



a. ~~$\sigma_A=\sigma_B$~~

b. ~~$d\Omega_A/dV_A=d\Omega_B/dV_B$~~

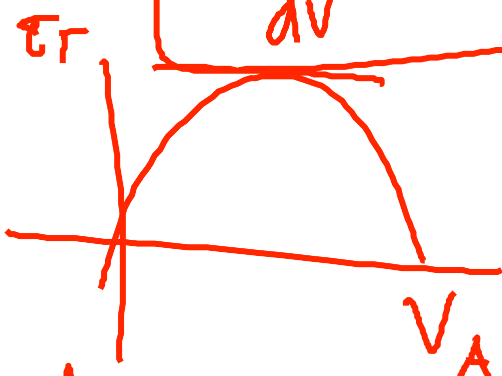
c. $d\sigma_A/dV_A=d\sigma_B/dV_B$

$\sigma_T = \sigma_A + \sigma_B$

$\sigma_A \equiv$ entropy of A $= \ln \Omega_A$

$\Omega_A \equiv$ # of microstates of A

$$\boxed{\frac{d(\sigma_T)}{dV} = 0}$$



$$\frac{d\sigma_T}{dV_A} = 0 = \frac{d\sigma_A}{dV_A} + \frac{d\sigma_B}{dV_A}$$

$$0 = \frac{d\sigma_A}{dV_A} - \frac{d\sigma_B}{dV_B}$$

$V = V_A + V_B$

$dV = 0 = dV_A + dV_B$

$dV_A = -dV_B$

The next two questions pertain to the following situation

An isolated system in internal thermal equilibrium can be in either macrostate A with entropy $\sigma_A = 27$ or macrostate B with entropy $\sigma_B = 20$. A particular microstate X is part of the A macrostate, and a particular microstate Y is part of the B macrostate.

4. What is the ratio of the probabilities, P_A/P_B, that the system will be found in the two macrostates?

microstates corresponding to a macrostate is just e^{σ}

a. $P_A/P_B = 0.2$
b. $P_A/P_B = 1.0$
c. $P_A/P_B = 1.3$
d. $P_A/P_B = 1097$
e. $P_A/P_B = 148$

$$P_A = \frac{\Omega_A}{\Omega_T} \qquad P_B = \frac{\Omega_B}{\Omega_T}$$

$$\frac{P_A}{P_B} = \frac{\Omega_A/\cancel{\Omega_T}}{\Omega_B/\cancel{\Omega_T}} = \frac{\Omega_A}{\Omega_B} = \frac{e^{\sigma_A}}{e^{\sigma_B}}$$

$$= e^{\sigma_A - \sigma_B} = e^{7} = 1097$$

The next two questions pertain to the following situation

An isolated system in internal thermal equilibrium can be in either macrostate A with entropy $\sigma_A = 27$ or macrostate B with entropy $\sigma_B = 20$. A particular microstate X is part of the A macrostate, and a particular microstate Y is part of the B macrostate.

5. What is the ratio of the probabilities, P_X/P_Y, that the system will be found in the two particular microstates?

a. $P_X/P_Y = 0.2$
b. $P_X/P_Y = 1.0$
c. $P_X/P_Y = 1.3$
d. $P_X/P_Y = 1097$
e. $P_X/P_Y = 148$

$x \quad \Omega_x = 1$

$y \quad \Omega_y = 1$

$$P_x = \frac{\Omega_x}{\Omega_{Tot}} = \frac{1}{\Omega_{Tot}} \qquad \frac{P_x}{P_y} = 1$$

