$$|\vec{v}| \equiv \sqrt{\vec{v} \cdot \vec{v}} \quad \vec{v} = \sum_{i=1}^{3} (\vec{v} \cdot \hat{r}_i)\, \hat{r}_i \quad d\vec{l}_{path} = \frac{d\vec{l}}{du}\, du \quad d\vec{A} = \left(\frac{\partial \vec{l}}{\partial u} \times \frac{\partial \vec{l}}{\partial v}\right) du\, dv$$

Conceptual version:

$$d\vec{l}_{path} = d\vec{l}_u$$
$$d\vec{l}_u \equiv \frac{\partial \vec{l}}{\partial u}\, du$$
$$d\vec{A} = d\vec{l}_u \times d\vec{l}_v$$

$$df(x_1, ..., x_n) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}\, dx_i \qquad dV = \left(\frac{\partial \vec{l}}{\partial u} \times \frac{\partial \vec{l}}{\partial v}\right) \cdot \frac{\partial \vec{l}}{\partial w}\, du\, dv\, dw$$
$$dV = (d\vec{l}_u \times d\vec{l}_v) \cdot d\vec{l}_w$$

Taylor

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

1st order approx for $x \ll 1$:

- $(1+x)^n \approx 1 + nx$
- $\sin x \approx x$
- $\cos x \approx 1 - \dfrac{x^2}{2}$
- $\tan x \approx x$
- $e^x \approx 1 + x$

- $\sin^{-1} x \approx x$
- $\cos^{-1} x \approx \dfrac{\pi}{2} - x$
- $\tan^{-1} x \approx x$
- $\ln(1+x) \approx x$

	0°	30°	45°	60°	90°
sin	0	$\dfrac{1}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{3}}{2}$	1
cos	1	$\dfrac{\sqrt{3}}{2}$	$\dfrac{1}{\sqrt{2}}$	$\dfrac{1}{2}$	0
tan	0	$\dfrac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$$\sin a \sin b = \tfrac{1}{2}[\cos(a-b) - \cos(a+b)]$$
$$\cos a \cos b = \tfrac{1}{2}[\cos(a+b) + \cos(a-b)]$$
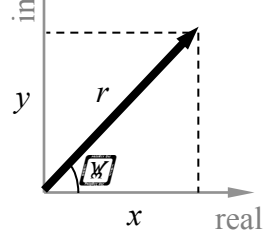$$\sin a \cos b = \tfrac{1}{2}[\sin(a+b) + \sin(a-b)]$$

Complex Numbers

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$\tilde{z} = x + iy = re^{i\theta}$$

$$\tilde{z}^* \equiv x - iy = re^{-i\theta}$$

$$|\tilde{z}| \equiv \sqrt{\tilde{z}^* \tilde{z}} = r$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos\theta}{2}} \qquad \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 - \cos\theta}{2}}$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$
$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

Integral Table

$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a}\cos^{-1}\left(\frac{a}{x}\right) \qquad \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right) \qquad \int \frac{x\, dx}{\sqrt{a^2 \pm x^2}} = \pm\sqrt{a^2 \pm x^2}$$

$$\int_0^{2\pi} \sin^2\phi\, d\phi = \int_0^{2\pi} \cos^2\phi\, d\phi = \pi \qquad \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \qquad \int \frac{x\, dx}{(a \pm x)^2} = \frac{a}{a \pm x} + \ln(a \pm x)$$

$$\int \sin^2\phi\, d\phi = \frac{\phi}{2} - \frac{\sin(2\phi)}{4} \qquad \int \frac{dx}{(a \pm x)^2} = \mp\frac{1}{a \pm x} \qquad \int \frac{x\, dx}{a^2 \pm x^2} = \pm\frac{1}{2}\ln\left(a^2 \pm x^2\right)$$

$$\int \cos^2\phi\, d\phi = \frac{\phi}{2} + \frac{\sin(2\phi)}{4} \qquad \int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \qquad \int \frac{x\, dx}{\left(a^2 \pm x^2\right)^{3/2}} = \mp\frac{1}{\sqrt{a^2 \pm x^2}}$$

$$\int \sin^3\theta\, d\theta = \frac{\cos^3\theta}{3} - \cos\theta \qquad \int \frac{dx}{a^2 - x^2} = \frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right)$$

$$\int \cos^3\theta\, d\theta = \sin\theta - \frac{\sin^3\theta}{3} \qquad \qquad \qquad \int \ln(ax)\, dx = x\ln(ax) - x$$

$$\int \cos^n\theta \sin\theta\, d\theta = -\frac{\cos^{n+1}\theta}{n+1} \qquad \int \frac{dx}{(a^2 \pm x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2 \pm x^2}} \qquad \int \frac{\ln(ax)}{x}\, dx = \frac{1}{2}[\ln(ax)]^2$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) \qquad \int \frac{x^2}{\sqrt{a^2 - x^2}}\, dx = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right)$$

$$\int \sqrt{x^2 \pm a^2}\, dx = \frac{x}{2}\sqrt{x^2 \pm a^2} \pm \frac{a^2}{2}\ln\left|x + \sqrt{x^2 \pm a^2}\right| \qquad \int \frac{(x - a\cos\theta)\sin\theta\, d\theta}{(x^2 + a^2 - 2ax\cos\theta)^{3/2}} = \frac{1}{x^2}\frac{a - x\cos\theta}{\sqrt{x^2 + a^2 - 2ax\cos\theta}}$$