Physics 325 – Homework #3 **due in 325 homework box by Fri, 1 pm**

All solutions must clearly show the steps and/or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning. Answers given without explanation will not be graded: **"NO WORK = NO POINTS"**. *However you may always use any relation on the 3D-calculus and 1D-math formula sheets without proof; both are posted in the same place you found this homework. Finally please write your* **NAME** *and* **DISCUSSION SECTION** *on your solutions.* ☺

For the rocket-type problems, you may use without proof the result derived in class:
Of course it's up to you to make sure you understand what the variables mean!

$$\boxed{m\dot{v}_x = -\dot{m}v^{\text{ex}} + F_x^{\text{EXT}}}$$

Problem 1 : Momentum Maxes Out

Consider a rocket (initial mass m_0, exhaust speed v^{ex}) accelerating from rest in free space. ("Free space" is convenient code for "no external forces". ☺) At first, as it speeds up, its momentum p increases, but as its mass m decreases, p eventually begins to decrease. For what value of m is p maximized?

Problem 2 : Multi-Stage Rockets

(a) A certain rocket carries $\lambda = 80\%$ of its initial mass as fuel. (That is, the total mass of fuel is λm_0). What is the rocket's final speed, accelerating from rest in free space, if it burns all its fuel in a single stage? Express your answer as a multiple of v^{ex}.

(b) Suppose instead that the rocket burns the fuel in two stages as follows: In the first stage, it burns a mass $0.4\ m_0$ of fuel. It then jettisons the first-stage fuel tank, which has a mass of $0.1\ m_0$, and then burns the remaining $0.4\ m_0$ of fuel. Find the final speed in this case, assuming the same value of v^{ex} throughout, and compare with your single-stage result. NOTE: when fuel tanks are "jettisoned", they are simply detached from the rocket, not ejected at high speed the way the fuel is.

Problem 3 : Rocket vs Air

Consider a rocket subject to a linear resistive force, $\vec{f} = -b\vec{v}$, but no other external forces. Show that if the rocket starts from rest and ejects mass at a constant rate $k = -\dot{m}$, its speed is $v = \frac{k}{b}v^{\text{ex}}\left[1 - \left(\frac{m}{m_0}\right)^{b/k}\right]$.

Problem 4 : Leaking Tanker → a rocket problem in disguise

A truck hauling a big tank of oil starts at position $x = D$ (Chicago) and heads due west ($-x$ direction) toward its destination at $x = 0$ (Des Moines). At Chicago, the total mass of the loaded truck is M and the mass of oil it is carrying is λM. (Thus $M(1-\lambda)$ is the "tare" mass = the mass of the truck when it is empty.) The driver starts from rest at time $t = 0$ with his engines set to deliver a constant force of magnitude F throughout the trip.

Unfortunately, the trucker's oil tank is **leaking**: it is losing oil at a constant rate-per-unit-distance of $dm/dx = \lambda M / D$. Here, m is the total mass of the truck and its load of oil. NOTE: the truck is losing oil, so dm is negative, but dx is negative too since the truck is heading west; that's why dm/dx is a positive constant. Ignore the small change in m from the truck's consumption of gasoline (it's *tiny* compared to the truck's mass).

(a) Calculate $m(x)$ in terms of x and the given constants D, M, λ, and/or F. Apply limiting-case checks to your result: stare at the expression for dm/dx and figure out how much oil will be left in the tanker by the time it reaches Des Moines (easy!). Does your expression for $m(x)$ agree with your conclusion? Also check that your $m(x)$ gives the correct result at Chicago. If those two limiting cases are correct, your $m(x)$ can hardly be wrong!

(b) Calculate the kinetic energy T of the truck as a function of x and the given constants. (The intention here is not to perform an energy analysis … though you can try one ... $mv^2/2$ is just a convenient quantity to obtain.)

(c) At what position x_V does the truck achieve its maximum speed, and at what position x_T does the truck achieve is maximum kinetic energy? (A crude plot is enough for one of them; the other requires a calculation.)

(d) A month later, the trucker repeats his Chicago-to-Des Moines run with the same parameters D, M, λ, and F. However, he is hauling a different oil tank, and this one loses oil at a constant rate-per-unit-time of $dm/dt = -M/T$ where T is a constant. Calculate the truck's position, $x(t)$, for values of $t < T$.
Guidance: (1) You may find it helpful at some point to switch variables from t to $u \equiv 1 - t/T$. (2) You will need the integral of $\ln(u)$; it's on your 1Dmath formula sheet, but it's worth memorizing as it appears rather a lot.

Problem 5 : Pickup Truck in the Rain

A pickup truck is driving at night through the rain. The driver can't see very well … by mistake, he drives onto a frozen lake. Oops! He immediately turns off his engine to save fuel (it wouldn't help anyway on frictionless ice) and lets the truck coast (move under no power). Unfortunately, the rain is accumulating in the back of his truck, at a constant rate of σ kg/sec. This makes the truck heavier … oh dear! The given parameters of this problem are the truck's initial mass m_0 and velocity $v_0\,\hat{x}$ when it hits the lake at time $t = 0$, and the rate σ of rain accumulation. Also, the rain is falling straight downwards in the reference frame of the ground. **Guidance** is provided in the footnote[1].

(a) Calculate the truck's speed $v(t)$ as a function of time.

(b) New situation: As soon as he hits the frozen lake at $t = 0$, the trucker realizes that he can help himself by bailing the rain from the back of his truck. The trucker is very fit: he is able to get rid of every raindrop as it falls by catching it in a bucket and hurling it backwards off the truck ($-x$ direction) at speed u (relative to the truck). Calculate the truck's speed $v(t)$ in this new situation.

(c) Final situation: This time, the trucker has some good luck → the rain is falling mostly downward but it also has a horizontal speed component of u (relative to the ground) in the $+x$ direction. The trucker lets the rain accumulate as in part (a), hoping that the rain's forward speed component will help push him across the lake. Calculate the truck's speed $v(t)$ in this last situation.

Problem 6 : Center-of-Mass (CM)

A uniform thin metal sheet is cut in the shape of a quarter circle of radius b. It lies in the xy plane, its center is at the origin, its straight edges lie along the x & y axes, and the metal is all in the $x,y > 0$ quadrant. Find the position $\vec{R}$ of the CM.

Problem 7 : Hemisphere CM

Find the CM position $\vec{R}$ of a solid hemisphere with radius b and uniform mass density. To supply a coordinate system: the flat base of the hemisphere coincides with the xy plane and is centered on the origin, and the hemisphere's bowl is "above" the plane ($z > 0$).

---

[1] Problem 5 is another rocket problem in disguise, but here we have *incoming* bits of mass to deal with, not only an *outgoing* exhaust. You must revisit the rocket-equation derivation in one/some/all of the parts (you decide). Write down "before" and "after" expressions, $P_x(t)$ and $P_x(t+dt)$. Each one may or may not include contributions from the three "objects" in the problem: (1) the incoming bit of rain (2) the truck, including any load of rain it's carrying (3) the outgoing bit of rain, if there is one. Use $P_x(t)$ and $P_x(t+dt)$ to build the equation of motion for object (2) = the truck and its contents → that is the object whose speed we need to find. Finally, these problems require the *time*-dependence of v, so there's a second equation you need → an expression for $m(t)$.