The new invariants

$E = mc^2$

What does “mass” mean?

World lines

4-dimensional physics

Causality

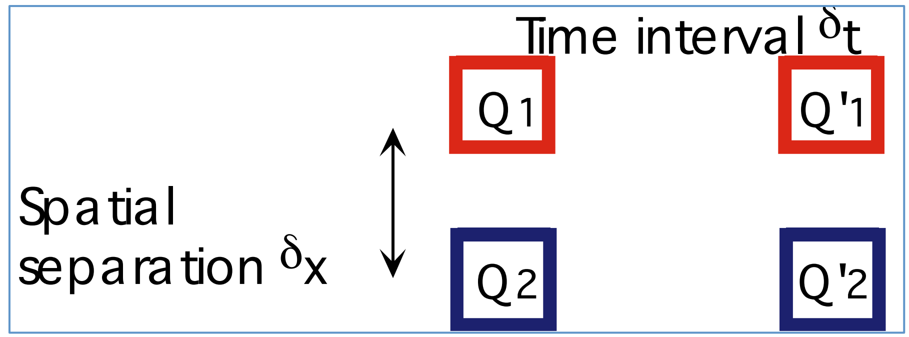
<u>Next</u>:

The twin "paradox"

Accelerated reference frames and general relativity

The locality of conservation laws

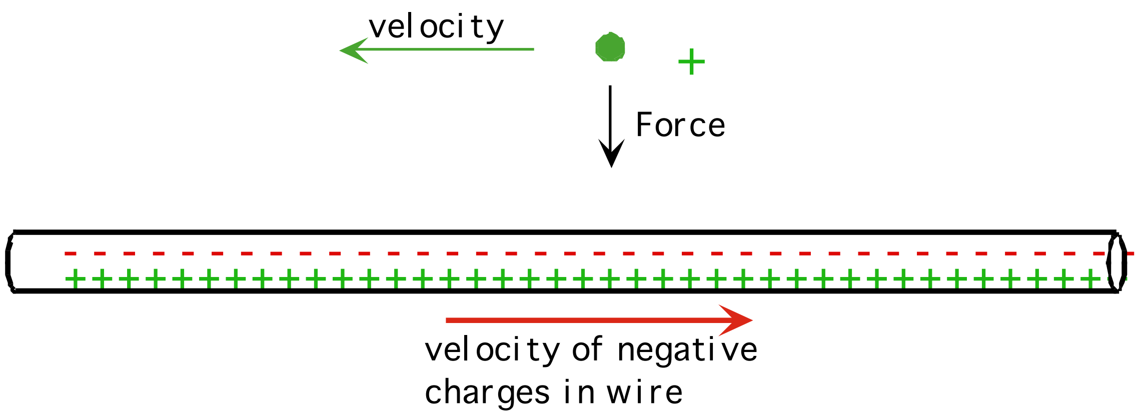
- We have seen various conserved quantities such as energy, momentum and electric charge. Their conservation laws remain valid (in new form) in special relativity. SR makes these laws *even more strict* than they were before.
- Consider a hypothetical process with conserved quantity, Q:



- Is it possible in a short time interval ($dx/dt > c$) for some Q to be transferred a long distance from box 1 to box 2?
  - We want to conserve Q: $Q_1+Q_2 = Q'_1+Q'_2$.
- Newton's physics does not forbid such a process, but SR does. This follows from the relativity of simultaneity. Suppose that I see the transfer happen at a particular time. An observer moving w.r.t. me will say that the Q changes at the two boxes at two different times. So he says there is a time interval during which Q was not conserved. This violates the principle of relativity – both observers must obtain the same laws.
- Thus, conservation laws only work if they are *local*. Q cannot hop around. It must move continuously from one place to another, no faster than c.