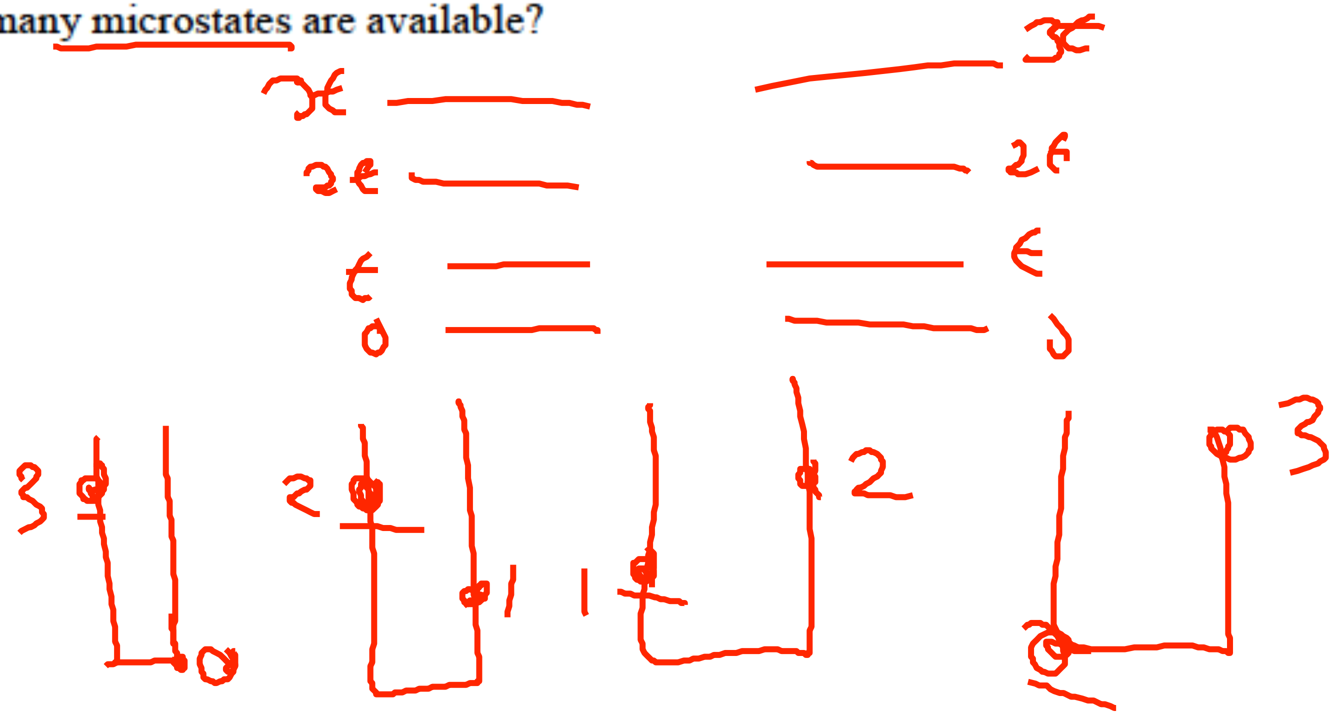
$$P_Y = \frac{\Omega_y}{\Omega_{Tot}} = \frac{1}{\Omega_{Tot}}$$

The next two questions pertain to the following situation

A total of 3 energy quanta are distributed in equilibrium within a collection of 2 oscillators, each with the same energy level spacing.

6. How many microstates are available?

a. Ω=3
b. Ω=4
c. Ω=6
d. Ω=8
e. Ω=9



The next two questions pertain to the following situation

A total of 3 energy quanta are distributed in equilibrium within a collection of 2 oscillators, each with the same energy level spacing.

7. What is the ratio of the average number of quanta to the most likely number of quanta in the first oscillator?

$\langle q \rangle$ = average number of quanta in the first oscillator.

q_{ml} = most likely number of quanta in the first oscillator.

a. $\langle q \rangle / q_{ml} = 0$

b. $\langle q \rangle / q_{ml} = 1$

c. $\langle q \rangle / q_{ml} = \infty$

$$\langle q \rangle = \sum_{q_1} q_1 P(q = q_1)$$

$$= 0P(0) + 1P(1) + 2P(2) + 3P(3)$$

$P(0) = \frac{1}{4}$

$P(1) = \frac{1}{4}$

$P(2) = \frac{1}{4}$

$P(3) \frac{1}{4}$

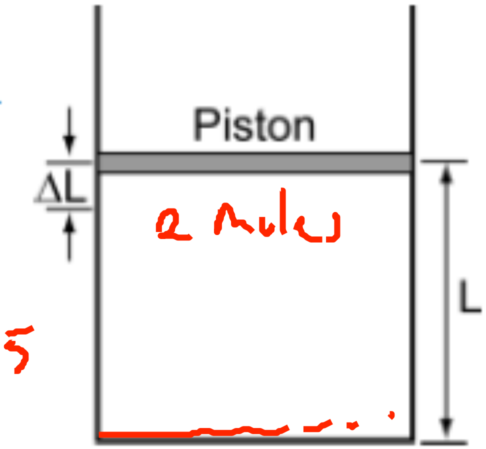
$$\langle q \rangle = \frac{1}{4} + 2/4 + \frac{3}{4} = \frac{3}{2} = 1.5$$

q_{ml}

$$\frac{1.5}{2} = \ldots$$

The next two questions pertain to the following situation

One end of a cylinder containing 2 moles of O_2 gas at 25 °C is sealed by a massless piston. The cylinder also contains a small piece of paper. You want to ignite the paper by quickly compressing the piston and adiabatically heating the O_2 gas. (Recall the fire starter demo from class.) Paper burns at 232 °C.



