The new invariants

World lines

4-dimensional physics

Causality

The twin "paradox"

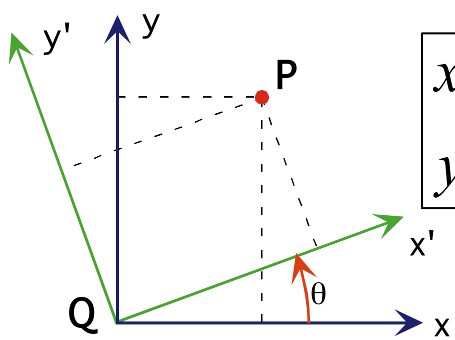
<u>Next</u>:

Accelerated reference frames and general relativity

Term paper topic due March 18 (see the edge)

4-dimensional spacetime

- Three-dimensional geometry becomes a chapter in four-dimensional physics. ... Space and time are to fade away into the shadows. (Minkowski, 1908)
- The geometrical interpretation of SR is based on the similarity between rotations and Lorentz transformations. Take two coordinate systems, rotated with respect to each other:



$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

Coordinate rotation doesn't change the distance between points **P** and **Q**: $x'^2 + y'^2 = x^2 + y^2$. $\sin^2 q + \cos^2 q = 1$ expresses this **invariance of distance under rotations**. The two people get different x and y, but agree about d.

The Lorentz transformation looks ~like a rotation:

(I'm ignoring y and z.)

$$x' = \gamma(x - \beta t) = \gamma x - \gamma\beta t$$
$$t' = \gamma(t - \beta x) = -\gamma\beta x + \gamma t$$

You can verify that $(t')^2 - x'^2 = (t)^2 - x^2$. Although two observers measure different lengths and time intervals, they agree on the value of this quantity, the "**interval**".

(Note the minus sign, it's defined to be more like a time interval than a space interval.)

Relativity is full of invariants

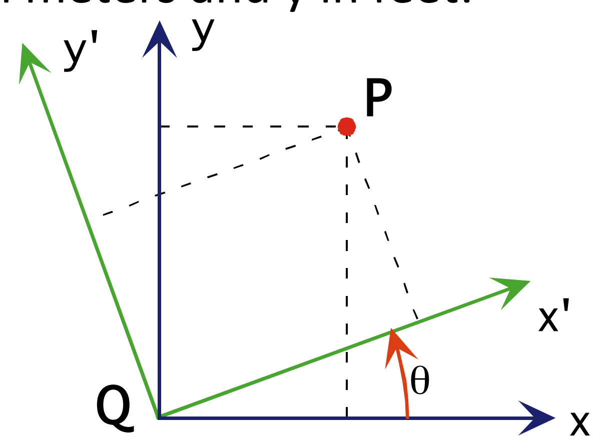
they just aren't the ones you expected.

- Minkowski interpreted the invariant interval as a geometrical quantity in a non-Euclidean geometry. It has quantities similar to the trigonometric functions, called hyperbolic trigonometric functions (*e.g.*, hyperbolic sine, *etc*.).
- Using this mathematics, we can interpret the Lorentz transformation as a non-Euclidean "rotation":

$$x' = x\cosh\theta - t\sinh\theta$$
$$t' = -x\sinh\theta + t\cosh\theta$$

Except for the - sign, this is like a rotation. That - sign is the indicator the time dimension is not like the 3 space dimensions.

