

Fall 2014 – Physics 325 Final Exam
Monday Nov 15, 1:30 pm – 4:30 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **3 hours** to work the problems.

At the beginning of the exam:

- 1) Write your **name** and **netid** on your answer booklet(s).
- 2) Turn your **cell phone off**.
- 3) Put away all calculators, phones, computers, notes, and books.

During the exam:

- 1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
- 2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you understand what you're doing and are **not just guessing**. Also a good **annotated sketch** is often the best explanation of all!
- 3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. Therefore you should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
- 4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. **IMPORTANT:** If you think you've made a mistake but can't find it, **explain what you think is wrong** → you may well get partial credit for noticing your error!
- 5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or $5/7$).
- 6) Specify the **units** for all numerical answers.

Remember: There are many **Math Tables** provided

When you're done with the exam:

Turn in everything : answer booklet, questions, and formula sheets

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.



You may use without proof these Moments of Inertia:

- **Thin rod** of uniform mass m and length d rotating around its **center**: $I = md^2 / 12$
- **Thin rod** of uniform mass m and length d rotating around its **end**: $I = md^2 / 3$

In both of the above cases, the axis of rotation $\hat{\omega}$ is perpendicular to the rod.

- **Solid cylinder** of mass m and radius r around its **axis**: $I = mr^2 / 2$
- **Solid sphere** of mass m and radius r around any diameter: $I = 2mr^2 / 5$

Problem 1 : What a Drag

A thin massless stick of length d is placed flat on a horizontal table. One end of the stick is secured to a pivot at the origin, while a small ball of mass m is glued to the other end. The table is frictionless. At time $t = 0$ the ball is given a kick so that its initial angular velocity around the origin is ω_0 . The ball is now subject to a **linear drag force** $\vec{F} = -b\vec{v}$ caused by air resistance. Calculate the total amount of angle, $\Delta\phi$, that the ball traverses before it comes to a stop.

Problem 2 : A Rocket Slows Down

A rocket ship of total mass m_0 (ship plus fuel) is traveling forward at constant speed v_0 through empty space, far from any sources of gravity. This ship has two rocket engines: one that emits fuel out the back end to speed the ship up, and another that emits fuel out the front end to slow the ship down. The engines both have exhaust speed v_{ex} relative to the rocket and both emit fuel at a constant rate of k kg/sec. The captain decides to reduce the ship's forward speed from v_0 to $v_0 / 2$. Calculate the amount of time Δt that the forward engine must be active to accomplish this speed reduction.

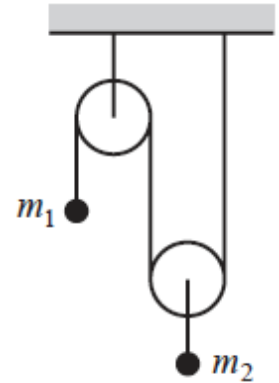
Problem 3 : No-Slip Pool

If you have ever played pool, you know that there is a perfect height at which you can strike the pool ball to make the ball roll with no initial slipping along the table, even if the pool table provides no frictional force at all. Calculate the height above the table at which you must strike the ball to make this happen in terms of the radius R of the pool ball. Assume that the strike you deliver to the ball is an impulse whose direction is horizontal = parallel to the table.

Problem 4 : Two Pulleys, One String

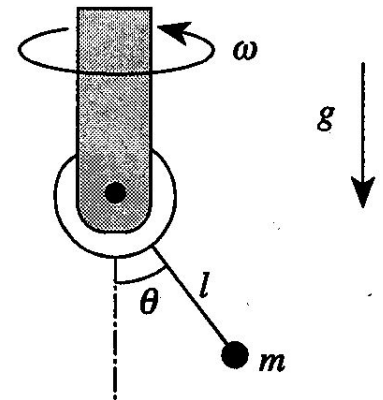
The figure at right shows two massless pulleys. A single massless string is threaded over / under the pulleys, with one end attached to the fixed platform at the top and the other end attached to the mass m_1 . The axle of the left-hand pulley is attached to the platform with a rigid massless rod. The axle of the right-hand pulley is attached to the mass m_2 with another rigid massless rod. Uniform gravity g points downward.

Calculate the acceleration of the mass m_2 .

**Problem 5 : Rotating Pendulum**

A pendulum consists of a rigid massless rod of length l with a mass m at one end. The other end is attached to a massless, frictionless pivot that only allows the pendulum to swing in one plane (i.e., the usual “plane-pendulum” configuration where only the angle θ shown in the figure can change).

However, this pivot can also rotate around the vertical axis, and it is forced by a motor to maintain a constant angular speed ω . Uniform gravity g is pointing down, as shown. The known quantities are thus m , l , g , and ω .



(a) Calculate the Lagrangian for this system using θ as your generalized coordinate.

