Phys 325 Discussion 8 – Energy : To Use or Not To Use?

In last week's lectures, we discussed the conservation of (T+U) → the version of energy conservation that is *useful* to us for doing calculations. (T+U)-conservation provides a very useful tool for calculating the behavior of a system. How many tools do we actually have now? In general, there are <u>three</u> tool groups that provide us with all the equations we need to solve for the unknowns in a mechanics problem:

- <u>force & torque</u>: the equations of motion $\vec{F}^{\text{EXT}} = d\vec{P}/dt$ and $\vec{\tau}^{\text{EXT,(A)}} = d\vec{L}^{(\text{A})}/dt$
- <u>energy</u>: $T+U$ conservation
- <u>constraints</u>: problem-specific restrictions on a system's motion, e.g. no-slip rolling, fixed string length, and the geometric restrictions imposed by rigid objects like ramps and sticks

Our energy tool requires some thought. What we learned is that there are three types of problems: those where (T+U)-conservation (a) cannot be used (b) must be used (c) may or may not be used. In the following problems, it is up to you to figure out in which category each problem falls. ☺

Problem 0 : The Meta Problem

Conservation laws can become confusing when there are too many of them. We have so far explored conservation of $\vec{P}$, $\vec{L}^{(A)}$, and (T+U). Let's set angular momentum aside and concentrate on just $\vec{P}$ and $T+U$. It is possible to write solvable mechanics problems where both, neither, or just one of them is conserved. That's 4 possibilities. The problems below cover 3 of those 4 possibilities (and none of them involves angular momentum at all). The "meta-problem" is to identify which problem covers which possibility, and which possibility is left out. This meta-problem will return at the end … or if you prefer, you can read through the problems without solving any of them and figure out the meta-problem at the end straight away.

Problem 1 : Rain in a Boat

Hints & Checkpoints [1]

A speedboat travels at constant speed v. A gentle rain is falling, and the rain accumulates in the boat at a rate k kg/sec. Assume that the speed of the rain is zero before it lands in the boat (it is a gentle rain ☺).

(a) How much force do the boat's engines have to exert to maintain the boat's constant speed of v?

(b) This problem is an *inelastic collision* in disguise: rain "fuses" with the boat, causing total kinetic energy to change abruptly. Thus T+U is not conserved → some of it is converted into *heat* (of the rain / boat).
At what <u>rate</u> (Joules / second) is energy lost to heat?

Problem 2 : Boom!

Hints & Checkpoints [2]

Some chemistry students build a small bomb of mass $3m$ and drop it off the chemistry department's roof.
Let $+y$ be the downward direction and let $+x$ be horizontal to the ground to the RIGHT. At the instant that the bomb has speed u, it explodes, <u>releasing an amount of energy equal to four times its kinetic energy</u>. (Thus energy released = 4 ($3mu^2/2$) = 6 mu^2). The bomb breaks up into 3 pieces of equal mass m. The first piece goes to the LEFT with speed $v_L = \sqrt{3}\,u$, the second piece goes straight DOWN with unknown speed v_D , and the third piece goes off in an undetermined direction with unknown speed v_3 . Calculate the unknown speeds

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[1] (a) Hint: This is a system with *changing mass*. That makes the problem similar to our work on rocket motion … but this problem is much simpler since the speed of the boat is constant and the initial speed of the rain is zero. ☺ Answer: $F = vk$ (b) $\frac{1}{2}kv^2$

[2] <u>Strategic suggestion</u>: Do *not* introduce an angle for the third piece, work with the <u>components</u> v_{3x} and v_{3y}. Get the angle at the end. <u>Caution</u>: Think carefully about where that "released energy" appears in $(T+U^{\text{INT}}+U^{\text{EXT}})_{\text{initial}} = (T+U^{\text{INT}}+U^{\text{EXT}})_{\text{final}}$, a sign error is very easy to make here. Answer 1: $v_D = 3u$, $\vec{v}_3 = \sqrt{3}\,u\,\hat{x}$; Answer 2: $v_D = 0$, $\vec{v}_3 = u(\sqrt{3}\,\hat{x} + 3\,\hat{y}) = 2\sqrt{3}\,u$ at 30° off vertical.

v_D and v_3 and figure out the direction of the third piece. There are two valid answers. Also, do not be afraid of the quadratic that appears: it will simplify very nicely and easily as long as you keep track of which variables you're solving for and which ones you already know.