The universal speed, c, now has a geometrical meaning: the conversion factor between space and time units. Suppose we measured x in meters and y in feet:



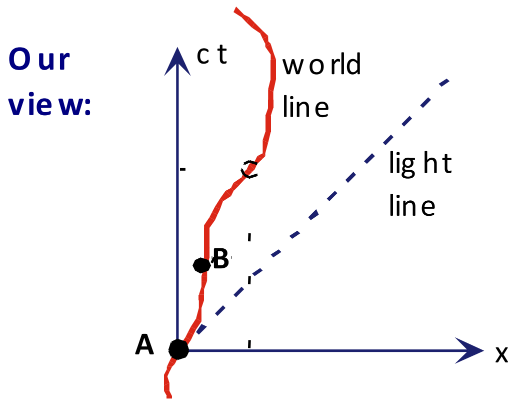
With these units, the quantity $x^2 + y^2$ is not invariant under rotations. In fact, until we make the units agree, we can't even combine them. We must multiply y by a conversion factor, k, which is the number of meters per foot. Then $x^2 + (ky)^2$ is invariant.

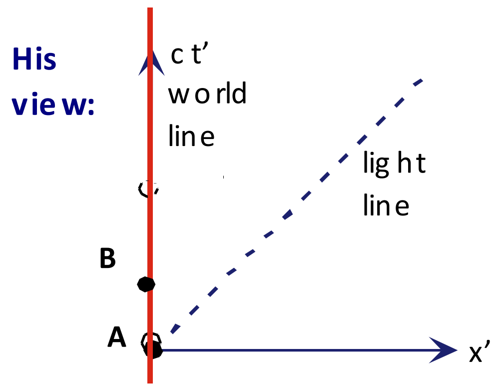
4-dimensional physics

- The principle of relativity requires that
  if the laws of physics are to be the same in every inertial reference frame, the quantities on both sides of an = sign must undergo the same Lorentz transformation so they stay equal.
  You cannot make any invariant from space or time variables alone.
  That's why we call the SR world 4-D, and call the old world 3-D + time. No true feature of the world itself is representable in the 3 spatial dimensions or the 1 time dimension separately.
- In Newtonian physics, **p**=m**v** (**bold** means vector). Momentum and velocity are vectors, and mass is a scalar (invariant) under 3-d rotations. This equation is valid even when we rotate our coordinates, because both sides of the equation are vectors.
- The new "momentum" is a 4-d vector (4-vector for short). It's fourth component is E/c, the energy.
  - The factor of c is needed to give it the same units as momentum.
- The lengths of 3-vectors remain unchanged under rotations. So does the invariant "length" of 4-vectors under *Lorentz transformations*. The length^2 of a 4-vector is the square of its "time" component minus the square of its space component: $(E/c)^2 - p^2 = (m_0c^2)^2$

4-D geometry

- In the geometrical interpretation of SR, c is just a *conversion factor*, the number of meters per second. The geometrical interpretation of SR helped lead Einstein to general relativity, although it didn't directly change the physics.
- World lines A graph of an object's position versus time:

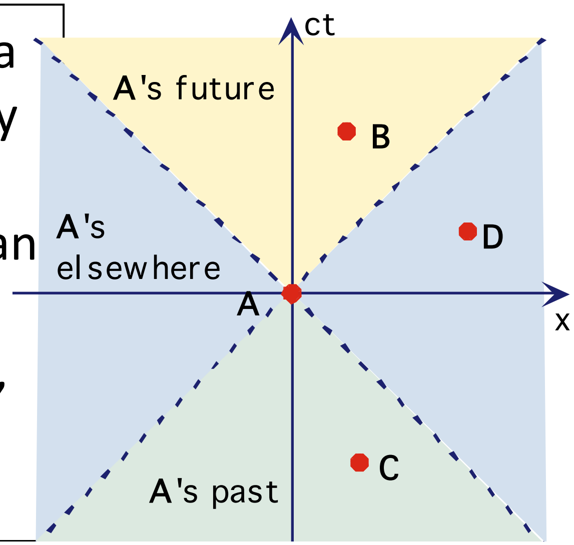




If an object is at rest in any inertial reference frame, its speed is less than c in every reference frame. The speed limit divides the spacetime diagram into causally distinct regions.