Unification of electricity and magnetism

Einstein's one simple postulate solves a lot of problems. Consider the magnetic force on a moving charge due to the electric current in an electrically neutral wire (no electric field):



The magnetic force occurs when the charge is moving. If we look at it from the charge's point of view (*i.e.*, in its own "rest frame"), there can't be a magnetic force on it, but there must be some kind of force, because the charge is accelerating.

So, the principle of relativity tells us that the charge must see an electric field in its rest frame. (Why must it be an electric field?) How can that be? The answer comes from Lorentz contraction. The distances between the + and - charges in the wire are Lorentz contracted by different amounts because they have different velocities. The wire appears to have an electrical charge density. (The net charge in a current loop will still be zero, but the opposite charge is found on the distant part of the loop, where the current flows the opposite direction.)

Relativity of Fields

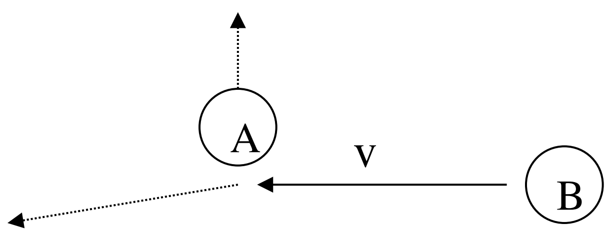
- When we change reference frames, electric fields partially become magnetic fields, and vice versa. Thus, they are merely different manifestations of the same phenomenon, called electromagnetism.
- The first oddity of Maxwell's equations was that the magnetic force existed between moving charges. But now we say that there's no absolute definition of moving.
- The resolution is that whether the force between two objects is called electric or magnetic is also not invariant.

Relativity is a Law

- Relativity might have sounded like some vague "everything goes" claim initially- at least in the popular press. Now we are deriving specific new physical laws from it.
- Relativity is a *constraint* on the physical laws. It says "No future physical law will be found which takes on different forms in different inertial frames."
- And "future laws" in 1905 include all the laws concerning nuclear forces, the form of quantum theory, … So far, the constraint holds!

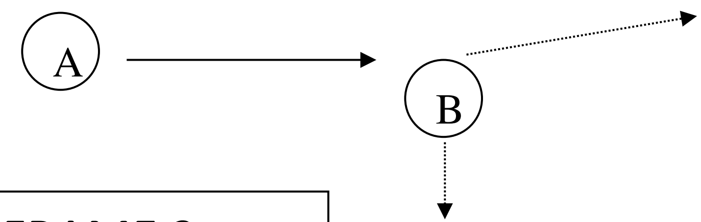
Conservation of momentum

- Consider a collision between two disks A and B, with the same rest mass, m_0. We will look at this collision in 2 frames:
  - The frame of A before the collision.
  - The frame of B before the collision, moving at some big v wrt. A.

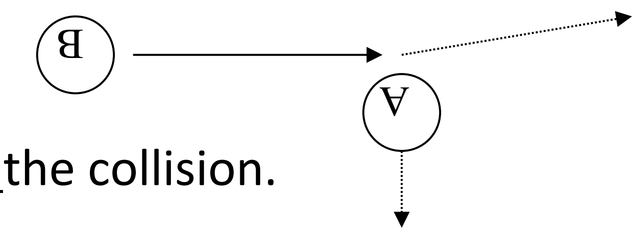


FRAME 1

We pay attention to momentum, p, only along this axis : Initially, there's no p on that axis.



FRAME 2



p_{A2} means momentum of A as seen in frame 2 <u>after</u> the collision.

$p_{A1} = - p_{B2}$ and $p_{A2} = -p_{B1}$ (symmetry)

but $p_{A1} = -p_{B1}$ and $p_{A2} = - p_{B2}$ (conservation)

so $p_{A1} = p_{A2}$ and $p_{B1} = p_{B2}$

How inertial m changes with v.

- Now we know that the momentum (along the deflection direction) of disk A is the same in the frame initially at rest with respect to A and the frame initially at rest with respect to B.

- But momentum is mass*distance/time.

- The distances must be the same in both frames. (Why?)