$\alpha = 5/2$

$T = 25 + 273{,}15$

$T_B = 232 + 273{,}15$

8. What is the smallest fractional change in the position of the piston to cause the paper to burn?

a. ΔL/L = 0.27
b. ΔL/L = 0.55
c. ΔL/L = 0.63
d. ΔL/L = 0.73
e. ΔL/L = 0.94

$Q = 0$

$\gamma = \frac{\alpha + 1}{\alpha}$

$$\boxed{V T^{\alpha} = \text{const}}$$

$pV^{\gamma} = \text{const}$

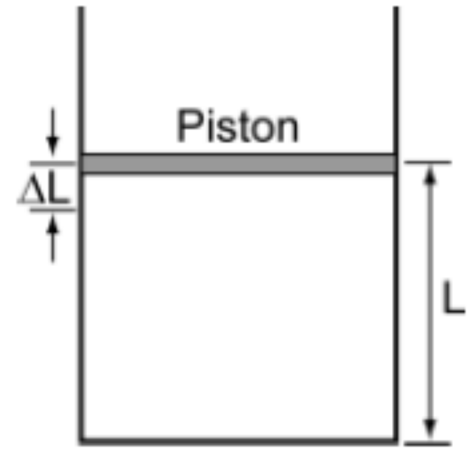
$V = AL$

$$V_i T_i^{\alpha} = V_f T_f^{\alpha}$$

$$\cancel{A} L (298)^{5/2} = \cancel{A}(L - \Delta L)(232 + 273{,}15)^{5/2}$$

$$\left(\frac{298}{232+273}\right)^{5/2} = \frac{L - \Delta L}{L} = 1 - \frac{\Delta L}{L} \Rightarrow \frac{\Delta L}{L} = 1 - \left(\frac{298}{232+273}\right)^{5/2}$$

One end of a cylinder containing 2 moles of O_2 gas at 25 °C is sealed by a massless piston. The cylinder also contains a small piece of paper. You want to ignite the paper by quickly compressing the piston and adiabatically heating the O_2 gas. (Recall the fire starter demo from class.) Paper burns at 232 °C.



9. How much work is done to compress the gas to 232 °C?

a. 5.2×10^3 J
b. 8.6×10^3 J
c. 1.2×10^4 J

$\Delta U = \cancel{Q}^{\,0} + W_{on}$

adiabatic ⇒ $Q = 0$

$W_{on} = \Delta U = C\Delta T$

$= \frac{5}{2} Nk\Delta T$

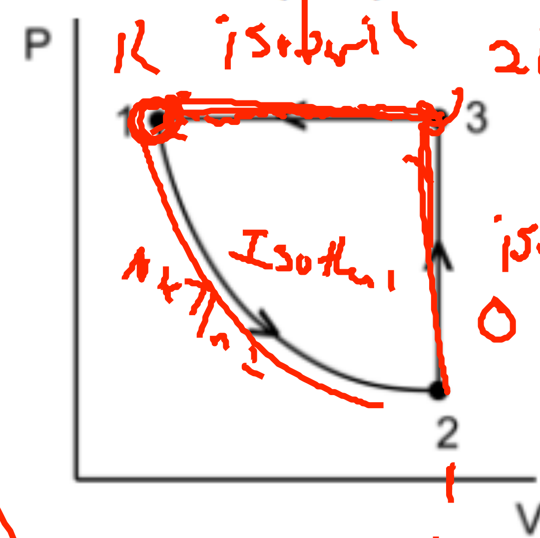
$= \frac{5}{2} Nk(232 - 25)$

$= \frac{5}{2} nR(207\,K) = 8600\,J$

$Q = \Delta U - W_{on}$

$\int p\,dV$

10. 1 mole of N_2 gas expands isothermally at 300 K from 1 liter to 2 liters (1→2). The gas is then compressed back to 1 liter along the path shown (2→3→1). Calculate the work done by the gas on the closed path (1→2→3→1).



