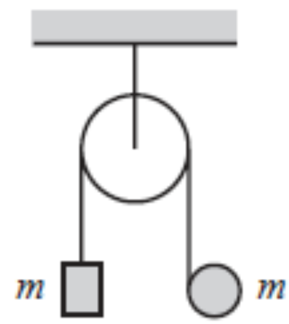
Physics 325 – Homework #6 due in 325 homework box by Fri, 1 pm

All solutions must clearly show the steps and/or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning. Answers given without explanation will not be graded: **"NO WORK = NO POINTS"**. *However you may always use any relation on the 3D-calculus and 1D-math formula sheets without proof; both are posted in the same place you found this homework. Finally please write your* **NAME** *and* **DISCUSSION SECTION** *on your solutions.* ☺

Problem 1 : Atwood Machine with a Twist

The figure shows an Atwood machine, with a massless pulley attached to an immovable platform, with a massless string running over the pulley. On the left side, the string is attached to a block of mass m … but on the right side, the string is wound around a cylinder that is solid, uniform, and has mass m (same as the block). Thus, as the masses move, we must also consider the possibility of the string *unwinding* from around the cylinder → how exciting! ☺ Your task is to calculate the accelerations of the block and the cylinder.



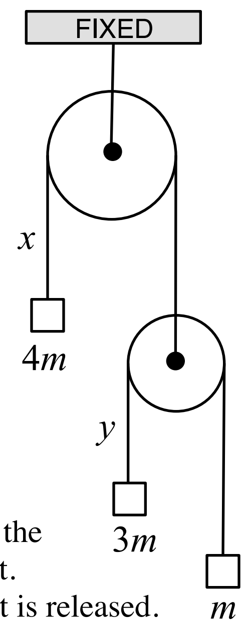
Hint: First demonstrate that the accelerations are the *same*, a task that requires only the force law $F = ma$, suitably applied.

Suggestion: If you didn't complete the third problem of Discussion 6, now would be a perfect time to go through it, as it contains an example of an unrolling string.

Problem 2 : Double Atwood Machine

At right, you see an Atwood machine attached to *another* Atwood machine. As usual, the pulleys and strings are massless. The total mass attached to both the left and right sides of the upper pulley is thus the same: $4m$ … and so our instincts are convinced from the start that the upper pulley and string will remain stationary. But no! This bizarre, mind-blowing system behaves differently. Your task is to calculate the accelerations $\ddot{x}$ and $\ddot{y}$ in terms of the given constants g and/or m.

Suggestion: Discussion 6 Problem 2 provides useful guidance for how to proceed.

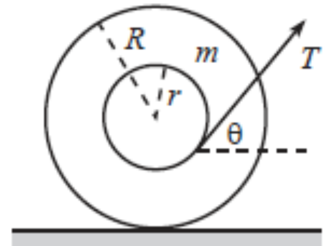


Problem 3 : Optimization

A thin massless stick of length d has a mass m attached to one end. The other end is attached to a fixed pivot, and the stick is held at rest in a horizontal position relative to the ground. A second mass m is now glued on the stick at a distance x away from the pivot. Calculate the value of x that will maximize the angular acceleration of the stick when it is released.

Problem 4 : Pulling on a Spool

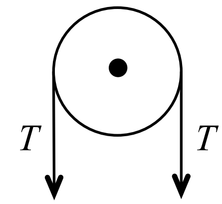
A spool of mass m and moment of inertia I is free to roll without slipping on a table. It has an inner radius r and and outer radius R. A string is attached to the inner part of the spool as shown (the string is tangential to the inner spool), and someone pulls on the string with tension T at an angle θ with respect to the horizontal.



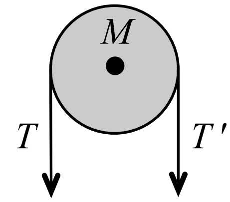
(a) Calculate the linear acceleration, a, of the center of the spool.

(b) What conditions do you have to place on the given parameters to ensure that the spool moves to the right?

