When to Use Energy

Proof of (T+U) conservation *used* $\vec{F} = m\vec{a}$ … so

$\dot{T} + \dot{U} = 0$ always **gives you back a force-based EOM**.

It's often a more efficient way to obtain the EOM than using forces or torques, but more strategy tips are needed:

When do we NEED to use d(T+U) = 0 ?

When force details are unknown.

- **Elastic Collisions**
  All we know about the forces is that "elastic" ≡ **T is conserved** in the collision.
- **Known ΔE added / subtracted from system**
  e.g. particle explodes, releasing known ΔE
  → must use energy analysis to include this info.

When can we NOT use d(T+U) = 0 ?

When it's not true OR can't be calculated.

- $U(\vec{r},t)$ with **explicit *t*-dependence**
- Forces that **can't be described** by $U(\vec{r})$ but **do work**
  e.g. kinetic friction (depends on normal force),
  drag force (depends on velocity),
  force fields with $\vec{\nabla} \times \vec{F} \neq 0$
- **Energy leaves / enters system** in **non-calculable way**
  e.g. inelastic collisions where $\Delta U^{INT} \neq 0$
- System is **losing mass**, e.g. rocket motion

Collisions

When details of interaction force not known, some other info must be provided.

Elastic Collision : total T is conserved

- Collision takes place in infinitesimal time interval $\Delta t \approx 0$
  ∴ **impulse** collision force : $F_{\text{impulse}} \cdot \Delta t = \Delta P$
  $\infty \cdot 0 = \text{finite}$
- $\boldsymbol{\Delta U^{EXT}(\vec{r}) = 0}$ ∵ $\Delta\vec{r} = 0$ (particles have no time to move!)
- **Particle structure unchanged** by collision ∴ $\boldsymbol{\Delta U^{INT} = 0}$
  i.e. no energy can escape into deforming / heating / fragmenting the particles
- "Elastic" also implies that energy **cannot escape** to **surrounding medium**, e.g. via sound waves

Inelastic Collision : total T+U^{EXT} *not* conserved

- **Total Mass is conserved** (non-relativistic systems)
- Classic example : **Fusion / Fission** of N ↔ 1 particles
  When particles fuse, KE of incoming particles converted into heat of fused particle ∵ $\boldsymbol{\Delta U^{INT} \neq 0}$
  ☞ Often useful : **go to CM Frame**
- **Abrupt ΔT_{total}** , i.e. that occurs over $\Delta t \approx 0$, is a signature of an inelastic process
  $U^{EXT}(\vec{r}) = 0$ can't change over $\Delta t \approx 0$ ∵ $\Delta\vec{r} \approx 0$
  ∴ $\Delta(T+U^{EXT}) = \Delta T \neq 0$ → inelastic