a. 324 J
b. 0 J
c. -765.4 J
d. -1245 J
e. -1547 J

$\alpha = \frac{5}{2}$

$PV = NkT \qquad P = \frac{NkT}{V}$

$P_1 = \frac{Nk(300K)}{1L}$

$1 \to 2 \quad W_{by} > 0$

$2 \to 3 \quad \Delta V = 0 \quad W_{by} = 0$

$3 \to 1 \quad \Delta V < 0 \Rightarrow W_{by} < 0$

$\Delta U = 0 \qquad Q = W_{on} = -W_{by}$

$W_{on} = -\int_{V_i}^{V_f} p\,dV \Rightarrow W_{by} = \int_{V_i}^{V_f} p\,dV$

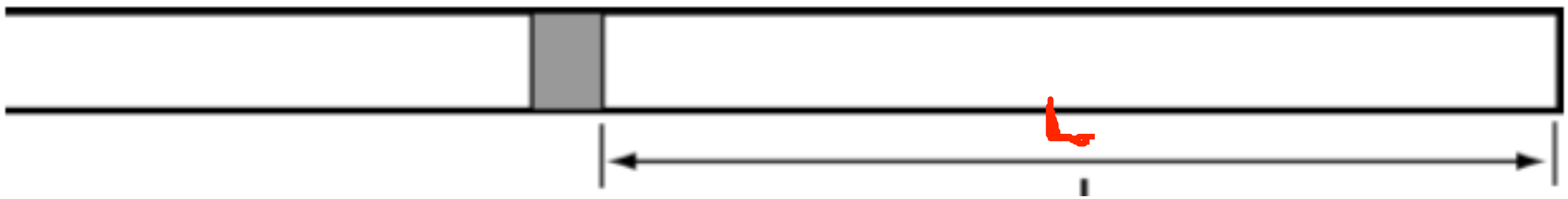
$W_{by} = W_{1\to2} + W_{2\to3} + W_{3\to1}$

$W_{by}^{1\to2} = \int_{V_i}^{V_f} \frac{NkT}{V} dV = NkT \ln\left(\frac{V_f}{V_i}\right) = NkT\ln 2$

$W_{by}^{3\to1} = P\int_{V_i}^{V_f} dV = P(V_f - V_i) = \frac{Nk(300K)}{1L}(-1L) = -Nk(300K)$

The next two questions pertain to the following situation

Suppose 1 mole of He is confined inside a long cylindrical vessel. The cross section of the vessel is so small that the He atoms can only move along the long axis of the cylinder. Thus, their motion is one dimensional. One end of the vessel is sealed with a movable piston.



11. If the He gas is at 300 K, estimate the average force acting on the piston when the piston is located a distance L = 3 m from the closed end of the cylinder.

a. <F> = 0.83 N
b. <F> = 8.3 N
c. <F> = 83 N
d. <F> = 830 N
e. <F> = 8300 N

Pressure on piston $= \dfrac{F}{A_{piston}}$

$$p = \frac{NkT}{V} = \frac{NkT}{L\,A_{piston}}$$

$$F = p\,A_{piston} = \frac{NkT}{L} = 831\ N$$

The next two questions pertain to the following situation

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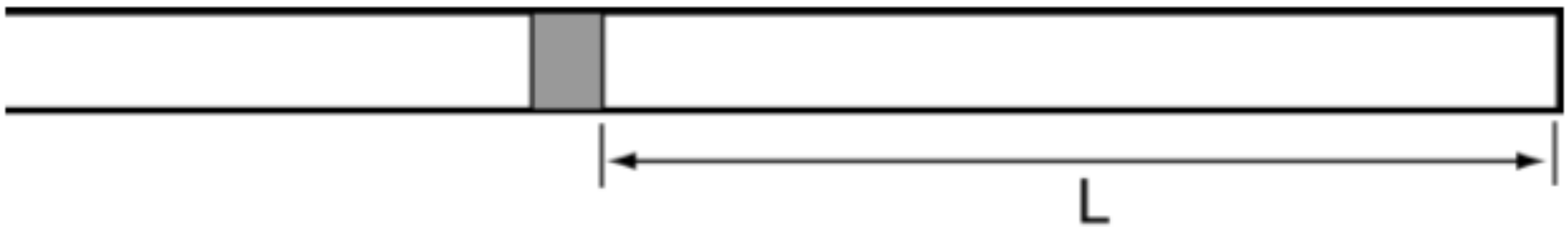


12. If we substitute the He gas in the cylinder with 1 mole of N_2, how does the average force change?

a. Increases, because N_2 is a diatomic molecule and therefore has a larger average thermal energy than He.
b. Stays the same, because N_2 and He have the same average translational kinetic energy.
c. Decreases, because N_2 has more degrees of freedom than He and therefore has less energy available for translational degrees of freedom.