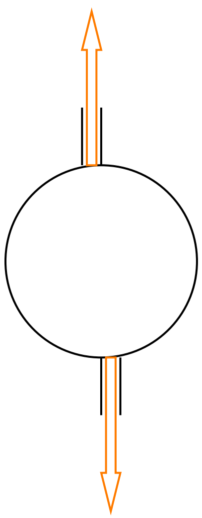
- The elapsed times in the two frames differ by a factor of g, $1/(1-\beta^2)^{1/2}$, so the mass assigned to disk A in the frame initially moving with respect to A must be γ times as big as the mass assigned to it by frame 1, initially at rest wrt A. So long as we consider the case where the deflection velocity is small, we don't have to distinguish between m in frame 1 and in the frame in which A is now at rest.
- So the frame moving at v wrt A sees A's inertial mass increased over the rest mass by the factor g.

Conservation of Momentum

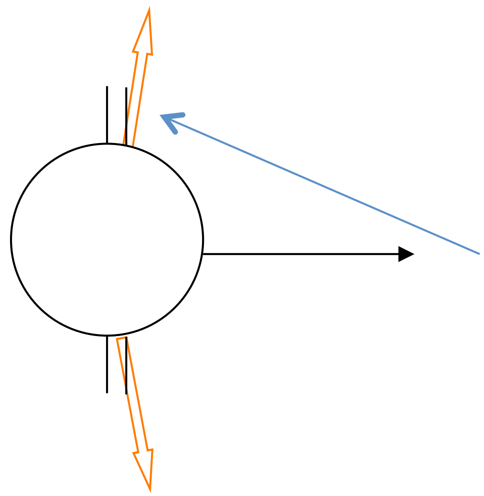
- Assuming conservation of momentum led to the requirement that the m in p=mv is not invariant. $m=\gamma m_0$
- m_0 is the "rest mass" seen by an observer at rest wrt the object.
- Warning: there's another convention, also in common use, to let "mass" mean what we here call the rest mass, not the inertial mass used above. If you see some apparently contradictory statements in texts, probably that's because they use this other convention.

Energy and momentum

- Consider a star, with two blackened tubes pointing out opposite directions. Light escapes out the tubes, but only if it goes straight out. It carries momentum and energy, known (by Maxwell) to obey $E=pc$.



Now what happens if the star is moving at velocity v to our right?



In order for the light to get out of the moving tubes without hitting the edges, it must be going forward a bit (angle v/c)
So net forward momentum is lost to the light. The lost momentum is: $(v/c)\,\Delta E/c$, where ΔE is the lost energy.

If we assume that total momentum is conserved:
$v(\Delta m) + (\Delta v)\, m = (v/c^2)\, \Delta E$.
So we have a relation between the lost mass Δm, the lost speed Δv and the lost energy ΔE. Can we scrounge another equation in these, so we can express Δm and Δv each in terms of just ΔE? What else do we know about these changes?

What's Δv?

- The one thing you know easily is $\Delta v = 0$, because our choosing some reference frame with can't make the star accelerate.
- There is no preferred reference frame:
  - we mean it and we use it all the time in calculations.

- So the Δv drops out of: $v(\Delta m) + (\Delta v)\, m = (v/c^2)\, \Delta E$.
- giving: $\Delta m = \Delta E/c^2$
- A reasonable extrapolation is to drop the delta so
- $m = E/c^2$

- <u>This does NOT say that "mass is convertible to energy". It says that inertial mass and energy are two different words for the same thing, measured in units that differ by a factor of c^2</u>.
- This applies to <u>inertial</u> m. *Rest mass* is only a part of that.