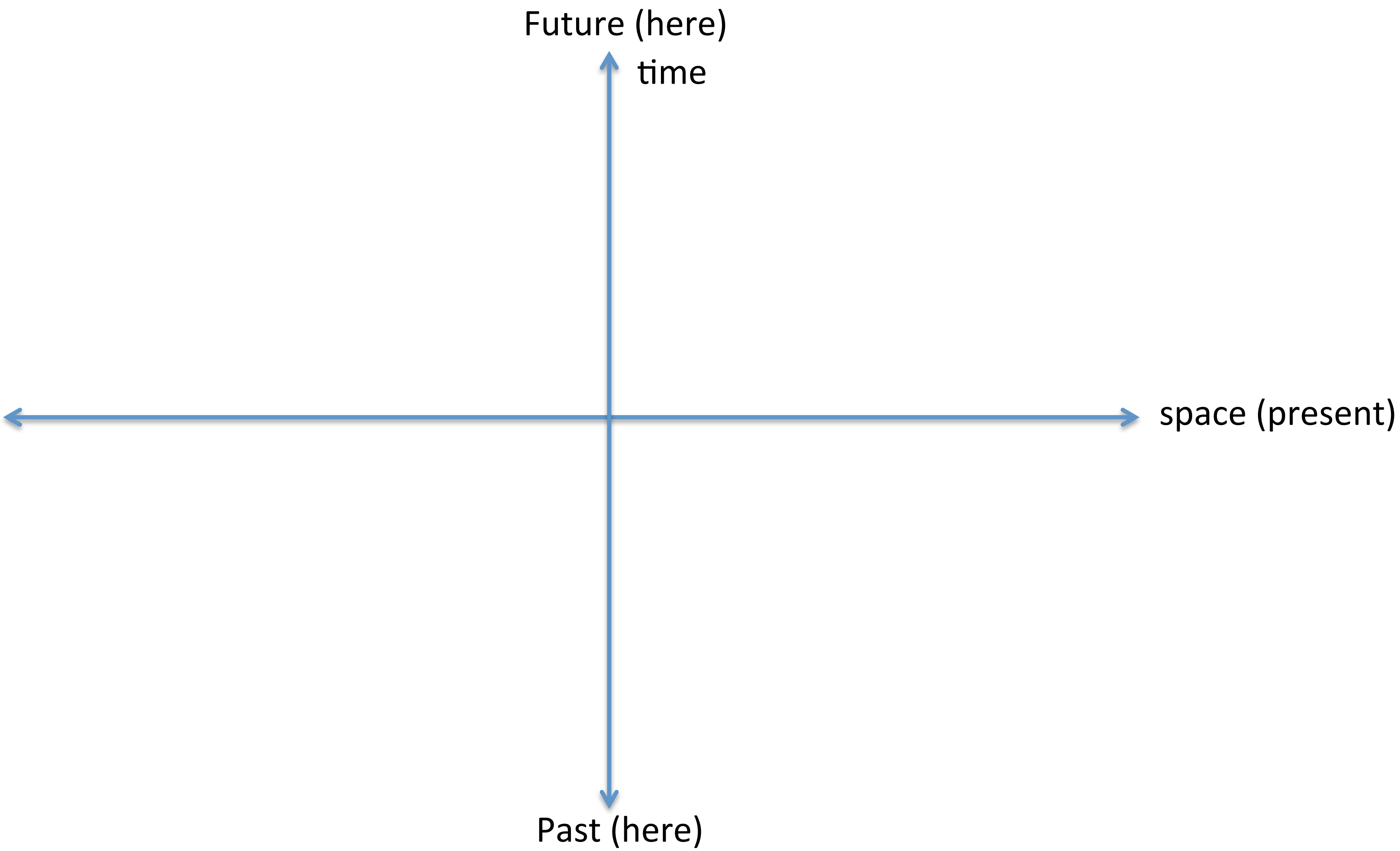
Lorentz_transform_of_world_line.gif

A, **B**, **C**, & D are events. **A** might be a cause of **B**, since effects produced by **A** can propagate to **B**. They cannot get to **D** without travelling faster than light, nor to **C** because it occurs before **A**. **C** might be a cause of **A**, **B**, and/or **D**. **D** could be a cause of **B**, since light can get from **D** to **B.**

