Fall 2014 – Physics 325 Midterm Exam #2
Thursday Nov 13, 11:00 am – 12:30 pm

This is a closed book exam. No use of calculators or any other electronic devices is allowed. Work the problems only in your answer booklets only. The exam questions will *not* be collected at the end, so **anything you write on these question pages will NOT be graded**

You have **80 minutes** to work the problems.

At the beginning of the exam:
1) Write your **name** and **netid** on your answer booklet(s).
2) Turn your **cell phone off**.
3) Put away all calculators, phones, computers, notes, and books.

During the exam:
1) **Show your work** and/or reasoning. **Answers with no work or explanation get no points.** But ...
2) **Don't write long essays** explaining your reasoning. We only need to see enough work to confirm that you understand what you're doing and are **not just guessing**. Also a good **annotated sketch** is often the best explanation of all!
3) **All question parts on this exam are independent**: you can get full points on any part even if your answers to all the other parts are incorrect. Therefore you should **attempt all the question parts!** If you get stuck, move on to the next one and come back later. The worst thing you can do is stall on one question and not get to others whose solution may be very simple.
4) Partial credit will be given for incorrect answers if the work is understandable and some of it is correct. IMPORTANT: If you think you've made a mistake but can't find it, **explain what you think is wrong** ➔ you may well get partial credit for noticing your error!
5) It is fine to leave answers as **radicals or irreducible fractions** (e.g. $10\sqrt{3}$ or 5/7).
6) Specify the **units** for all numerical answers.

Remember: There are many **Math Tables** provided

When you're done with the exam:
Turn in your **answer booklet**. (You can keep the exam questions).

Academic Integrity:

The giving of assistance to or receiving of assistance from another person, or the use of unauthorized materials during University Examinations can be grounds for disciplinary action, up to and including expulsion from the University.

Please be aware that prior to or during an examination, the instructional staff may wish to rearrange the student seating. Such action does not mean that anyone is suspected of inappropriate behavior.

You may use without proof these Moments of Inertia:

- **Thin rod** of uniform mass m and length d rotating around its **center**: $I = md^2/12$
- **Thin rod** of uniform mass m and length d rotating around its **end**: $I = md^2/3$

  In both of the above cases, the axis of rotation $\hat{\omega}$ is perpendicular to the rod.
- **Solid cylinder** of mass m and radius r around its **axis**: $I = mr^2/2$

Problem 1 : Bead on a Spinning Parabolic Wire

A bead of mass m is threaded onto a wire that is bent into the shape of a parabola: $z = s^2/a$ with a constant. (s is the usual radial coordinate from the cylindrical coordinate system.) The bead moves along this wire without friction. The wire is rotated around the z axis with constant angular speed Ω, while uniform gravity g points in the $-z$ direction.

(a) Calculate the Lagrangian $L(s,\dot{s},t)$ for this system.

(b) No matter what you got for (a), **use the following Lagrangian for the remaining parts**:

$$L(s,\dot{s},t) = A\left[\dot{s}^2 + Bs^2\dot{s}^2 + Cs^2\right]$$

Using this Lagrangian, show that $s = 0$ is an equilibrium position.

(c) Determine any condition(s) that must be imposed on the values A, B, and/or C to make $s = 0$ a stable equilibrium position, and assuming those conditions are true, find the frequency of small oscillations around $s = 0$.

Problem 2 : Geodesics on the Earth's Surface

(a) Consider a spherical surface that is centered on the origin and has radius R (e.g. the surface of the Earth). Find but **DO NOT SOLVE** the differential equation that describes the geodesic path $\theta(\phi)$ on this surface between generic endpoints (θ_1,ϕ_1) and (θ_2,ϕ_2). Also, you **DO NOT HAVE TO PERFORM ALL DERIVATIVES**, e.g. leaving your answer in the form $f = \frac{d}{dx}[g]$ is fine, where x is one of your coordinates and f and g are functions of your coordinates and/or their derivatives.

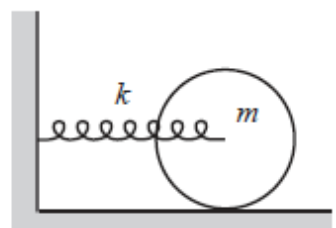
(b) Write down a specific path $\theta(\phi)$ that you *know* to be a geodesic of the spherical surface, and show that the differential equation you found in (a) does work for this path. If you could not answer (a) at all, then explain in some other way why the specific path you chose is a geodesic.

Problem 3 : The Cost of Doing Business

A factory has to produce N widgets in an amount of time T. Let $n(t)$ be the total number of widgets produced by time t. The production costs are $(\alpha + \beta^2 \dot{n})$ per widget plus $(\gamma^2 n^2 + \delta)$ per unit time, where $\alpha, \beta, \gamma, \delta$ are constants. Calculate the production schedule $n(t)$ that will minimize the total cost, given that production starts at time $t = 0$.

Problem 4 : Spring and Rolling Cylinder

The axle of a solid cylinder of mass m and radius r is connected to a spring with spring-constant k and an unstretched length of zero (i.e., the magnitude of the spring force is kx, where x is the length of the spring). The cylinder rolls without slipping on the surface beneath it.



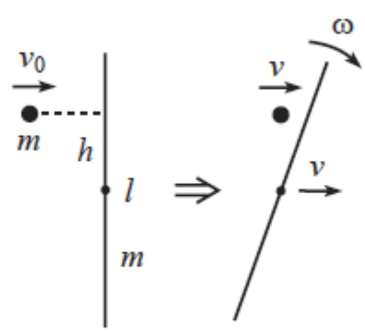
(a) Write down the Lagrangian $L(x, \dot{x}, t)$ for this system where x is the length of the spring.

(b) **Based ONLY on your Lagrangian** — not your knowledge of the system — calculate one constant of motion in terms of x, $\dot{x}$, and the given constants, and describe its physical meaning in words. If your Lagrangian does not give you any constant of motion, explain why.

(c) Using any method you want, determine the general solution $x(t)$ for the motion of the system.

Problem 5 : An Elastic Collision

A small ball of mass m travels to the right at speed v_0 toward a vertical stick of length l and mass m (same mass as the ball). The stick is initially at rest. The ball collides **elastically** with the stick, striking it at a position that is a distance h above the stick's center.



Calculate the value of h that causes both the stick and the ball to move to the right at the same speed after the collision (denoted v in the right-hand part of the figure).