Kinetic energy

- So a moving object has energy $E = \gamma m_0 c^2$.
- How does this connect with our usual conception of energy? In classical physics, the kinetic energy is $KE = 1/2\ mv^2$.
- The time dilation factor : $\gamma = \frac{1}{\sqrt{1-\beta^2}}$
- For small b, this is approximately, $\gamma \approx 1 + \frac{1}{2}\beta^2$
- Thus: $E \approx m_0 c^2 + \frac{1}{2} m_0 v^2$
- The second term is just Newtonian KE. The m_0c^2 term is the energy a massive object has just by existing.
- As long as rest mass is conserved, the m_0c^2 energy is constant and therefore hidden from view. We'll see that rest mass can change, so this energy can be significant.
- Remember that as $\beta \rightarrow 1, \gamma \rightarrow \infty$. So an object's energy $\rightarrow \infty$. This is a reason why c is the speed limit. It takes an ∞ amount of energy to get there. As you push on an object, $v \rightarrow c$ asymptotically. That is, $F \neq ma$.

An invariant is lost and another gained

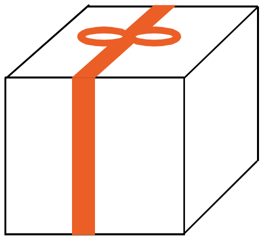
- By assuming the correctness of Maxwell's equations and the principle of relativity we have shown inertial m must depend on the reference frame.
- Lorentz and Poincare' got their speed-dependent m's from essentially the same argument, but using that the laws of physics "look" the same in either frame, not that they are the same.

- So inertial mass is not an invariant.
  - So what's "real" about an object, i.e. not dependent on how you look at it?
- Old invariant: m
- New invariant: $E^2 - p^2c^2 = m_0^2c^4$

Photons (light) have no rest mass

- Newtonian physics does not allow massless objects. They would always have zero energy and momentum, and would be unobservable.
- Now in SR imagine an object with zero invariant mass:
  $E^2 = c^2p^2$ so E=pc, like for Maxwell's light. Any object with zero invariant mass moves at the speed of light. Gluons are also supposed to be massless.
- Any object moving at the speed of light has *zero* invariant mass, otherwise its energy would be infinite.
- All colors of light (and radio pulses, etc.) from distant objects (e.g. quasars) are found to get to us after the same transit time.
  - But now some versions of string theory claim that very high-frequency light might show some slight frequency-dependent speed, in a testable range!)

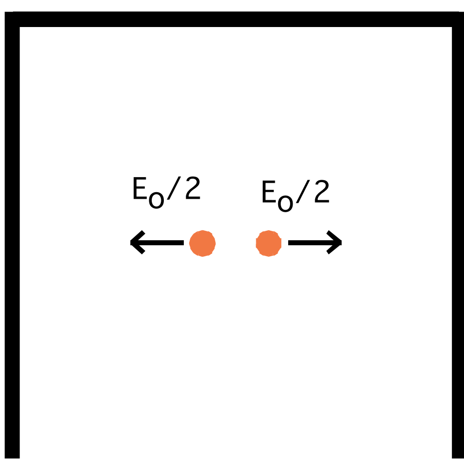
What does "rest mass" mean?



Suppose I have a box with some unknown stuff inside. I want to learn something about what that stuff is by measuring its properties, but I'm not allowed to open the box until my birthday. What can I learn?

- I can measure the energy and momentum of the stuff inside by letting the box collide with other objects (assume the box itself to be very light so we can ignore its energy and momentum). Suppose that when the box is at rest (p=0), I measure energy E_o. So the "rest mass" of the stuff is given by $E_o/c^2 = m_0$.

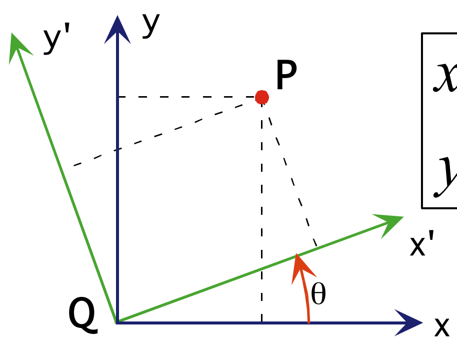
I open the box, only to find two photons bouncing back and forth. Each photon has energy $E = E_0/2$, and since they are moving opposite directions, their momenta cancel ($\mathbf{p} = 0$).