Problem 3 : Sliding Off a Sphere *Hints & Checkpoints* [3]

A sphere of radius R has a frictionless surface and is fixed in place on a table. A small block of mass m is perched precariously on top of the sphere. A tiny disturbance causes the block to start sliding down the sphere.

(a) Calculate the angle ϕ^* where the block leaves the surface of the sphere → this will be the point at which the normal force between the block and the sphere goes to zero. (The angle ϕ is defined with respect to the upward vertical; the block thus starts at $\phi = 0$.) For reference, here is the formula for acceleration in 2D polar coordinates = cylindrical coordinates with the z part removed: $\vec{a} = \hat{s}\left(\ddot{s} - s\dot{\phi}^2\right) + \hat{\phi}\left(s\ddot{\phi} + 2\dot{s}\dot{\phi}\right)$.

(b) Did you use (T+U)-conservation to solve that? I hope so! Well, remember in lecture when we discussed how the conservation statement $d(T+U) = 0$ is a consequence of the force law $F=ma$? As it happens, this is one of those problems where the use of energy is actually *optional*. To see how $F=ma$ is embedded in $d(T+U) = 0$, take the time-derivative of the energy-conservation equation you used in part (a) and show that $\dot{T} + \dot{U} = 0$ is equivalent to a component of $\vec{F} = m\vec{a}$.

(c) To solve the original problem, you undoubtedly used the radial component, $F_s = ma_s$, of the force law along with energy conservation. See if you can now completely avoid energy conservation by solving the problem entirely using the two force-law components $F_s = ma_s$ and $F_\phi = ma_\phi$. A hint is in the footnote if you need it.

(d) What about problem 2? Could you have chosen *not* to use energy conservation in problem 2? Why or why not?

Problem 0 : The Meta Problem Returns *Checkpoints* [4]

(a) Which of the quantities $\vec{P}$ and/or (T+U) are conserved in each problem?

(b) For the conservation-combination that is left out, create a solvable problem with the required properties. (It is *incredibly* useful to invent problems. If only you could *write* the exams as well as solving them! ☺ Really!)

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[3] (a) $\cos\phi^* = 2/3 \rightarrow \phi^* \approx 48°$ (b) You get the ϕ component: $F_\phi = ma_\phi$ (c) Hint: The F_ϕ equation involves $\ddot{\phi}$ while the F_s equation involves only $\dot{\phi}$. One way to combine them is to take the *time-derivative* of the F_s equation, then use both equations to get rid of $\ddot{\phi}$, then integrate the result using your known boundary conditions to find the answer. (d) No. As we discussed in lecture, the situations where it is *essential* to use energy conservation are those where *we don't know all the forces*. In those cases, we obviously cannot use a force-based approach! In problem 2, we knew nothing about the chemical forces responsible for the explosion, we knew only how much energy was released.

[4] (a) What's conserved in each problem → **1**: neither P nor T+U **2**: T+U and P **3**: T+U, not P (b) You need a problem where total P is conserved but T+U is not. To have P conservation, we must have no external forces. But we need T+U to change, so forces *internal* to the system must cause energy to escape to some other form. The classic example of this type of problem is fusion: a collision problem where two or more particles stick together. The unspecified internal forces that cause the particles to fuse cause an abrupt change in kinetic energy, and where does it go? Into the heat of the fused particle. Problem 1 is similar to a fusion problem, except there *is* an external force acting (to keep the boat at constant speed despite its increasing mass) so P is not conserved. Problem 2, by the way, also has an external force (gravity) but it does *not* disturb P-conservation. Reason: problem 2 takes place over an *infinitesimally small amount of time* (an impulse situation), so $\Delta P = F^{\text{GRAVITY}}\,\Delta t = 0$, and P is conserved during the explosion.