Phys 325 Discussion 2 – Drag Force Intuition

The resistance exerted by fluids on moving objects is well described by a **linear drag force** that's proportional to the object's speed plus a **quadratic drag force** that's proportional to the object's speed-squared:

$$\vec{f}_{\text{air}} = -(bv + cv^2)\,\hat{v} = \vec{f}_{\text{lin}} + \vec{f}_{\text{quad}}$$

(Note that this expression is only valid as long as the object moving through the air is travelling at speeds well below the speed of sound.) For **spherical objects** of **diameter *D*** the constants b and c have the forms $b = \beta D$ and $c = \gamma D^2$, where β and γ properties of the medium through which the object is travelling. When that medium is **air at STP**[1] we get these specific values:

linear constant:	$b = \beta\, D$	where	$\beta_{\text{air}} = 1.6 \times 10^{-4}$ N·s/m^2
quadratic constant:	$c = \gamma\, D^2$	where	$\gamma_{\text{air}} = 0.25$ N·s^2/m^4

Let's use these relations to develop some intuition about air resistance in everyday situations!

Q1. Baseballs and Beach Balls

Taylor 2.1; Checkpoints [2]

(a) Consider a ball of diameter D that flies through the air at STP. Show that the ratio of the quadratic to the linear drag force is $f_{\text{quad}} / f_{\text{lin}} = \left(1.6\times10^3 \text{ s/m}^2\right) D\,v$.

(b) Taking the case of a <u>baseball</u>, which has regulation size D = 7 cm, find the approximate "threshold" speed v_{thresh} of the baseball at which the two drag forces are equally important. Since baseball speeds are usually given in miles per hour (mph), you might want to use that unit to get a feel for your results → use the convenient approximate relation **1 m/s ≈ 2 mph** to convert from metric. Use your finding to address these questions:

- For what approximate range of speeds is it safe to treat the drag force as purely quadratic?
- Under normal conditions is it a good approximation to ignore the linear term?

(c) Answer the same questions for a beach ball of diameter 70 cm.

Q2. Origin of the Linear Term

Taylor 2.2

The origin of the linear drag force on a sphere in a fluid is the **viscosity** of the fluid. According to Stokes' Law, the viscous drag on a sphere is $f_{\text{lin}} = 3\pi\eta D v$ where v is the viscosity (see footnote[3]) of the fluid, D is the sphere's diameter, and v is its speed (as before).

(a) Use Stokes' Law to derive the form given above: $f_{\text{lin}} = bv$ with $b = \beta D$. (You'll get a formula for β.)

(b) Given that the viscosity of air at STP is $\eta = 1.7 \times 10^{-5}$ N·s/m^2, verify the value of β_{air} given above.

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[1] STP stands for "Standard Temperature and Pressure" and refers to T = 0° C and P = 100 kPa ≈ 1 atmosphere.

[2] Q1 (b) $v_{\text{thresh}} \approx 0.02$ mph → quadratic totally dominates for typical baseball speeds of 80–100 mph
(c) $v_{\text{thresh}} \approx 0.001$ m/s → quadratic totally dominates for typical beach ball speeds of a few m/s

[3] Viscosity is not an easy concept to understand. For now, you can't do better than to read Taylor's textbook: he has a wonderful footnote to this question (problem 2.2) that provides an excellent physical definition of viscosity.

Q3. Origin of the Quadratic Term *Taylor 2.4*

The origin of the quadratic drag force on any projectile in a fluid is the **inertia** of the fluid that the projectile has to push as it moves forward.

(a) Assuming the projectile has a speed v and a cross-sectional area A (i.e. the area perpendicular to its velocity), and that the fluid through which it moves has a mass density ρ (units: mass/volume), show that the rate at which the projectile displaces fluid (units: mass/time) is $\rho A v$. Hint: make a sketch!

(b) Making the simplifying assumption that all of this fluid is accelerated to the speed v of the projectile, show that the net drag force on the projectile is $\rho A v^2$.

Summary: We now have a decent understanding of the quadratic v^2 dependence of "everyday" air resistance: one factor of v comes from the amount of fluid that must be pushed per unit time, and the other comes from the speed to which that fluid must be accelerated.

FYI: It is not true that *all* the fluid the projectile encounters is accelerated to the full speed v, but one might reasonably guess that the actual force would have the form $f_{\text{quad}} = \kappa \rho A v^2$, where κ is a number less than 1 that would depend on the shape of the projectile (e.g. κ would be small for a streamlined body and larger for a body with a flat front end). As it happens, this guess is correct, and for a sphere the factor κ is found to be 1/4.

(c) Show that the expression above for f_{quad} reproduces the form at the top of the previous page, with the constant $c = \gamma D^2$ as advertised. Your work will give you an expression for γ; use it to verify our value for γ_{air} given that air at STP has density $\rho = 1.29\ \text{kg/m}^3$ and that a sphere has $\kappa = 0.25$.

Q4. A Brief Interlude: Magnetism and Math *≈ Taylor 1.45*

Another velocity-dependent force is the **Lorentz force**, $\vec{F} = q\vec{v} \times \vec{B}$, that a magnetic field $\vec{B}$ exerts on a moving particle of charge q and mass m. Applying $\vec{F} = m\vec{a}$, we get $\vec{a} = (q\vec{v} \times \vec{B})/m$. Prove these useful results:

- If $\vec{a}(t)$ is always orthogonal to $\vec{v}(t)$ — as in the magnetic-force case — then $|\vec{v}(t)|$ never changes;
- and the converse: if $|\vec{v}(t)|$ is constant, then $\vec{a}(t)$ must be orthogonal to $\vec{v}(t)$.

Stuck? …. Are you sure? ☺ … see the file "From225-6BasicPieces.pdf" on our website, and look at pieces #0 and #5.

Q5. The Reynolds Number *Taylor 2.3; Checkpoints* [4]

(a) Show that the ratio of the quadratic and linear drag forces on a moving sphere is $f_{\text{quad}} / f_{\text{lin}} = R/48$, where the dimensionless **Reynolds number** is $R \equiv Dv\rho/\eta$. FYI: The Reynolds number is a common measure of the relative importance of linear and quadratic drag, so you should know about its existence. When R is very large (e.g. for big, fast objects) the quadratic force dominates; when R is very small (e.g. for small, slow objects) the linear force dominates … more or less. ☺ The factor of 48 between R and the quadratic-to-linear drag ratio is for spherical objects; objects of other shapes have different constants of proportionality.

(b) Find the Reynolds number for a baseball (diameter 7 cm) that is thrown through STP air (density 1.29 kg/m^3 and viscosity 1.7×10^{-5} N·s/m^2) at a world-class pitching speed of 100 mph.

(c) Find the Reynolds number for a steel ball bearing (diameter 2 mm) that moves at 5 cm/s through glycerin (density 1.3 g/cm^3 and viscosity 12 N·s/m^2 at STP).

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[4] Q5 (b) $R \approx 3 \times 10^5 >> 48 \rightarrow$ quadratic term dominant (c) $R \approx 0.01 << 48 \rightarrow$ linear term dominant