Physics 325 – Homework #7 **due in 325 homework box by Fri, 1 pm**

All solutions must clearly show the steps and/or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning. Answers given without explanation will not be graded: **"NO WORK = NO POINTS"**. *However you may always use any relation on the 3D-calculus and 1D-math formula sheets without proof. Please write your* **NAME** *and* **<u>DISCUSSION SECTION</u>** *on your solutions.*

Problem 0 : Elastic Collision (was on Homework 3 briefly, if you solved it then use your solution now!)

An **elastic collision** between objects is defined as a collision in which the *nature* of the each object involved is unchanged by the collision. By the *nature* of the objects we mean their *internal* properties such as their shape, heat, and mass. These properties are the same before and after an elastic collision; only the objects' velocities change. The classic example is billiard balls colliding: they are rigid so their shapes do not deform and they don't break apart into pieces (during a normal game of pool, anyway ☺). The consequence of all this is the following operational definition:

> An **elastic collision** is one in which the <u>total kinetic energy</u> of the colliding objects is the <u>same</u> before and after the collision.

This result emerges from the original definition for this reason: when the objects cannot change their nature in any way, their kinetic energy has *no other form into which it can transform*. (e.g. no heat or sound can be generated if the objects cannot deform or vibrate).

Consider an elastic collision between two objects of equal mass, one of which is initially at rest. Let their velocities be $\vec{v}_1$ (non-zero) and $\vec{v}_2 = 0$ before the collision, and $\vec{v}_1^{\,'}$ and $\vec{v}_2^{\,'}$ after the collision. The collision is not necessarily "head-on", so the particles can emerge at all sorts of different angles. Write down the vector equation representing conservation of momentum and the scalar equation that expresses the elastic nature of the collision. Use these to prove that the angle between $\vec{v}_1^{\,'}$ and $\vec{v}_2^{\,'}$ is 90°.

HINT: You can solve this in two ways: by working with vector algebra or by introducing a coordinate system. The first method is more elegant while the second method is more straightforward. If you chose to go with a coordinate system, remember this oft-forgotten truth:

A coordinate system is just a tool, and you are Free to Choose the best one for the job!

In this problem, a moving beam hits a stationary target. How tempting it is to line up one of your axes with the beam – that's the usual choice. You then have two angles to deal with in the final state: for each particle relative to the beam axis. But you don't care about those two angles! You only want the angle *between* the two particles. So make your coordinate system work for you: *run your main axis along one of the <u>final-state</u> particles, not the beam.* ☺

FYI: This result was important in the history of atomic and nuclear physics: two bodies emerging from a collision on perpendicular paths strongly suggests that they had equal mass and scattered elastically.

Problem 1 : Irrotational or Not?

For each of the three force fields below:

- First determine if it is conservative.
- If it *is* conservative, find the corresponding potential energy $U(x,y,z)$ by explicit integration (i.e. using the formula $U(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} \vec{F}\cdot d\vec{l}$), then verify that $\vec{F} = -\vec{\nabla}U$.

In all cases, k is a constant. Also, you are free to choose any point $\vec{r}_0$ for the reference point where U=0, so why not pick the origin. (Usually makes life easier. ☺)

(a) $\vec{F}(\vec{r}) = k(x\,\hat{x} + 2y\,\hat{y} + 3z\,\hat{z})$ (b) $\vec{F}(\vec{r}) = k(y\,\hat{x} + x\,\hat{y})$ (c) $\vec{F}(\vec{r}) = k(-y\,\hat{x} + x\,\hat{y})$

Problem 2 : Clay and Wheel

A circular wheel has radius R, mass M, and moment of inertia I (for rotation around its center). The wheel's mass distribution is not precisely known, but it does have cylindrical symmetry (as a well-fashioned wheel should) so its center-of-mass coincides with its geometric center. The wheel is placed flat on a frictionless table, at rest, with its center at the origin. A small clay ball of mass m is sent toward the wheel with velocity v_0 in the $+x$ direction, traveling along along the line $y = b$. When the ball of clay strikes the edge of the wheel, the two objects stick together.

(a) Calculate the rotational speed ω of the wheel after the clay sticks to it.
Major Hint: At some point you will need the moment of inertia I' of the fused (clay+wheel) system relative to its CM. You must *simplify* this quantity algebraically or the rest of the problem becomes a real mess. Fortunately, it *does* simplify down to something really elegant: $I' = I + \mu R^2$ where $\mu \equiv mM/(m+M)$.

(b) Calculate the initial and final kinetic energies of the system. Is kinetic energy conserved? If it is not conserved in general, find a *limiting case* — a special value for one or more of the given parameters m, M, R, b — where it *is* conserved, and explain why it is conserved in that case. Some notes:

- We are not given enough information to calculate the wheel's moment of inertia I from its mass M and radius R, but for any object, I will clearly be proportional to M and will increase with R. You may thus write $I = M f(R)$ where f is some increasing function of R.
- You can't run b to values greater than R or your solution will be invalid. (We assumed the clay *hit* the wheel.)
- The parameter v_0 is not in the list to avoid the trivial limiting case $v_0 = 0$ (where nothing happens ☺).