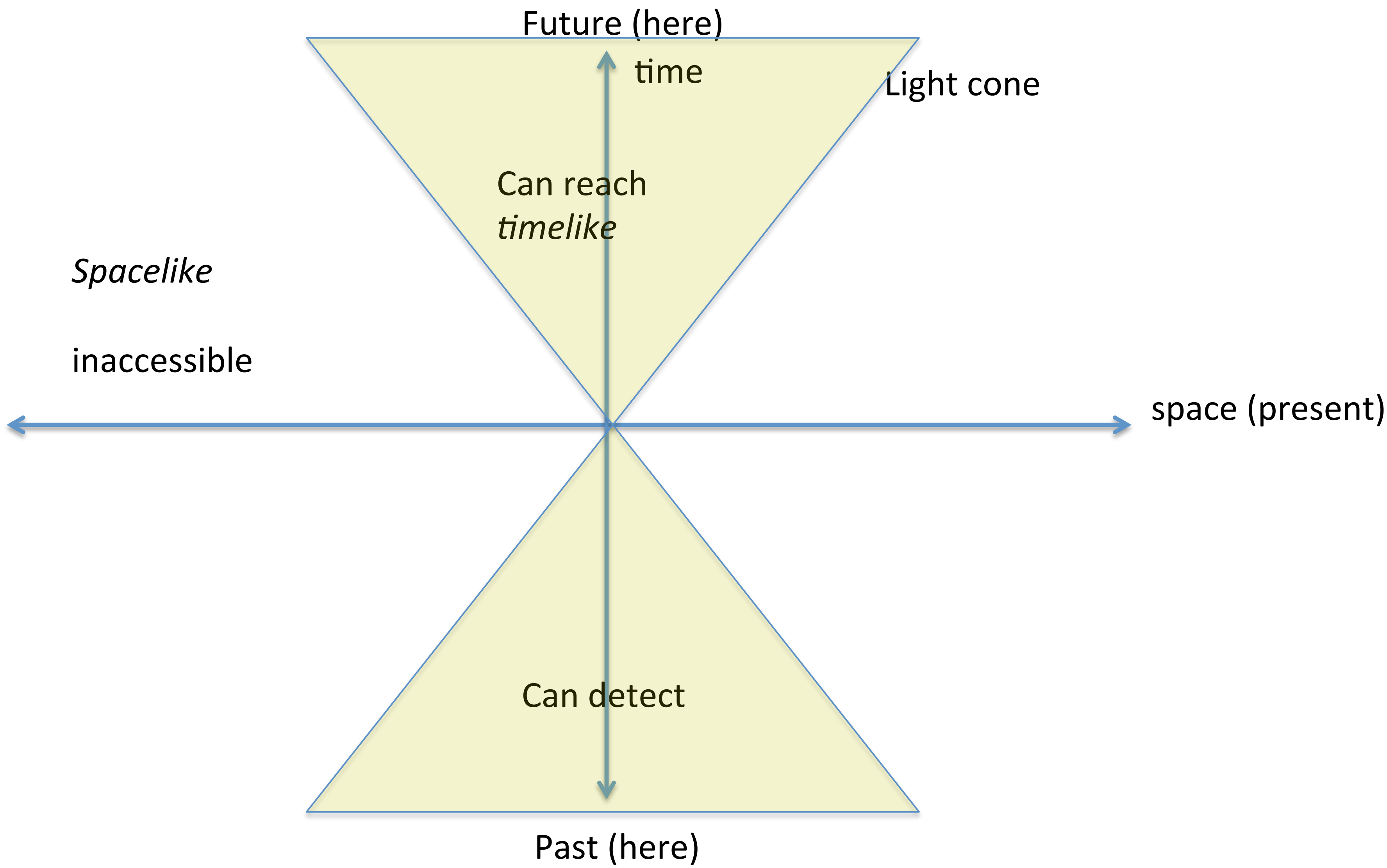


If the interval, $(ct)^2 - x^2$, between pairs of events is positive ("timelike"), then a causal connection is possible. If it is negative ("spacelike"), then not.