(b) No matter what you obtained for part (a), use the following Lagrangian for the remaining calculations:

$$L(\theta, \dot{\theta}, t) = A \left(\frac{1}{2} \dot{\theta}^2 + \frac{1}{2} B^2 \sin^2 \theta + C \cos \theta \right)$$

where A , B , and C are constants. Using this Lagrangian, determine the one equilibrium value of θ that is in the range $0 < \theta < \pi$ and calculate the frequency of small oscillations around this value.

Problem 6 : Math Time

- (a) Consider a Lagrangian $L(q, \dot{q}, t)$ for a system with one degree of freedom. Starting from the definition of the Hamiltonian, use your math skills to prove that $\frac{dH}{dt} = -\frac{\partial L}{\partial t}$.
- (b) Interpret the relation $\frac{dH}{dt} = -\frac{\partial L}{\partial t}$: how can we use it to sometimes identify a useful property of our system?

Problem 7 : Quickies

For each of the following short problems, remember to include some explanation for your answer. Don't write a paragraph, just write enough to make it clear you understand the problem.

- (a) Three point masses are placed on a flat table and allowed to move anywhere on its surface. A rigid rod connects mass #1 to mass #2 while a second rigid rod connects mass #2 to mass #3. The connections are completely flexible, meaning that the angle between the two rods can take any value. How many degrees of freedom does this system have?
- (b) Invent a system — i.e. invent a short mechanics problem — where the system's total momentum \vec{P} is conserved but where the total kinetic-plus-potential energy ($T+U$) is not, even when U includes all conservative forces acting on the system.
- (c) Newton's third law says that the force exerted by object 1 on object 2 is equal and opposite to the force exerted by object 2 on object 1. Does this law always hold? If not, give an example where it doesn't hold.
- (d) If Newton's third law doesn't hold, our equation $\vec{F}^{\text{EXT}} = d\vec{P} / dt$ relating the total force and total momentum of a multi-particle system will not hold either. Why not?

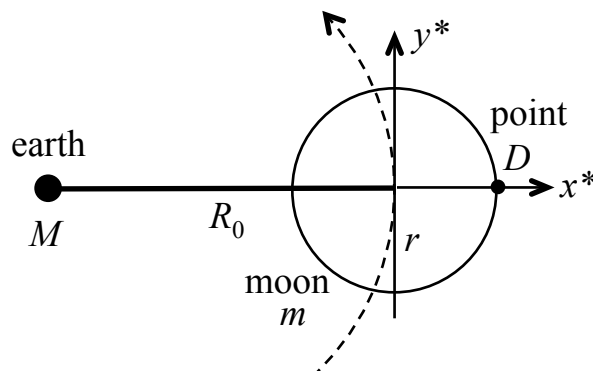
Problem 8 : Mass on a Train

A high-speed train is traveling at a constant speed v across the surface of the earth, where v is measured in the earth's non-inertial reference frame. Let earth's rotational velocity be ω in the $+z$ direction. We will use the polar angle θ measured with respect to the north pole to describe the train's position on the earth's surface. (Thus $\theta = 0^\circ$ is the north pole, $\theta = 90^\circ$ is the equator, etc.)

- (a) The train travels on a straight, horizontal track across the South Pole. Find the angle between a plumb line (i.e. a mass on a string) that is suspended from the ceiling inside the train and another plumb line inside a hut on the ground.
- (b) Find all possible combinations of the train's location θ and velocity-direction \hat{v} that will make the Coriolis force on a mass at rest in the train's frame equal to zero. (i.e. If you are sitting still on the train, where does the train have to be and in what direction does it have to be moving in order for you to experience zero Coriolis force?)

Problem 9 : The Dark Side of the Moon

The moon has mass m and radius r . It orbits around the earth, whose mass is M , in a circular orbit of radius R_0 (where R_0 is the distance between the centers of the earth and moon). As you may know, the moon is in a so-called "tidally locked orbit", meaning that it **always presents the same face to the earth**. To clarify, let D denote the point on the moon's surface that is the furthest from the earth at this moment; the nature of the moon's motion is such that this unique "darkside" point D is **always** the point furthest from the earth. The final variable we need is Ω , which is the angular speed of the moon's orbit around the earth (i.e., $\Omega = 2\pi / 30$ days).



Your task is to calculate g_{eff} at D = the "darkest" point on the moon. Here, g_{eff} is the total force-per-unit-mass experienced by objects at rest in the moon's non-inertial reference frame; x^* and y^* axes have been provided in the figure to help specify the moon's non-inertial frame. Note that g_{eff} includes (among other things) the gravitational pulls from both the moon *and* the earth (i.e. don't forget about the earth!).

Express your answer for g_{eff} at the point D in terms of the three quantities g_{moon} , ε , and δ , defined as follows:

$$g_{\text{moon}} = \frac{Gm}{r^2}, \quad \varepsilon \equiv \frac{r}{R_0} \approx 4 \times 10^{-3}, \quad \delta \equiv \frac{m}{M} \approx 10^{-2}. \quad \text{Also use the (easily-obtained) relation } \Omega^2 = \frac{GM}{R_0^3}.$$

Finally, approximate your answer to lowest non-vanishing order in the tiny parameter ε .