(c) Let $t = 0$ be the moment when the clay hits the wheel. Calculate the coordinates $x(t)$ and $y(t)$ of the wheel's geometric center for times $t > 0$.

Problem 3: Falling and Sliding Cube

A cube of side b is balanced on its edge in unstable equilibrium. The cube is resting on a frictionless surface. A tiny (negligible) impulse causes the cube to start to topple: it starts at rest, then under the influence of uniform gravity, it rotates until one of its sides hits the table. Calculate ω^2 = the final angular-velocity squared of the cube just before it hits the table. In this problem and the next, you may use without proof that the moment of inertia of a cube rotating around its center, with rotational axis perpendicular to any of the cube's faces, is $Mb^2/6$. (If you need any other moments of inertia for the cube, you'll have to do a little bit of work.)

HINT: The hard part about this problem is the relation between V, the speed of the center of mass, and ω, the rotational speed of the cube. For rolling cylinders and such, we would get that from a "no-slip rolling" condition like $V = \omega R$ … except this cube *is* slipping and it is not round. The "rolling" condition you need here is — as it always is — purely geometric in nature so what you need is a good sketch of the cube at some intermediate orientation between its initial and final positions. V is the rate of change of the CM position Y, and ω is the rate of change of the cube's angle of rotation ϕ, so label your sketch with the CM coordinate Y and the rotation angle ϕ (defining them relative to whatever you want → who cares what reference position or angle you use since you want a relation between their rates of change?) The sketch will quickly give you the relation you need between V and ω.

Problem 4: Tripping Cube

A cube of side b slides along a flat frictionless table with speed v. The cube then hits a "bump": a very low step that causes the leading lower edge of the cube to come to an abrupt stop. After our discussion in lecture, the phrase "abrupt stop" should immediately tell you that the cube-bump collision is *inelastic* in nature.

(a) Calculate the cube's angular velocity ω immediately after the cube hits the step.

(b) Calculate the minimum speed v for which the cube rolls over after hitting the step (instead of falling back to its original position). HINT: This question concerns the situation *after* the collision with the step, so restart your strategy thinking from the start.

Problem 5 : Compound Pendulum and the "Sweet Spot"

A thin (width-zero) rod with a non-uniform mass density has one end attached to a fixed pivot at the origin. The pivot allows the rod to swing freely in the xy plane, with $+x$ pointing to the right and $+y$ pointing downward (in the same direction as the uniform gravitational field $\vec{g}$). To complete our coordinate system, we also introduce the angle ϕ to denote the distance that the rod makes with the $+y$ axis, with positive ϕ indicating angles in the $+x$ quadrant (i.e., positive ϕ means the rod has swung to the right of vertical). Here is what we know about the rod: its total mass is M, its moment of inertia around the pivot is I, and its center of mass position is a distance R below the pivot.

(a) The term **compound pendulum** is traditionally used to mean any pendulum whose mass is distributed in some way along the length of the pendulum, in contrast to a **simple pendulum** where the mass is concentrated at a single point and connected to the pivot by a massless arm. In Discussion 7, you calculated the small-oscillation frequency of the generic compound pendulum described above and found it to be $\omega = \sqrt{RMg / I}$.
What is the length d of the equivalent simple pendulum with the same mass M and small-oscillation frequency ω? Your answer must of course be expressed in terms of the known parameters of our rod: M, I, and/or R.

(b) You put the rod to rest in a vertical position. Then, at time $t = 0$, you hit the rod sharply at the point $(x, y) = (0, b)$, i.e. at a distance b below the pivot. Your blow delivers an impulse $k = F^{IMP} \Delta t$ to the rod in the $+x$ direction. Calculate the angular momentum $L\big|_{t=0+}$ where "t=0+" is convenient notation for "at the moment immediately after t=0". You may calculate L around any point you choose, just specify the reference point.

(c) Using the angular momentum you just obtained, determine $P\big|_{t=0+}$. Remember that capital P is a gloriously unambiguous variable: as we proved in lecture a while back, it refers to both the "total momentum" *and* the "CM momentum" of a multi-particle system, as they are always the *same thing*.

(d) Calculate the force $\vec{F}^P$ exerted by the pivot during the brief moment that the impulse force, $\vec{F}^{\text{IMP}}$, is active. Express your answer in terms of $\vec{F}^{\text{IMP}}$, the striking-distance b where the impulse is applied, and/or the parameters M, I, R of the rod.

(e) Calculate the striking-distance b_0 where you would have to apply the impulse in order for the pivot to have to deliver no force whatsoever. This special distance is familiar to you if you've ever played baseball: if the baseball hits your bat at its "sweet spot", you won't feel any impulse on your hand.

(f) Take the special case of a rod with uniform mass density. Calculate the sweet-spot b_0 for such a rod in terms of the rod's total length d and/or its total mass M.