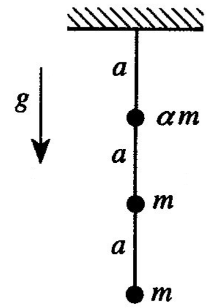
Physics 326 – Homework #2 due Friday, 1 pm

All solutions must clearly show the steps and/or reasoning you used to arrive at your result. You will lose points for poorly written solutions or incorrect reasoning. Answers given without explanation will not be graded: ***"No Work = No Points"****. However you may always use any relation on the 1DMath, 3DMath or exam formula sheets or derived in lecture / discussion. Write your* **NAME** *and* **DISCUSSION SECTION** *on your solutions.*

Problem 1 : Triple Pendulum (a classic qual-exam problem)

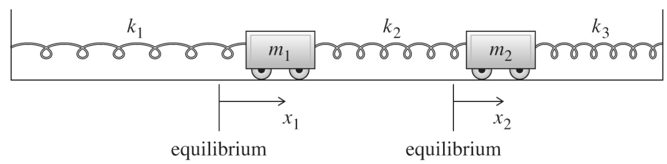


A triple pendulum consists of masses αm, m, and m suspended to each other in a line by three massless rods of length a. The whole thing is suspended from a stationary pivot, as shown in the figure. Throughout this problem, you may assume that the displacements of the masses from equilibrium are small.

(a) Find the value of α such that one of the normal frequencies of this system will equal the frequency of a simple pendulum of length $a/2$ and mass m.

(b) Find the mode corresponding to this frequency and sketch it.

Problem 2 : Coupled and Damped



In general, the analysis of couple oscillators with dissipative forces is much more complicated than the conservative cases we have considered. However, there are a few cases where the same methods still work.

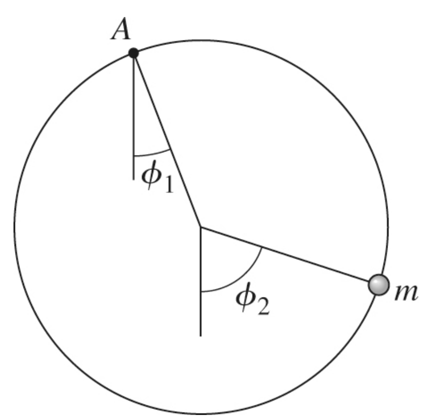
(a) Write down the equations of motion for the standard 2-mass-3-spring system, with equal masses $m_1=m_2=m$ and equal springs $k_1=k_2=k_3=k$, but with each mass subject to a linear damping force $-b\vec{v} = -2\beta m\vec{v}$ (same β for both masses).

(b) Use the method of normal coordinates to solve this problem, i.e. identify two coordinates ξ_+ and ξ_- that are linear combinations of x_1 and x_2 that decouple the equations of motion.

(c) Using damped 1D-oscillator skills, solve your decoupled equations from (b) to obtain the general solutions for the normal coordinates, $\xi_+(t)$ and $\xi_-(t)$. Assume that b is small, so that the oscillations are underdamped.

(d) Find $x_1(t)$ and $x_2(t)$ for the initial conditions $x_1(0) = A$ and $x_2(0) = v_1(0) = v_2(0) = 0$.

Problem 3 : Bead on a Swinging Hoop



A bead of mass m is threaded on a frictionless circular wire hoop of radius R and the same mass m as the hoop. The hoop is suspended at the point A and is free to swing in its own vertical plane as shown in the figure.

(a) Using the angles ϕ_1 and ϕ_2 as generalized coordinates, solve for the normal frequencies of small oscillations, and find and describe (e.g. with sketches) the motion in the corresponding normal modes.

(b) Find two sets of initial conditions that allow the system to oscillate in each of its two normal modes.

Problem 4 : Pendulum on a Cart

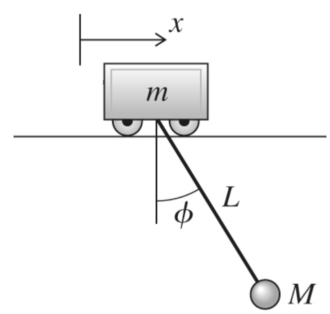
A simple pendulum (mass M and length L) is suspended from a cart of mass m that moves freely along a horizontal track. You will find it helpful to introduce the dimensionless parameters $\eta \equiv m/M$ and $\omega_0 = \sqrt{g/L}$.

(a) What are the normal frequencies of small oscillations of the system?

(b) Find and describe (e.g. with sketches) the corresponding normal modes of the system.
If you are doing this before the lecture on **DC modes**, please read the brief Appendix.

(c) The cart / pendulum system is held at rest in the configuration $x = 0$ and $\phi = \phi_0$, where ϕ_0 is small. At time $t = 0$, the system is released from rest. Write down the subsequent motion $x(t)$ of the cart and $\phi(t)$ of the pendulum.

Appendix : DC Modes



If you find that one of the eigenfrequencies, ω, of your system is <u>zero</u> then your system has a **zero-frequency mode**, a.k.a. a **DC mode**. The normal-mode solution form is $q_i(t) = A_i\cos(\omega t - \delta)$; if you plug ω=0 into that, you get a solution of all constants: $q_i(t) = A_i\cos(\delta) = B_i$. That's not enough free parameters: we need *two* free parameters *per coordinate*, and we only have one! What to do? → Go back to one step *after* we postulated normal mode form, when we wrote down $\ddot{\vec{q}} = -\omega^2\vec{q}$. If you have a DC mode, then $\ddot{\vec{q}} = 0$ → none of the coordinates are accelerating at all in this mode. Well we know the solution of $\ddot{q}_i = 0 \;\rightarrow\; q_i(t) = A_i + B_i t$.
That's got the two free parameters we need, great. That's the solution form you use for a DC mode.