

$$|\vec{v}| \equiv \sqrt{\vec{v} \cdot \vec{v}} \quad \vec{v} = \sum_{i=1}^3 (\vec{v} \cdot \hat{r}_i) \hat{r}_i \quad d\vec{l}_{path} = \frac{d\vec{l}}{du} du \quad d\vec{A} = \left(\frac{\partial \vec{l}}{\partial u} \times \frac{\partial \vec{l}}{\partial v} \right) du dv$$

Conceptual version: $d\vec{l}_{path} = d\vec{l}_u$
 $d\vec{A} = d\vec{l}_u \times d\vec{l}_v$
 $dV = (d\vec{l}_u \times d\vec{l}_v) \cdot d\vec{l}_w$

$$df(x_1, \dots, x_n) = \sum_{i=1}^n \frac{\partial f}{\partial x_i} dx_i \quad dV = \left(\frac{\partial \vec{l}}{\partial u} \times \frac{\partial \vec{l}}{\partial v} \right) \cdot \frac{\partial \vec{l}}{\partial w} du dv dw$$

Taylor

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

1st order approx for $x \ll 1$:

- $(1+x)^n \approx 1+nx$
- $\sin x \approx x$
- $\cos x \approx 1 - \frac{x^2}{2}$
- $\tan x \approx x$
- $e^x \approx 1+x$
- $\sin^{-1} x \approx x$
- $\cos^{-1} x \approx \frac{\pi}{2} - x$
- $\tan^{-1} x \approx x$
- $\ln(1+x) \approx x$

	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

$$\sin a \sin b = \frac{1}{2} [\cos(a-b) - \cos(a+b)]$$

$$\cos a \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$\sin a \cos b = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$$

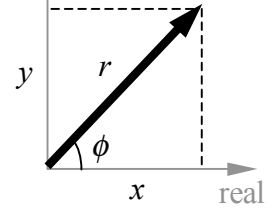
$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

Complex Numbers

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\tilde{z} = x + iy = r e^{i\theta}$$



$$\tilde{z}^* \equiv x - iy = r e^{-i\theta}$$

$$|\tilde{z}| \equiv \sqrt{\tilde{z}^* \tilde{z}} = r$$

$$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}} \quad \sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{1-\cos\theta}{2}}$$

Integral Table

$$\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a} \cos^{-1}\left(\frac{a}{x}\right) \quad \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left(x + \sqrt{x^2 \pm a^2}\right) \quad \int \frac{x dx}{\sqrt{a^2 \pm x^2}} = \pm \sqrt{a^2 \pm x^2}$$

$$\int_0^{2\pi} \sin^2 \phi d\phi = \int_0^{2\pi} \cos^2 \phi d\phi = \pi \quad \int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \quad \int \frac{x dx}{(a \pm x)^2} = \frac{a}{a \pm x} + \frac{1}{2} \ln(a \pm x)$$

$$\int \sin^2 \phi d\phi = \frac{\phi}{2} - \frac{\sin(2\phi)}{4} \quad \int \frac{dx}{(a \pm x)^2} = \mp \frac{1}{a \pm x} \quad \int \frac{x dx}{a^2 \pm x^2} = \pm \frac{1}{2} \ln(a^2 \pm x^2)$$

$$\int \cos^2 \phi d\phi = \frac{\phi}{2} + \frac{\sin(2\phi)}{4} \quad \int \frac{dx}{a^2+x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \quad \int \frac{x dx}{(a^2 \pm x^2)^{3/2}} = \mp \frac{1}{\sqrt{a^2 \pm x^2}}$$

$$\int \sin^3 \theta d\theta = \frac{\cos^3 \theta}{3} - \cos \theta \quad \int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln\left(\frac{a+x}{a-x}\right) \quad \int \ln(ax) dx = x \ln(ax) - x$$

$$\int \cos^3 \theta d\theta = \sin \theta - \frac{\sin^3 \theta}{3} \quad \int \frac{dx}{(a^2 \pm x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 \pm x^2}} \quad \int \frac{\ln(ax)}{x} dx = \frac{1}{2} [\ln(ax)]^2$$

$$\int \sqrt{a^2-x^2} dx = \frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right) \quad \int \frac{x^2}{\sqrt{a^2-x^2}} dx = -\frac{x}{2} \sqrt{a^2-x^2} + \frac{a^2}{2} \tan^{-1}\left(\frac{x}{\sqrt{a^2-x^2}}\right)$$

$$\int \sqrt{x^2 \pm a^2} dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln|x + \sqrt{x^2 \pm a^2}| \quad \int \frac{(x-a \cos \theta) \sin \theta d\theta}{(x^2+a^2-2ax \cos \theta)^{3/2}} = \frac{1}{x^2} \frac{a-x \cos \theta}{\sqrt{x^2+a^2-2ax \cos \theta}}$$