The rest mass of a collection of objects does not equal the sum of their individual rest masses, even if they don't interact. (unlike inertial mass)
Newton's concept of mass as "quantity of matter" is gone, although it often remains a good approximation. It's replaced by a Lorentz invariant relationship between energy and momentum.

4-dimensional spacetime

- Three-dimensional geometry becomes a chapter in four-dimensional physics. ... Space and time are to fade away into the shadows. (Minkowski, 1908)
- The geometrical interpretation of SR is based on the similarity between rotations and Lorentz transformations. Take two coordinate systems, rotated with respect to each other:



$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

Coordinate rotation doesn't change the distance between points **P** and **Q**: $x'^2 + y'^2 = x^2 + y^2$. $\sin^2 q + \cos^2 q = 1$ expresses this **invariance of distance under rotations**. The two people get different x and y, but agree about d.

The Lorentz transformation looks ~like a rotation:

(I'm ignoring y and z.)

$$x' = \gamma(x - \beta ct) = \gamma x - \gamma\beta ct$$
$$ct' = \gamma(ct - \beta x) = -\gamma\beta x + \gamma ct$$

You can verify that $(ct')^2 - x'^2 = (ct)^2 - x^2$. Although two observers measure different lengths and time intervals, they agree on the value of this quantity, the "**interval**".

(Note the minus sign, it's defined to be more like a time interval than a space interval.)

Relativity is full of invariants

they just aren't the ones you expected.

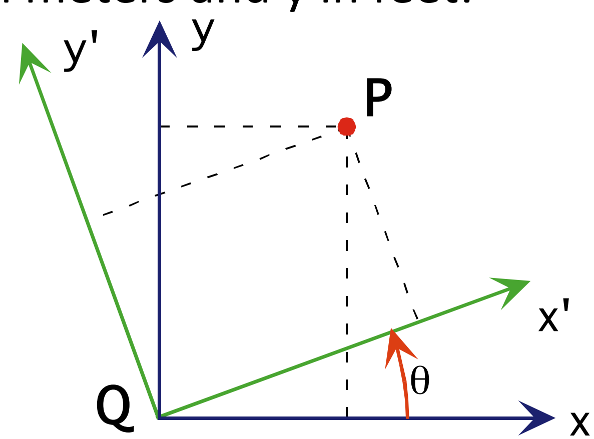
- Minkowski interpreted the invariant interval as a geometrical quantity in a non-Euclidean geometry. It has quantities similar to the trigonometric functions, called hyperbolic trigonometric functions (*e.g.*, hyperbolic sine, *etc*.).
- Using this mathematics, we can interpret the Lorentz transformation as a non-Euclidean "rotation":

$$x' = x\cosh\theta - ct\sinh\theta$$
$$ct' = -x\sinh\theta + ct\cosh\theta$$

Except for the - sign, this is like a rotation. That - sign is the indicator the time dimension is not like the 3 space dimensions.

The universal speed, c, now has a geometrical meaning: the conversion factor between space and time units. Suppose we measured x in meters and y in feet:

With these units, the quantity $x^2 + y^2$ is not invariant under rotations. In fact, until we make the units agree, we can't even combine them. We must multiply y by a conversion factor, k, which is the number of meters per foot. Then $x^2 + (ky)^2$ is invariant.