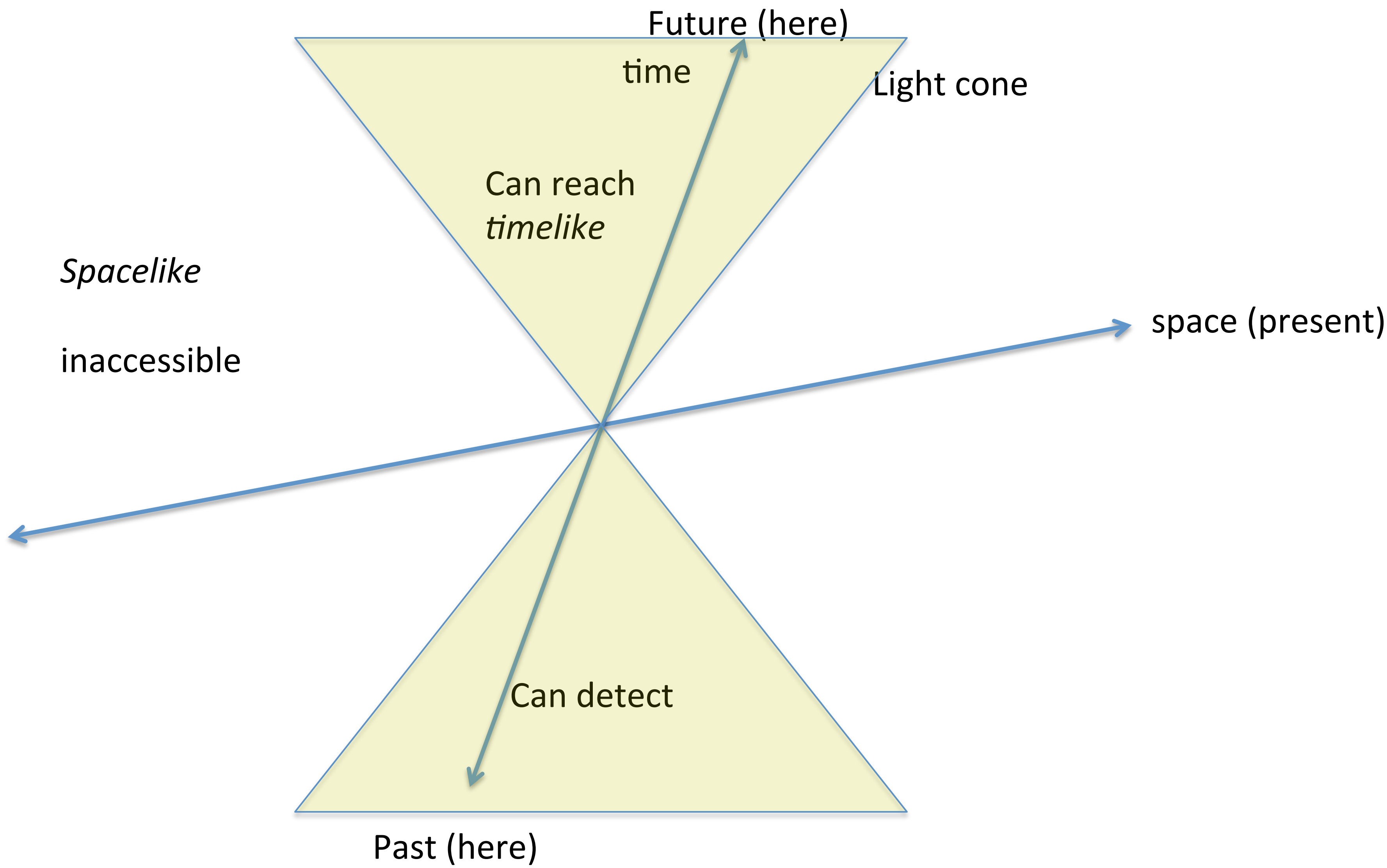
Space-time diagrams



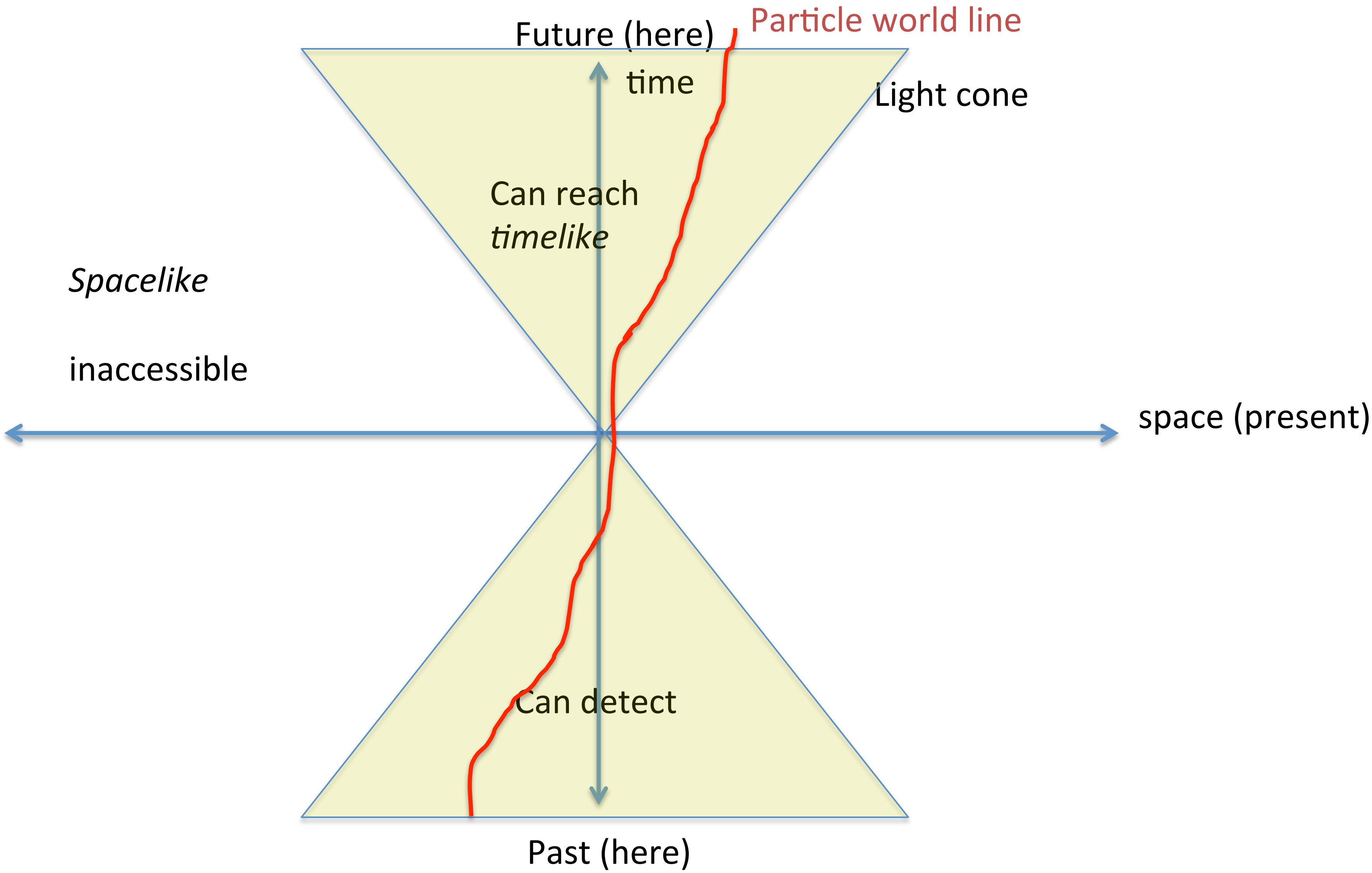
Space-time diagrams



Spacetime from rocket



Space-time diagrams



Causality in Special relativity

- Strong form: No event can be affected in any way by events outside its past light cone.
- Weak form: No information may be transmitted except forward within a light cone.
- Weaker form: No information can be transmitted except within a light cone.
- You may wonder why we make such pointless distinctions. Can't *any* "effect" be used to transmit information? Stay tuned.
- In a deterministic world, the Strong form would mean that an event would be completely predictable on the basis of knowledge of its past cone alone. Observations OUTSIDE the past light cone might provide the same info in more convenient form, but would never be *needed*, because everything knowable about the event would be determined by the preceding events in the light cone.
- What about in a world where things are not completely predictable on the basis of *anything*? The Strong form would mean that one could find within the past light cone enough information to obtain *as much predictive accuracy as possible* about an event.

