Physics 325 – Homework #2 due in 325 homework box[1] by Friday, 1 pm

All solutions must clearly show the steps and/or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning. Answers given without explanation will not be graded: **"NO WORK = NO POINTS"**. *However you may always use any relation on the 3D-calculus and 1D-math formula sheets without proof; both are posted in the same place you found this homework. Finally please write your* **NAME** *and* **DISCUSSION SECTION** *on your solutions.* ☺

Problem 1 : A Force with Mixed Dependences[2]

A particle of mass m moves under the influence of the following force: $\vec{F}(t,v) = -k\,t\,v\,\hat{v}$, where k is a positive constant. At time $t = 0$ the particle passes through the origin with velocity $v_0\,\hat{x}$, where v_0 is a positive constant.

(a) Calculate the particle's velocity $\vec{v}(t)$ for times $t > 0$.

(b) How much time does it take for the particle to come to a stop?

(c) What is the maximum x-position reached by the particle? Hint: you will need to look up $\int_0^\infty (\text{something})\,dt$, e.g. at http://wolframalpha.com , which is a very valuable online tool that you should learn to use!

Problem 2 : A Position-dependent Force

A particle of mass m is constrained to move along the positive x-axis and is under the influence of the following force: $\vec{F}(x) = -\frac{K}{x^2}\hat{x}$, where K is a positive constant. At time $t = 0$ the particle is released from rest at position $x = x_0$, where x_0 is positive. Calculate the time at which the particle (first) reaches the origin.
Integration assistance: The trig substitution $x = x_0 \sin^2 \alpha$ will help, as will the identity $\sin^2\theta = (1-\cos 2\theta)/2$.
(That last identity is very useful, by the way; I strongly recommend you derive it, just for your own knowledge! There's no need to memorize it ➔ just know that switching from trig[θ] to trig[2θ] is a helpful trick, then derive the switch when you need it.) Caution: When you calculate velocity, you will have to take a square root … and every time you do that, you must think carefully — physically! — about which **sign** to choose for the result.

Problem 3 : Drag to Any Power

A block with mass m slides on a horizontal surface. At time $t = 0$, the block is at position $x = 0$ and has speed v_0 in the $+x$ direction. (Note: "speed" means "*magnitude* of velocity", so v_0 is implicitly a positive number.)
Let's explore a generic form for the drag force that the block experiences: $\vec{F}(v) = -k\,v^\beta\,\hat{v}$, where k and β are positive constants. The air-resistance cases $\beta = 1$ and $\beta = 2$ are derived in Taylor; let's try other options.

(a) At what position does the block stop and how long does it take to get there in the case $\beta = 0$?

(b) At what position does the block stop and how long does it take to get there in the case $\beta = 3/2$?

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[1] All physics homework boxes are in the 2nd floor overpass between Loomis and MRL (to the north of Loomis).

[2] Thank you to Richard Weaver, from whom problems #1-3 are adapted.

(c) Consider the case $\beta = 1/2$, and change the experiment so that the block is now dropped vertically: it will now experience a downward gravitational force as well as the drag force. Experimenters drop the block rest at $t = 0$ and determine from their measurements that the block's terminal velocity is $v_{\text{TER}} = 50$ m/s. Using $g = 10$ m/s^2, calculate the times (in seconds) at which the block reached these speeds: 50%, 90%, 99%, and 99.9% of its final speed v_{TER}. You may use WolframAlpha.com to do any integrals you encounter. ☺

Problem 4 : Particle in a Magnetic Field

A particle has charge q and mass m moves through a region with a constant magnetic field $\vec{B} = B\hat{z}$ where B is a positive constant. The particle thus experiences the magnetic Lorentz force, $\vec{F} = q\vec{v} \times \vec{B}$. (Please neglect gravity.) The particle's initial velocity is $\vec{v}\big|_{t=0} = v_{0x}\hat{x} + v_{0y}\hat{y} + v_{0z}\hat{z}$ and its initial position is $\vec{r}\big|_{t=0} = x_0\hat{x} + y_0\hat{y}$.

Like the drag force, the Lorentz force is velocity-dependent … but as its *direction* is completely unlike that of the drag force, it produces very different effects. Our task, as always, is be to calculate the particle's trajectory $\vec{r}(t)$, i.e. to determine its coordinates as a function of time: $x(t)$, $y(t)$, and $z(t)$. Since Newton's equations of motion are second-order in the coordinates $x_i(t)$, we will as usual proceed in two steps: first calculate velocity $\vec{v}(t)$, then calculate $x_i(t)$. However in the first step, we will encounter a new challenge: solving **coupled differential equations**. Let's proceed in steps to see how this works.

(a) Warmup: show that any function $f(t)$ that can be written in the form $A\cos\omega t + B\sin\omega t$, with two arbitrary constants A and B, can always be rewritten in the <u>alternate</u> form $f(t) = C\sin(\omega t + \phi)$, with two different constants C and ϕ. To accomplish your demonstration, find the transformation equations $C(A,B)$ and $\phi(A,B)$ that allow you to determine the second set of constants (C,ϕ) from the first set (A,B).
FYI: Why did we do this warmup? ➔ Because that second, alternate form is often forgotten but is usually much easier to interpret and work with than the first form!

(b) Apply Newton's second law to obtain the three differential equations describing the particle's velocity components, $v_x(t)$, $v_y(t)$ and $v_z(t)$.

(c) … and now it gets interesting! The equation for $v_z(t)$ is easy to solve, but the equations for $v_x(t)$ and $v_y(t)$ are **coupled**: each one involves **both** functions. There is no way we can separate-and-integrate until we obtain **separated = uncoupled** equations for $v_x(t)$ and $v_y(t)$: one equation involving *only* v_x and its derivatives, and another equation involving *only* v_y and its derivatives. How do we do this? See if you can figure it out on your own … if you're stuck, after a seemly amount of time ☺ check the footnote[3] to learn the technique you need. To finish this part, obtain solutions for $v_x(t)$, $v_y(t)$ and $v_z(t)$ that satisfy the initial conditions given at the top. One final hint is to remember part (a) → either of the two forms presented there will help you find the solutions you seek, but do consider using the "alternate" form with the phase factor: it will help with part (e) where you must *interpret* your final solution.

(d) Step 1 is complete! Now perform step 2: calculate the particle's trajectory $x(t)$, $y(t)$, and $z(t)$.

(e) The trajectory you just obtained is a <u>helix</u>. Sketch it, and calculate its two key dimensions: the **diameter** of each turn and the "**pitch**" = vertical rise (Δz) per turn.

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[3] You should have one equation involving $\dot{v}_x$ & v_y and another equation involving $\dot{v}_y$ & v_x. To turn the first one into a separated equation for v_x alone you must substitute in v_y from the second equation … but the second equation gives you $\dot{v}_y$ not v_y! What to do? We must alter the first equation so it, too involves $\dot{v}_y$ ➔ *Take the time derivative of the first equation*. Then you can substitute in $\dot{v}_y$ from the second equation and obtain a diffEq for v_x alone. It is a famous and very common 2nd-order equation. You can obtain the general solution for $v_x(t)$ by that most noble of techniques: **Guess** the form ☺and **Plug** it back in to your original ODEs.