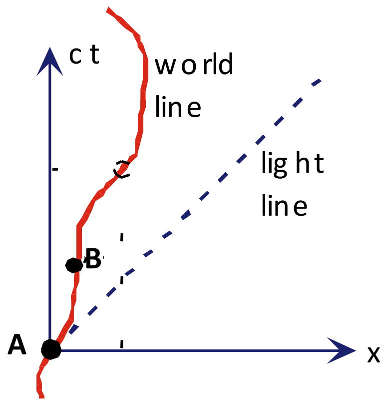
<u>4-dimensional physics</u>

- The principle of relativity requires that
  if the laws of physics are to be the same in every inertial reference frame, the quantities on both sides of an = sign must undergo the same Lorentz transformation so they stay equal.
  <u>You cannot make any invariant from space or time variables alone</u>.
  That's why we call the SR world 4-D, and call the old world 3-D + time. No true feature of the world itself is representable in the 3 spatial dimensions or the 1 time dimension separately.
- In Newtonian physics, **p**=m**v** (**bold** means vector). Momentum and velocity are vectors, and mass is a scalar (invariant) under 3-d rotations. This equation is valid even when we rotate our coordinates, because both sides of the equation are vectors.
- The new "momentum" is a 4-d vector (4-vector for short). It's fourth component is E/c, the energy.
  - The factor of c is needed to give it the same units as momentum.
- The lengths of 3-vectors remain unchanged under rotations. So does the invariant "length" of 4-vectors under *Lorentz transformations*. The length$^{\mathbf{2}}$ of a 4-vector is the square of its "time" component minus the square of its space component: $(E/c)^2 - p^2 = (m_0c^2)^2$

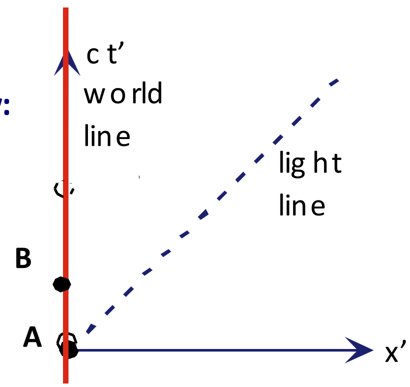
4-D geometry

- In the geometrical interpretation of SR, c is just a *conversion factor*, the number of meters per second. The geometrical interpretation of SR helped lead Einstein to general relativity, although it didn't directly change the physics.
- World lines A graph of an object's position versus time:

Our view:



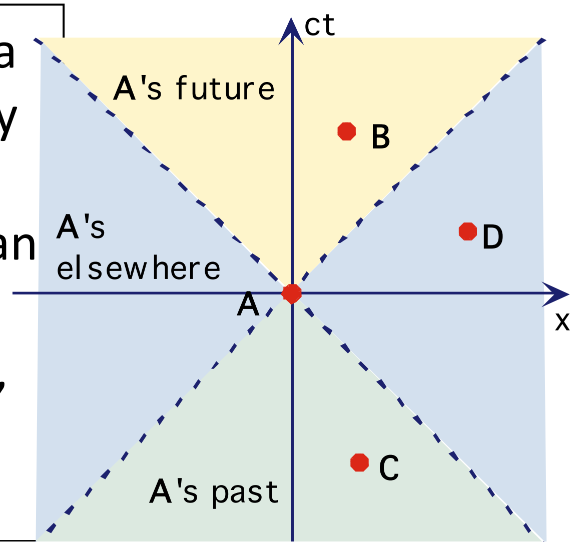
His view:



If an object is at rest in any inertial reference frame, its speed is less than c in every reference frame. The speed limit divides the spacetime diagram into causally distinct regions.

Lorentz_transform_of_world_line.gif

A, **B**, **C**, & D are events. **A** might be a cause of **B**, since effects produced by **A** can propagate to **B**. They cannot get to **D** without travelling faster than light, nor to **C** because it occurs before **A**. **C** might be a cause of **A**, **B**, and/or **D**. **D** could be a cause of **B**, since light can get from **D** to **B**.



If the interval, $(ct)^2 - x^2$, between pairs of events is positive ("timelike"), then a causal connection is possible. If it is negative ("spacelike"), then not.

Causality in Special relativity

- Strong form:
  No event can be affected in any way by events outside its past light cone.

- Weak form: No information may be transmitted except forward within a light cone.

- Weaker form: No information can be transmitted except within a light cone.