What does "Nothing can travel faster than the speed of light" mean?

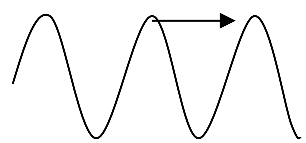
We know that

- no ordinary mass can go faster, because that would require infinite energy.
- no conserved quantity can go faster, because then it would not be conserved in some reference frames.
- If we believe that causation must go forward in time, then we know that no "information" can go faster than c, because that would allow backwards-in-time causation.
    - What happens if you can send info backward? Say you send your grandma info that somebody much cuter than your grandpa was about to move into her neighborhood. Then you aren't born. Then the info doesn't get sent. So you are born, so …….

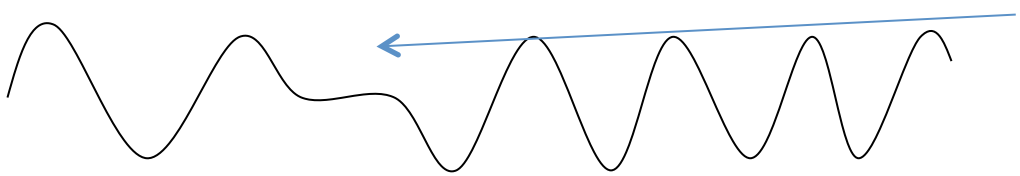
What does "no object travels faster than c“ mean?

If "no object travels faster than c", then the following aren't objects:

- The bright spot made by a beacon shining on a wall.
- The cutting point of a scissors.
- The crest of an E-M wave in matter. (Certain materials have index of refraction less than 1 over some frequency range, hence a "phase velocity" greater than c for some light.)



The repetitive pattern carries no info!



Only the *breaks* in the repeating pattern must travel slower than c.

What are we then claiming?

If we are to describe the world as having some primary constituents, with various higher-level phenomena just being patterns in the constituents' behavior, we want to restrict the primary constituents to those which don't travel faster than light. We claim there exists *some* complete description of the world in terms of constituents which don't travel faster than c.

Causality in Special relativity

- Things:
  - One version of positivism tried to reduce all statements to simple relations among "things".
  - You are all familiar with statements such as "No two things can be in the same place at the same time."
  - We see statements like "No thing can travel faster than the speed of light."
- So what is a "thing"?
  - Is the Mississippi river a thing? (What would Heraclitus have said?)
  - Is a person a thing?
  - Is a moving bright spot on the wall a thing?
- If you believe in external reality, is it necessary to believe it consists of well-defined things?
  - If not, what becomes of statements like those above?
  - Do things exist outside our description of events?