$P = \frac{NkT}{V}$

13. The heat capacity of a particular solid has the following form $C(T) = \alpha T^3$, where $\alpha = 3\times10^{-4}$ J K^{-4}. Calculate the internal energy of the solid at T = 100 K.

a. 5.0×10^3 J
b. 7.5×10^3 J
c. 9.4×10^3 J
d. 1.3×10^4 J
e. 3.2×10^4 J

$$C = \left.\frac{\partial U}{\partial T}\right|_V$$

$$U = \int_{T=0}^{T=100\,K} C\,dT$$

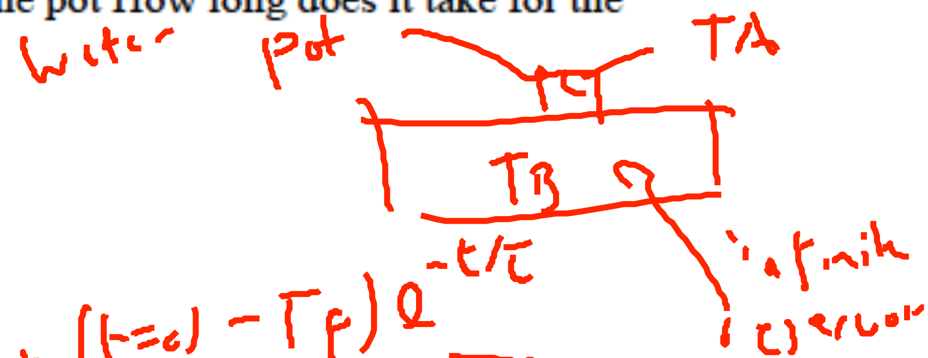
$$U = \int_{T=0}^{T=100\,K} \alpha T^3\,dT = \frac{\alpha}{4}\left[(100\,K)^4 - (0\,K)^4\right]$$

$$= \frac{10^8}{4}\,3\times10^{-4}$$

$$dU = C\,dT$$

14. A pot containing 3 kg of water is heated to 350 K and placed on a countertop at 300 K. Here, we treat the countertop as an infinite heat reservoir. The specific heat of water is 4184 J kg^{-1}K^{-1}. The pot is made from copper with κ = 400 W m^{-1} K^{-1}. The contact area between the pot and the countertop is 0.09 m^2, and the copper on the bottom portion of the pot is 2-mm thick. Neglect the heat capacity of the pot How long does it take for the temperature of the water to cool by 5 K?

a. 0.074 s
b. 0.700 s
c. 2.450 s
d. 4.720 s
e. 7.000 s



$$T_{water}(t) = T_f + (T_{water}(t=0) - T_f)\, e^{-t/\tau}$$

$T_f = 300\,K$

$T_{water}(t=0) = 350\,K$

Find time t when $T_{water}(t) = 345\,K$

$\tau = R_{TH} C$, $C = cm = 12{,}6\ldots \, J/K$

$$R_{TH} = \frac{d}{A\kappa} = \frac{2\times10^{-3}\,m}{(9\times10^{-2}\,m^2)(4\times10^{2}\,W\,m^{-1}K^{-1})} = \frac{2\times10^{-3}}{36}\,\frac{K}{W\cdot m}$$

$345\,K = 300\,K + (350\,K - 300\,K)\, e^{-t/\tau}$

$\frac{45}{50} = e^{-t/\tau} \Rightarrow \ln(9/10) = -t/\tau$

$\Rightarrow t = \boxed{\tau \ln(10/9)} = 73\times10^{-3}\,s$

The next three problems are related

15. Cells (e.g., animal cells) are complex; survival requires transport of various components throughout the cell. For small cells (e.g., bacteria), this occurs via simple diffusion. If the diffusion constant for a protein in the cell is 10 μm^2/s, approximately how long will it take a protein to diffuse from the center of the cell to any point on the cell wall (assume a spherical cell), assuming the cell radius is 1 μm?

a. ~~0.5 milliseconds~~
b. 20 milliseconds
c. 50 milliseconds
d. 200 milliseconds
e. 500 milliseconds

$$\langle r^2 \rangle = 6Dt$$

$$r_{rms} = \sqrt{\langle r^2 \rangle} = \sqrt{6Dt} = 1 \times 10^{-6}\ m$$

$$D = 10\ \frac{\mu m^2}{s} = 10 \times 10^{-12}\ \frac{m^2}{s}$$

$$t = \frac{r_{rms}^2}{6D} = \frac{10^{-12}\ m^2}{6(10)\times 10^{-12}\ \frac{m^2}{s}}$$

$$= \frac{1}{60}\ s$$

16. How much longer will this take in a typical mammalian cell (radius ~10 μm)?

a. 3.16 times longer
b. 10 times longer
c. 100 times longer

$t \propto r_{rms}^2$

$t \rightarrow$ 100 times longer

17. Assuming the diffusion is driven simply by thermal random motion, by what factor would the diffusion time across the bacteria change if we were to raise the temperature from 23°C to 30°C (and assuming the mean free path of the protein is the same)?

