Phys 325 Discussion 1 – Some Math Review

a.k.a. "Post-vacation Reboot"

Q1. Accelerating Bee *Checkpoints* [1]

A bee moves along a helical path given by the equation $\vec{r}(t) = \hat{x}\ b\sin\omega t + \hat{y}\ b\cos\omega t + \hat{z}\ ct^2$.
Calculate the magnitude the bee's acceleration in terms of the constants b, ω, and c.

Q2. Taylor Series *Checkpoints* [1]

Consider the definite integrals $I_1(a) = \int_0^a \frac{1-\cos x}{x^2}dx$ and $I_2(a) = \int_0^a \frac{\sin x}{x}dx$

(a) Calculate the <u>**two** leading non-vanishing terms</u> in the Taylor expansion of $I_1(a)$ and $I_2(a)$ for small $a \ll \pi$. (You will need the Taylor series for sine and cosine for small arguments)

(b) Using the Taylor expansion of part (a), estimate the values of $I_1(1)$ and $I_2(1)$.

Q3. Div & Curl *Checkpoints* [1]

Here is a vector field: $\vec{F}(x,y,z) = x^3z^4\ \hat{x} + xyz^2\ \hat{y} + x^2y^2\ \hat{z}$.

(a) Calculate the divergence $\vec{\nabla}\cdot\vec{F}$ of the field.

(b) Calculate the curl $\vec{\nabla}\times\vec{F}$ of the field.

(c) Evaluate the divergence of your result from part (b): $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{F}) = ?$ Thereby confirm, at least for this case, the useful identity $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0$, which is true for any vector field $\vec{A}$.

Q4. Hill Geography *Checkpoints* [2]

The height of a hill is given by $z(x,y) = \eta\ (2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$, where $\eta = 0.02$ meters^{-1}, and where x is the distance east of the origin and y is the distance north of the origin (both in meters).

(a) At what point (x,y) is the top of the hill located?

(b) How steep is the hill at the point $(x, y) = (1,1)$? That is, what is the angle between a vector perpendicular to the hill and the z-axis?

(c) You are at the point $(x, y) = (1,1)$ and you drop a ball from your hand. Naturally, the ball rolls in the downhill direction. In which compass direction does the ball roll?

Q5. Mystery Vector *Checkpoints* [2]

An unknown vector $\vec{v}$ satisfies these two equations: $\vec{b}\cdot\vec{v} = \lambda$ and $\vec{b}\times\vec{v} = \vec{c}$ where λ, $\vec{b}$, and $\vec{c}$ are known constants. What is $\vec{v}$ in terms of λ, $\vec{b}$, and $\vec{c}$? One possible hint: $\vec{A}\times(\vec{B}\times\vec{C}) = (\vec{A}\cdot\vec{C})\vec{B} - (\vec{A}\cdot\vec{B})\vec{C}$.

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[1] Q1 $\sqrt{b^2\omega^4 + 4c^2}$ Q2 (b) $I_1(1) \approx 35/72$, $I_2(1) \approx 17/18$ Q3 (a) $3x^2z^4 + xz^2$ (b) $(2x^2y - 2xyz)\hat{x} + (4x^3z^3 - 2xy^2)\hat{y} + yz^2\ \hat{z}$

[2] Q4 (a) $(-2,3)$ (b) $\cos^{-1}\left(1/\sqrt{1+2(22\eta)^2}\right) = 31.9°$ (c) direction $(\hat{x} - \hat{y})$, i.e. southeast Q5 $\vec{v} = \left(\lambda\vec{b} + \vec{c}\times\vec{b}\right)/b^2$