What has SR changed philosophically?

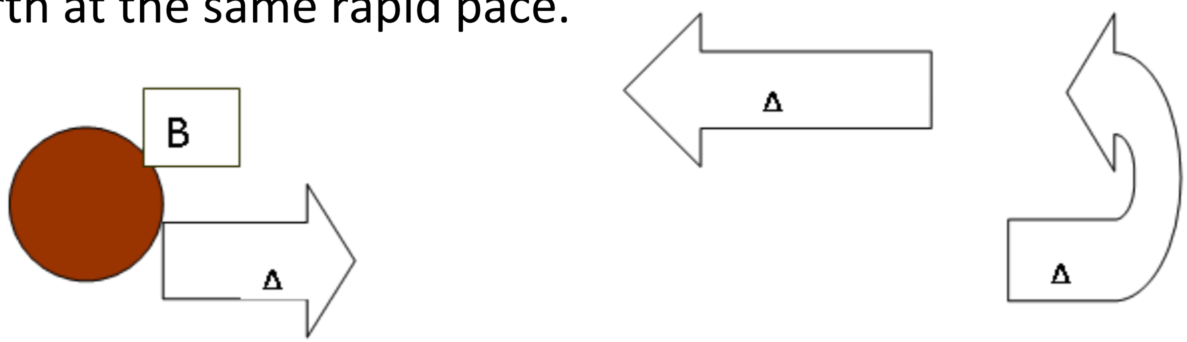
- The old invariants (t, lengths, m …) (things which were "real" in that they were observer-independent) have been tossed out. They are replaced with new invariants (c, $d^2-c^2t^2$, $E^2-c^2p^2$…) which have a slightly more complicated relation to our customary observations.

- If we had evolved experiencing many relative speeds close to c, there would be absolutely nothing philosophically exotic or particularly "relativistic" about "relativity". The Lorentz transformations would make sense to us in the same way that the Galilean transformations make sense. We would just have a different set of invariants.
  That's why Einstein wanted to name the theory "Invariants theory."

- The philosophical excitement comes from the transformation from one theory to the other- ideas that seemed immutable turned out to be mutable, and there's a lesson to be learned from that process.

The twin paradox

Suppose Alice and Beth are twins. Alice sets off in her rocket so fast that the time dilation factor becomes 10. She travels away from Earth for 10 years, as measured by Beth, who has remained on Earth. Alice then turns around and returns to Earth at the same rapid pace.



When Alice returns home, Beth has aged 20 years. How much has Alice aged? There appears to be a paradox. According to the Lorentz transformation, during the time Alice is travelling:

Beth says: I measure Alice's clock to be running slow by a factor of ten, so **she** has aged only two years.

Alice says: My clock is fine. I measure Beth's clock to be running slow by a factor of ten, so she has aged only 2 years.

They start and end standing right next to each other, so a direct comparison of clocks is possible. Who is correct?

Twin Non-Paradox

- The answer is that Alice, the twin who turned around, has aged less.
- The situation is not symmetrical, because in order to return to Earth, Alice must have accelerated. Our descriptions of how things looked to different observers (Lorentz transformations) so far do not describe accelerated observers, so we only know how things look to Beth.
- Of course Alice must agree that Beth is older, when they now stand side-by side. Now we can put together a conclusion about how Beth must have looked to Alice while Alice was accelerating. While turning back (accelerating toward earth), Alice must observe Beth's clock to be running fast, not slow.
- So this is <u>not a paradox at all</u> but just a reminder that the SR transformations only work between reference frames which are not accelerating (*at least* with respect to each other, leaving aside the question of absolute acceleration.) But you can also see that from SR we can draw conclusions about how things *must* look to accelerating observers.
- Let's go further in seeing how things look to accelerating observers. In particular, let's look for ways in which the simple laws of physics might get messed up in their frames.