a. ~~The time becomes longer by a factor of 1.017.~~
b. ~~The time becomes longer by a factor of 1.034.~~
c. The time becomes shorter by a factor of 0.967.
d. The time becomes shorter by a factor of 0.988.
e. The time becomes shorter by a factor of 0.876.

$$D = \frac{v\ell}{3}$$

$$\frac{1}{2} m |\vec{v}|^2 = \frac{3}{2} kT$$

$$v = \sqrt{\frac{3kT}{m}}$$

$$t = \frac{r_{rms}^2}{6D}$$

$$t_{23} = \frac{r_{rms}^2}{6\left(\frac{v_{23}\ell}{3}\right)} = \frac{r_{rms}^2}{2\ell v_{23}}$$

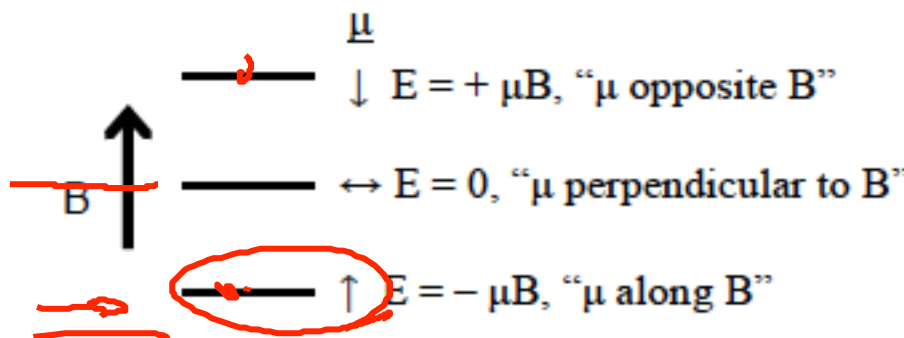
$$t_{30} = \frac{r_{rms}^2}{2\ell v_{30}}$$

$$\frac{t_{30}}{t_{23}} = \frac{v_{23}}{v_{30}} = \sqrt{\frac{23+273}{30+273}}$$

$$= 0.988$$

The next four questions are related

18. Consider a collection of atoms, each of which, according to quantum mechanics, can have precisely three values of the z-component of angular momentum ($-h/2\pi$, 0, and $+h/2\pi$), and thus can have a magnetic moment with the three possible values, $m\mu_z$, where $m = +1, 0,$ and -1. In a magnetic field (along the z-direction), these have an energy given by $-m\mu_z B$, as shown in the diagram.

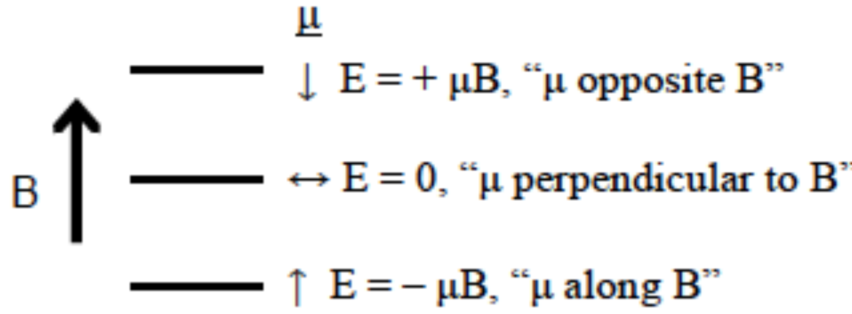


Assuming $\mu_z = 9.27 \times 10^{-24}$ J/T, and B = 1 Tesla, at T = 2 K what is the likelihood P(0) to find an atom in the middle state (i.e., with m = 0)?

a. 0.003
b. 0.03
c. 0.32
d. 0.5
e. 0.67

$$Z = \sum_i e^{-E_i/kT} = e^{-(-\mu B)/kT} + e^{0/kT} + e^{-\mu B/kT} = e^{\mu B/kT} + 1 + e^{-\mu B/kT}$$

$$P(E=0) = \frac{e^{-0/kT}}{Z} = \frac{1}{1 + e^{\mu B/kT} + e^{-\mu B/kT}} = 0.32$$



