Physics 325 – Homework #1 **due in 325 homework box[1] by Friday, 1 pm**

All solutions must clearly show the steps and/or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning. Answers given without explanation will not be graded: our master rule for homework and exams is **NO WORK = NO POINTS**. *However you may always use any relation on the 3D-calculus and 1D-math formula sheets without proof; both are posted in the same place you found this homework. Finally please write your* **NAME** *and* **DISCUSSION SECTION** *on your solutions.* ☺

Problem 1 : Puck on a Board

You lay a rectangular board on the horizontal floor and then tilt the board about one edge until it slopes at an angle θ with the horizontal. Choose your origin at one of the two corners that touch the floor, the x axis pointing along the bottom edge of the board, the y axis pointing up the slope, and the z axis normal to the board. You now kick a frictionless puck that is resting at the origin so that it slides across the board with initial velocity $\vec{v}\big|_{t=0} = v_{0x}\,\hat{x} + v_{0y}\,\hat{y}$.

Write down Newton's second law using the given coordinate system and use it to calculate how far the puck is from the origin when it returns to floor level.

Problem 2 : Tossing a Ball

A ball is thrown with initial speed v_0 up an inclined plane. The plane is inclined at an angle ϕ above the horizontal, and the ball's initial velocity is at an angle θ above the plane. Choose axes with x measured up the slope and y normal to the slope. You can neglect the z coordinate as the ball remains in the xy-plane ($z = 0$).

(a) Write down Newton's second law using these axes and find the ball's position as a function of time. (Your solution only needs to be valid for times *before* the ball lands back on the inclined plane.)

(b) Show that the ball lands on the plane a distance $R = 2v_0^2 \sin\theta \cos(\theta+\phi)\,/\,(g\cos^2\phi)$ from its launch point.

(c) Show that for given values of v_0 and ϕ, the maximum possible range up the inclined plane is $R_{\text{max}} = v_0^2\,/\,[g(1+\sin\phi)]$.

Problem 3 : Boom Goes the Dynamite

A cannon shoots a cannon ball at a launch angle of θ above the horizontal ground with initial speed v_0.

(a) Neglecting air resistance, use Newton's second law to find the ball's position as a function of time. Please use the following coordinates: choose the cannon as the origin, use x to measure horizontal position, and use y to measure vertical position.

(b) Let $r(t)$ denote the ball's distance from the cannon. What is the largest possible value of the cannon's launch angle θ if $r(t)$ is to increase throughout the ball's flight?
Hint 1: Write down $r^2(t) = x^2(t) + y^2(t)$ then impose the condition that $r^2(t)$ is always increasing with time.
Hint 2: You will soon get an expression like this: $\sin\theta < f(t)$, i.e. the right-hand-side of your constraint will be *time-dependent*. But we want a constraint on θ that holds for *all* times t ! ➔ What you must do to finish the problem is find the special time t^* where that right-hand-side, $f(t)$, imposes the *tightest* constraint on θ.

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[1] All physics homework boxes are in the 2nd floor overpass between Loomis and MRL (to the north of Loomis).

For the next problems, you may use the formulae for velocity and acceleration in cylindrical coordinates from Lecture 1B or from the textbook ➔ you do not have to re-derive those expressions.

Problem 4 : Twirling in Space ('cause why not?)

An astronaut in gravity-free space is twirling a mass m on the end of a string of length R in a circle, with constant angular velocity ω. Define your coordinate system so that the fixed end of the string is at the origin and the twirling mass's orbit is in the xy-plane ($z = 0$).

(a) Write down Newton's second law in polar coordinates (s, ϕ) and calculate the string tension (see footnote[2]).

(b) Let's set our clock so that the mass is at $\phi = 0$ at time $t = 0$, and let's set the twirling direction so that the ϕ coordinate is always increasing. At time $t = 2\pi / 3\omega$ the string breaks! Once that happens, the mass is no longer under the influence of any force. Write down a complete description of the mass' trajectory in Cartesian coordinates — i.e. find $x(t)$ and $y(t)$ for times $t > 2\pi / 3\omega$. Hint: Resist the temptation to return to the familiar: do *not* start solving $F=ma$ in Cartesian coordinates yet again! Instead, first write down the mass' trajectory in *polar* coordinates – $s(t)$, $\phi(t)$ – before the string breaks. (The trajectory is supremely simple in that system!) Next consider what will happen when the string breaks ... Once you know what you want to do, get everything you need in polar, then convert from polar to Cartesian and you'll be done very quickly. ☺

Problem 5 : Trapped on a Cylindrical Surface

Two concentric cylinders are centered on the vertical z axis and have radii $R \pm \varepsilon$, where $\varepsilon \ll R$. A small frictionless puck of mass m and thickness 2ε is inserted between the two cylinders; the puck can thus be considered a point mass that can move freely at a fixed distance R from the z axis.
Notational suggestion: As you know, the symbol v_i is universally used to denote the time-derivative of a position coordinate, e.g. $v_z = \dot{z}$. The time-derivatives of *angular* coordinates, however, are *not velocities* so a different symbol is appropriate. That symbol is usually ω, e.g. $\omega_\phi = \dot{\phi}$. This entire problem is in cylindrical coordinates; since there is only angle, drop the subscript for simplicity and just use ω to represent $\dot{\phi}$.

(a) Write down and solve Newton's second law in cylindrical coordinates (s, ϕ, z) to determine the general motion of the puck, including the effects of gravity. Note that your general solution for $s(t)$, $\phi(t)$, $z(t)$ will necessarily involve some unspecified constants as we have not been given any boundary conditions — we don't know the position or velocity of the puck at any point in time.

(b) The Starship Enterprise (NCC 1701-D, of course) transports our apparatus to an interesting region of space where the gravitional acceleration $\vec{g}$ is not a constant but grows with time! The gravitational force on the puck is now $\vec{F}_g = -mKt\,\hat{z}$ where K is a constant. Calculate the puck's new trajectory $z(t)$ (z-coordinate only) using these initial conditions: at time $t = 0$ the puck is at position $z = 0$ and the z-component of its velocity is v_{0z}.

(c) The Enterprise escapes the dangerous gravitational field and takes us to a gravity-free region of space. We now fill the gap between the cylinders with a liquid that exerts a velocity-dependent drag force on the puck: $\vec{F}_{\text{drag}} = -K\,\vec{v}$ where K is a constant. At time $t = 0$ we start the puck at position $(s, \phi, z) = (R, 0, 0)$ and give it an initial velocity $\vec{v}\big|_{t=0} = R\omega_0\,\hat{\phi}$, where ω_0 is constant. Calculate the puck's trajectory (all coordinates) for $t > 0$.

(d) Using your answer to (c), determine if the puck ever comes to a stop. If it does, find out at what time t this occurs and calculate the total distance travelled by the puck before it comes to a stop.

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[2] **Tension** means the *pulling force* exerted by a solid object (e.g. a string) on another object. The term is most often used when the object under tension does not deform, and thus prevents the attached object from moving in some direction. In this context "tension" is completely analogous to "normal force", except that normal forces *push* (e.g. tables) while tension forces *pull* (e.g. strings).