<u>Massive Pulleys</u>: How to handle them



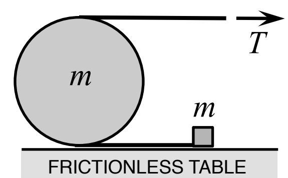
For the next problem, we must discuss something closely related to our work in Discussion 6 where we learned that there must always be **no net force on a massless object,** and similarly, **no net torque on a massless pulley**. For a massless pulley, it has zero moment of inertia so it had *better* have no net torque on it or its angular acceleration will be infinite. FYI: **angular acceleration** is traditionally given the symbol α and is defined to be $\alpha \equiv \dot{\omega} = \ddot{\phi}$. Thus, if you have a massless string running over a <u>massless pulley</u>, the net torque on the pulley must be zero, so the tensions on the emerging string segments must be the <u>same</u>, as shown above.



Suppose instead that you have a massless string running over a <u>massive pulley</u>. For the string to turn this pulley – i.e. change its rotation rate, ω – a <u>non-zero torque</u> is required. Thus for a massive pulley, the tensions on the string segments leaving the pulley are in general **<u>not the same</u>**, hence the labels T and T' in the figure at right.

You may be wondering how we can have unequal tensions on either end of the **massless string** that runs over the pulley. Do these not exert a net force ($T - T'$) on the massless string, causing it to have infinite acceleration?? The answer is no, and the reason is the **static friction** between the massless string and the pulley. Without friction, the string would just slide over the pulley and not cause any rotation. That would be a useless pulley. ☺ Static friction essentially *glues* the string to pulley at the points where they're in contact, and that literally makes the in-contact string segment *part of the pulley*. The significance of the string being *massless* is that it does not alter the pulley's moment of inertia. Hurray! ☺

Problem 5 : Pulling a Mass and a Cylinder



(a) A solid cylinder of mass m and radius r lies on a <u>frictionless</u> horizontal table, with a massless string running halfway around it. A small block of mass m is attached to one end of the string, and you pull on the other end with a horizontal force T. (Since the table is frictionless, it has no effect on the problem except to make gravity irrelevant; you could equally well put the cylinder and block in interstellar space. ☺) The circumference of the cylinder is sufficiently rough that the string does not slip with respect to it. Let x be the horizontal position of the block**,** with the $+x$ direction pointing to the right and the origin placed at any fixed location on the table. What is $\ddot{x}$?

<u>Hint</u>: The most challenging thing about this problem is the **rolling / string-length condition**. It is not the usual no-slip condition because the cylinder <u>*is*</u> *slipping* (it's on a frictionless surface!). The cylinder's motion is a <u>combination</u> of the linear motion of its CM ("sliding" motion) and its rotation around the CM ("rolling" motion). A good way to think about the rolling / string-length condition you need is to think about the amount of string being rolled-up or unrolled-from each side of the cylinder – much as you did in problem 1 or discussion 6 problem 3. Think about the quantity $\dot{x}$ = the rate at which the bottom length of string changes:

- If the cylinder was <u>not rotating at all</u>, $\dot{x}$ would be entirely due to the speed $\dot{X}$ of the cylinder's CM.
- If the cylinder was <u>only rotating</u>, with its CM fixed, $\dot{x}$ would be entirely due to the rate at which string was being rolled up onto the cylinder = removed from the bottom segment.

Turn those words into math, add the two effects together, and you'll have the rolling / string-length condition.

(b) It would be great to use the cylinder's **contact point with the table** as a reference point for $\vec{\tau} = d\vec{L}/dt$ since the torque around that point is simply $2rT$, with no unknowns. Let's see if it works …

- Let (A) be the point <u>on the cylinder</u> that is in instantaneous contact with the table
- Let (B) be the point <u>on the table</u> that is in instantaneous contact with the cylinder

These points are at the same location, so $\tau^{(A)} = \tau^{(B)} = 2rT$. However these points are *not* the same. Using all the information you found in part (a), calculate $dL^{(A)}/dt$ and $dL^{(B)}/dt$. Only <u>one</u> of them will match the torque $2rT$. Explain <u>why</u> the one that works succeeds, and why the one that doesn't work fails.