- You may wonder why we make such pointless distinctions. Can't *any* "effect" be used to transmit information? Stay tuned.

- In a deterministic world, the Strong form would mean that an event would be completely predictable on the basis of knowledge of its past cone alone. Observations OUTSIDE the past light cone might provide the same info in more convenient form, but would never be *needed*, because everything knowable about the event would be determined by the preceding events in the light cone.

- What about in a world where things are not completely predictable on the basis of *anything*? The Strong form would mean that one could find within the past light cone enough information to obtain *as much predictive accuracy as possible* about an event.

What does "Nothing can travel faster than the speed of light" mean?

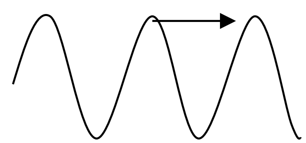
We know that

- no ordinary mass can go faster, because that would require infinite energy.
- no conserved quantity can go faster, because then it would not be conserved in some reference frames.
- If we believe that causation must go forward in time, then we know that no "information" can go faster than c, because that would allow backwards-in-time causation.
  - What happens if you can send info backward? Say you send your grandma info that somebody much cuter than your grandpa was about to move into her neighborhood. Then you aren't born. Then the info doesn't get sent. So you are born, so ....

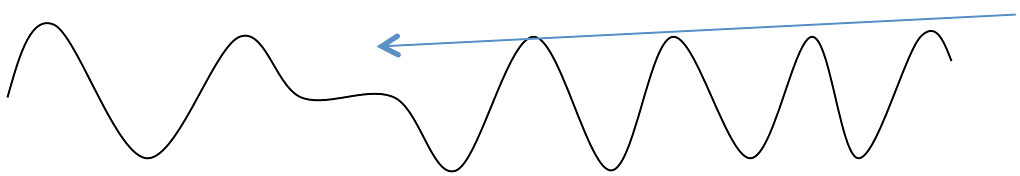
What does "no object travels faster than c“ mean?

If "no object travels faster than c", then the following aren't objects:

- The bright spot made by a beacon shining on a wall.
- The cutting point of a scissors.
- The crest of an E-M wave in matter. (Certain materials have index of refraction less than 1 over some frequency range, hence a "phase velocity" greater than c for some light.)



The repetitive pattern carries no info!



Only the *breaks* in the repeating pattern must travel slower than c.

What are we then claiming?

If we are to describe the world as having some primary constituents, with various higher-level phenomena just being patterns in the constituents' behavior, we want to restrict the primary constituents to those which don't travel faster than light. We claim there exists *some* complete description of the world in terms of constituents which don't travel faster than c.

Causality in Special relativity

- <u>Things:</u>
  - One version of positivism tried to reduce all statements to simple relations among "things".
  - You are all familiar with statements such as "No two things can be in the same place at the same time."
  - We see statements like "No thing can travel faster than the speed of light."
- <u>So what is a “thing”?</u>
  - Is the Mississippi river a thing? (What would Heraclitus have said?)
  - Is a person a thing?
  - Is a moving bright spot on the wall a thing?
- If you believe in external reality, is it necessary to believe it consists of well-defined things?
  - If not, what becomes of statements like those above?
  - Do things exist outside our description of events?

What has SR changed philosophically?

- The old invariants (t, lengths, m …) (things which were "real" in that they were observer-independent) have been tossed out. They are replaced with new invariants (c, $d^2-c^2t^2$, $E^2-c^2p^2$…) which have a slightly more complicated relation to our customary observations.

- If we had evolved experiencing many relative speeds close to c, there would be absolutely nothing philosophically exotic or particularly "relativistic" about "relativity". The Lorentz transformations would make sense to us in the same way that the Galilean transformations make sense. We would just have a different set of invariants.
  That's why Einstein wanted to name the theory "Invariants theory."

- The philosophical excitement comes from the transformation from one theory to the other- ideas that seemed immutable turned out to be mutable, and there's a lesson to be learned from that process.