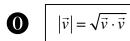
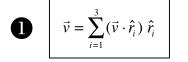
6 Basic Pieces : math that tends to get lost somewhere

Piece #0: Vector Magnitudes & Dot Products



Tools for Dot Products, \bullet distribute the dot \bullet geometry $\rightarrow A B \cos \theta$ esp. of <u>vector sums</u>: • orthonormal components $\rightarrow \sum_{i} A_{i}B_{i}$ & Pythagoras

Piece #1: Expressing Vectors in Component Form



for any complete, orthonormal set of unit vectors $\hat{r}_1, \hat{r}_2, \hat{r}_3$

Piece #2: Playing with Differentials

 $\frac{df}{dx} = g$ is equivalent to df = g dx

i.e.: You can algebraically manipulate differentials df like finite numbers Δf (rearrange, regroup, etc), then mentally return them to d's & derivatives at the end.

Tactic for changing variables in differential equations: multiply by d<blah> / d<blah>, then regroup differentials into new derivatives.

Piece #3: The 3D Chain Rule



 $df(x_1,...,x_n) = \sum_{i=1}^n \frac{\partial f}{\partial x_i} dx_i$ where the variables $x_1,...,x_n$ are all *independent*

In words: The change df that occurs in a function f when the independent coordinates x_i on which it depends are changed by amounts dx_i is just sum of each change dx_i times the relevant slope $\partial f / \partial x_i$.

Piece #4: Functions Do Not Commute With Operators

$$e.g. \quad \frac{d}{dx} f \neq f \frac{d}{dx} \quad \& \quad \vec{\nabla} f \neq f \vec{\nabla}$$

In both examples, the object on the left is just a function: a derivative of f. The object on the right is an operator: it will act on whatever comes next.

Piece #5: 1D Derivatives "Push Through" Dot- and Cross-products

e.g.

$$\frac{\partial}{\partial q} \left(\vec{A} \cdot \vec{B} \right) = \frac{\partial \vec{A}}{\partial q} \cdot \vec{B} + \vec{A} \cdot \frac{\partial \vec{B}}{\partial q} \qquad \& \qquad \left(\vec{A} \frac{\partial}{\partial q} \right) \times \vec{B} = \vec{A} \times \left(\frac{\partial \vec{B}}{\partial q} \right)$$

for any vector fields \vec{A} and \vec{B} that depend on any variable *a*.

Not obvious? Try it in Cartesian coordinates ... you'll see. ⁽²⁾ Since the final result makes no reference to any coordinate system, your Cartesian proof will be perfectly general.