$e^{ix} + e^{-ix} = 2\cos x$

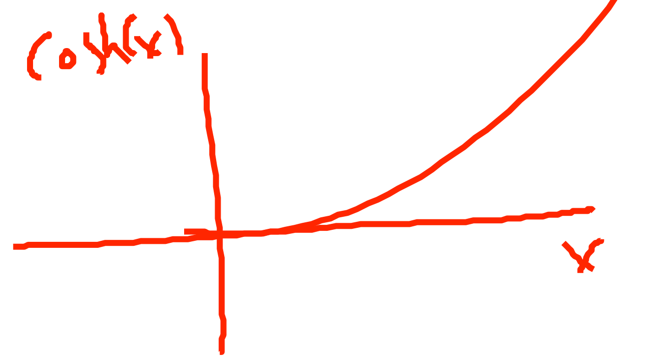
$e^{x} + e^{-x} = 2\cosh$

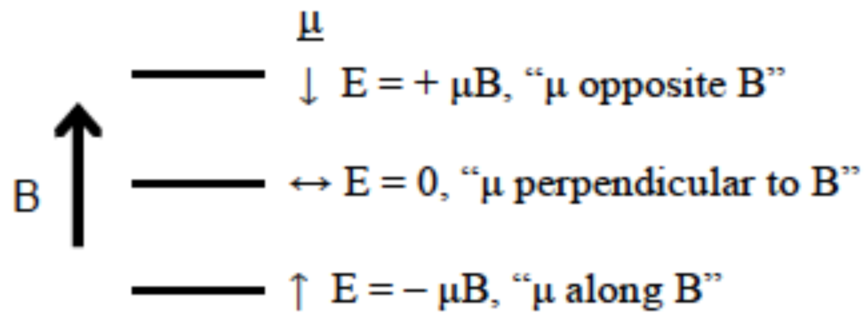
19. Now we increase the magnetic field to 10 T. What happens to P(0)?

a. P(0) decreases
b. P(0) increases
c. P(0) stays the same

$$\frac{1}{1 + \underbrace{e^{-\mu B/kT} + e^{\mu B/kT}}}$$

$$\rightarrow \frac{1}{1 + 2\cosh\left(\frac{\mu B}{kT}\right)}$$





20. Now (with the field at 10 T) we increase the temperature T (a lot). In the limit as the temperature goes to infinity, what happens to P(0)?

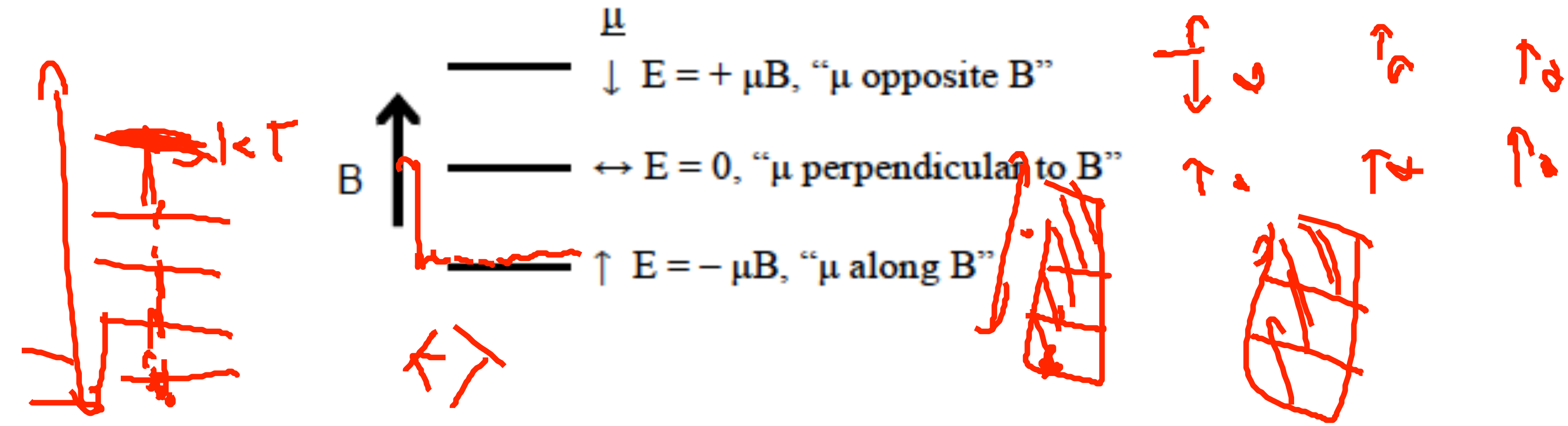
a. $P(0) \to 1/3$
b. $P(0) \to 1/2$
c. $P(0) \to 1$
d. $P(0) \to 0$
e. Cannot tell from the information given

$$\frac{1}{1+e^{\mu B/kT}+e^{-\mu B/kT}}$$

$$\frac{\mu B}{kT} \to 0$$

$$e^{\pm \mu B/kT} \to 1$$

$$P(0) = \frac{1}{1+1+1} = \frac{1}{3}$$



21. Consider N such atoms (located on a lattice). What is the ratio of the entropy of these atoms at high temperature, to the entropy at low temperature?

