Physics 326 – Homework #13 Part B due **WEDNESDAY May 4, 5 pm**

Fourier Series as an Inner Product Space

- **Space** : $|f\rangle \equiv$ **τ-periodic functions** $f(t)$ that are periodic over $t=[-\tau/2 \to \tau/2]$, with $\omega = 2\pi/\tau$
- $\mathbb{R}$ **Real Basis** : $|n\rangle \equiv \begin{cases} \sin(n\omega t) & n=1,\ldots,\infty \\ 1/\sqrt{2} & n=0 \\ \cos(n\omega t) & n=-1,\ldots,-\infty \end{cases}$
- $\mathbb{R}$ **Inner Product** : $\langle g|f\rangle \equiv \frac{2}{\tau}\int_{-\tau/2}^{\tau/2} g(t)f(t)dt$
- $\mathbb{C}$ **Complex Basis** : $|n\rangle = e^{in\omega t}$
- $\mathbb{C}$ **Inner Product** : $\langle \tilde{g}|\tilde{f}\rangle \equiv \frac{1}{\tau}\int_{-\tau/2}^{\tau/2} g^*(t)f(t)dt$

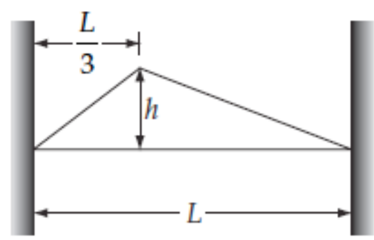
➔ **Basis is Orthonormal** : $\langle n|m\rangle = \delta_{nm}$

➔ **Completeness** : any $|f\rangle = \sum_{n=-\infty}^{+\infty} |n\rangle\langle n|f\rangle$

An instructive way to solve problems 1 and 2 is to use **Mathematica**: download the Mathematica example notebook from lecture 14A and use it as a template for your solutions. **SADLY** it appears that webstore.illinois.edu is **CHARGING** $17.50 for the program. :-(Of course you can use wolframalpha.com to do the calculations and plots as usual, but if you would like to try out the more powerful program behind the web interface, you can access Mathematica for free using the EWS (Engineering WorkStation) machines – thanks to Yifan for the tip! You can access these machines at various physical computer labs or remotely; instructions at https://it.engineering.illinois.edu/ews.

Problem 1 : Guitar String

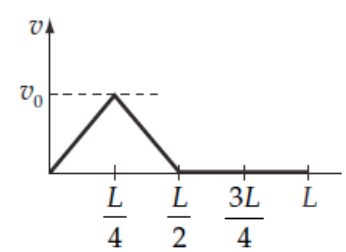
(a) At time $t=0$, a guitar string of length L is pulled into the shape $y(x,t=0)$ shown in the figure with a velocity of zero at all points. Given that the speed of transverse-wave propagation along this string is c, calculate the time-dependent motion of the string up to the fifth harmonic, i.e. calculate the amplitudes of the first five normal modes, of frequency ω_m where $m = 1$ through 5.
(Jargon: The first harmonic of an vibrating string is called the "fundamental"; the mode of next higher frequency is the second harmonic.)



(b) Plot the result at these four times: $t = 0, \tau/4, \tau/2, 3\tau/4$ where τ is the period of the string's vibration.

Problem 2 : Piano String

(a) A piano string of length L is struck by a hammer one quarter of the way along its length so that its initial transverse position $y(x,t=0)$ is zero everywhere, but its initial velocity $\dot{y}(x,t=0)$ is given by the distribution shown in the figure. Calculate the time-dependent motion of the string up to the fifth harmonic, given that the speed of transverse wave propagation is c.



(b) Plot the motion of the string by showing its shape in eight time-steps: at $t = 0, \tau/8, 2\tau/8, 3\tau/8, \ldots 7\tau/8$.