a. S(T→∞)/S(T→0) = ∞
b. S(T→∞)/S(T→0) = 0
c. S(T→∞)/S(T→0) = Nkln3

$3 \cdot 3 \cdot 3 \cdots$

as $T \to \infty$ $\quad 3^N$

$\sigma = \ln 3^N = N \ln 3$

$S = k\sigma = Nk \ln 3$

$\sigma = \ln 1 = 0 \Rightarrow S = k\sigma = 0$

$$\frac{S(T\to\infty)}{S(T\to 0)} = \frac{Nk\ln 3}{0}$$

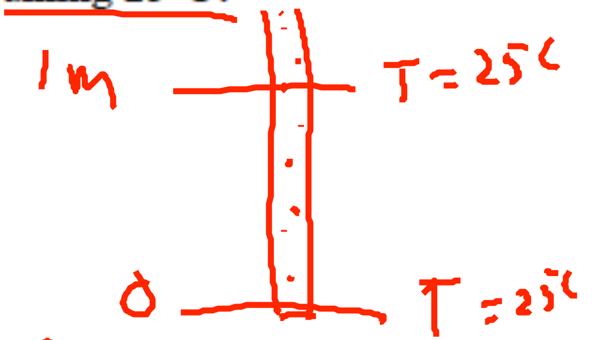
The next three questions are related

22. We want to design an experiment to demonstrate the law of atmospheres. We will do this by creating tiny water droplets of a fixed size, letting them fly around in a tall tube, and observing the decrease in their density (how many droplets per m^3) as a function of height in the tube.

What should be the approximate mass of each water droplet, such that the density of the water droplets at 1-meter height is half of what it is at 0 height, assuming 25°C?

a. 1×10^{-25} kg
b. 3×10^{-24} kg
c. 2×10^{-23} kg
d. 3×10^{-22} kg
e. There is no choice of mass for which this is possible.

$$\frac{P(z)/kT}{P(0)/kT} = \frac{n_p(z)}{n_p(0)}$$



$$P(z) = P_0 e^{-mgz/kT}$$

$$\frac{P}{kT} = \left(\frac{N}{V}\right) = n_p$$

$$\frac{P(z)}{P(z=0m)} = \frac{n_p(z)}{n_p(z=0m)}$$

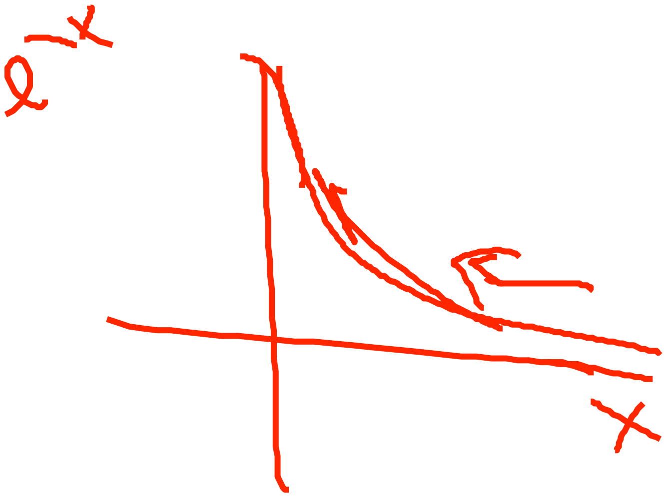
$$\frac{P_0 e^{-mg1m/kT}}{P_0 e^{-0/kT}} = \frac{n_p(1m)}{n_p(0)} = e^{-\frac{mg}{kT}}$$

$$\frac{n_p(1m)}{n_p(0)} = \frac{1}{2} = e^{-mg/kT}$$

$$m = \frac{kT\ln 2}{g} = 2.9\times10^{-22}\,kg$$

23. Now we add some heat into the container, and allow it to re-equilibrate (at a higher average temperature). What will happen to the density of the water droplets near the top of the tube?

a. the density will decrease
b. the density will stay the same
c. the density will increase



$$\frac{n_p(z)}{n_p(0)} = e^{-mgz/kT}$$

$T \uparrow$

$\frac{mgz}{kT} \downarrow$

$n_p(z) \uparrow$

24. Compare the temperature near the top of the tube to that near the bottom.

a. hotter at the top
b. hotter at the bottom
c. same temperature