

What sort of things happen in accelerated frames?

Why use them?

Next Topics:

Is curvature necessary? Conventionalism.

Gravitational waves – space is real

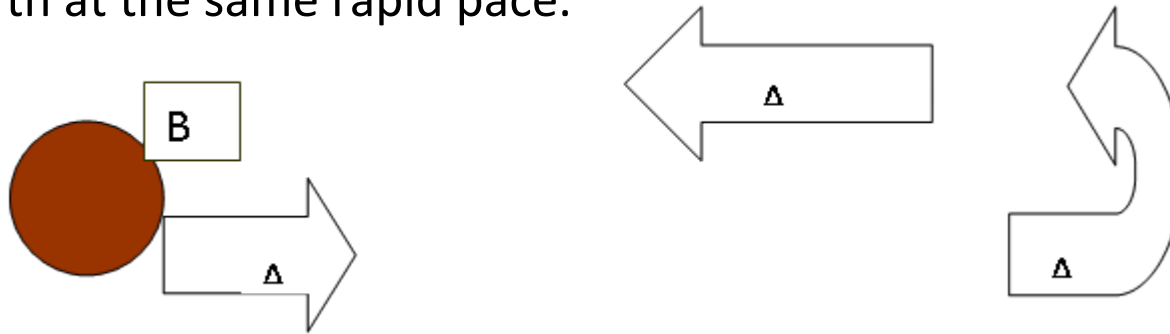
Singularities

Global properties of GR – cosmology

Topology, time travel, and other oddities

The twin paradox

Suppose Alice and Beth are twins. Alice sets off in her rocket so fast that the time dilation factor becomes 10. She travels away from Earth for 10 years, as measured by Beth, who has remained on Earth. Alice then turns around and returns to Earth at the same rapid pace.



When Alice returns home, Beth has aged 20 years. How much has Alice aged?

There appears to be a paradox. According to the Lorentz transformation, during the time Alice is travelling:

Beth says: I measure Alice's clock to be running slow by a factor of ten, so **she** has aged only two years.

Alice says: My clock is fine. I measure Beth's clock to be running slow by a factor of ten, so she has aged only 2 years.

They start and end standing right next to each other, so a direct comparison of clocks is possible. Who is correct?

Twin Non-Paradox

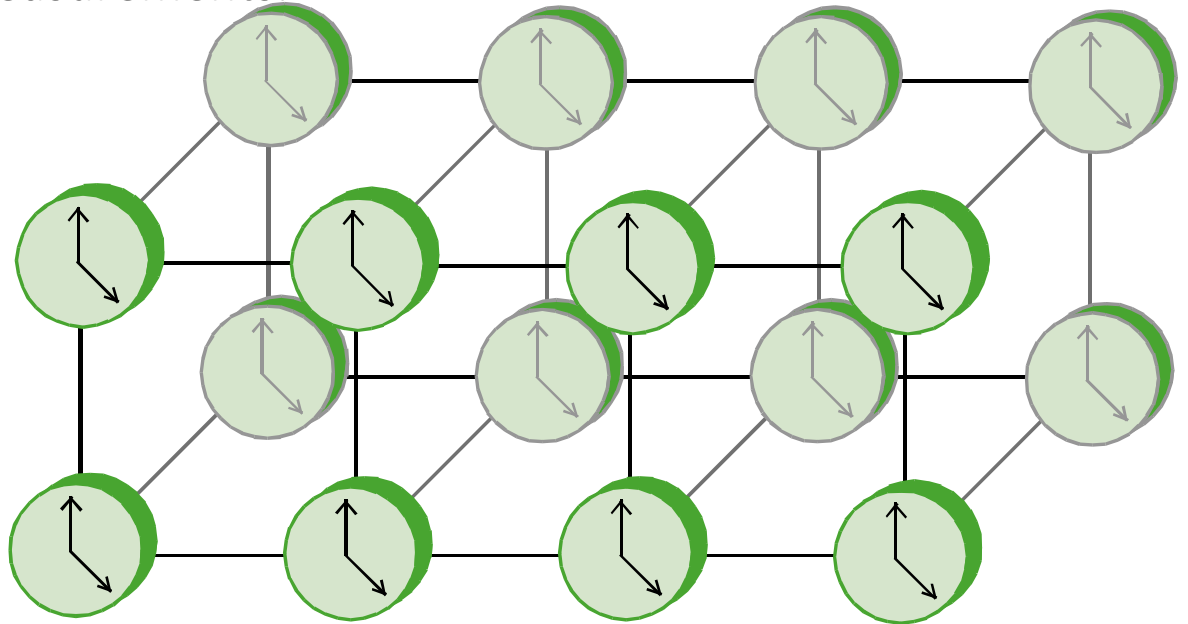
- The answer is that Alice, the twin who turned around, has aged less.
- The situation is not symmetrical, because in order to return to Earth, Alice must have accelerated. Our descriptions of how things looked to different observers (Lorentz transformations) so far do not describe accelerated observers, so we only know how things look to Beth.
- Of course Alice must agree that Beth is older, when they now stand side-by-side. Now we can put together a conclusion about how Beth must have looked to Alice while Alice was accelerating. While turning back (accelerating toward earth), Alice must observe Beth's clock to be running fast, not slow.
- So this is not a paradox at all but just a reminder that the SR transformations only work between reference frames which are not accelerating (*at least* with respect to each other, leaving aside the question of absolute acceleration.) But you can also see that from SR we can draw conclusions about how things *must* look to accelerating observers.
- Let's go further in seeing how things look to accelerating observers. In particular, let's look for ways in which the simple laws of physics might get messed up in their frames.

Defining Coordinates

Einstein insisted on the (Machian) idea that each observer must construct his own view of space and time by some actual *observable operations*: “we entirely shun the vague word ‘space,’ of which, we must honestly acknowledge, we cannot form the slightest conception, and we replace it by ‘motion relative to a practically rigid body of reference.’ ” With no absolute space or time to rely on, we need an operational definition of position and time measurements.

So let’s replace Newtonian space & time by an ensemble of meter sticks and clocks

Distribute a set of clocks on a lattice. Clock synchronization is done by sending a round trip signal between two clocks. The signal is assumed to arrive at the second clock halfway between emission and reception by the first. The position of each clock is measured similarly. Half of the round trip time, divided by c , gives the distance.



How do we test if our frame is inertial?

Do Inertial Frames Exist?

- Einstein's operational recipe (make a bunch of identical rods and clocks, build a lattice of the rods to mount the clocks on, check the clock synchrony with light rays) also allows you to *check if Special Relativity is correct*. It *assumes* that once the clocks are synchronized by this procedure, they will stay synchronized. (It also makes other assumptions, but we needn't think about them.)
- So while you are getting used to the ideas that:
 - Aristotle and instinct are wrong, there is no absolute rest frame.
 - Newton and Galileo are wrong, there is no absolute time or distance.
- Einstein has at least opened the possibility that his Special Relativity also is wrong, there may be no inertial frames.
- Einstein's recipe for building a coordinate frame from rigid sticks already has a problem:

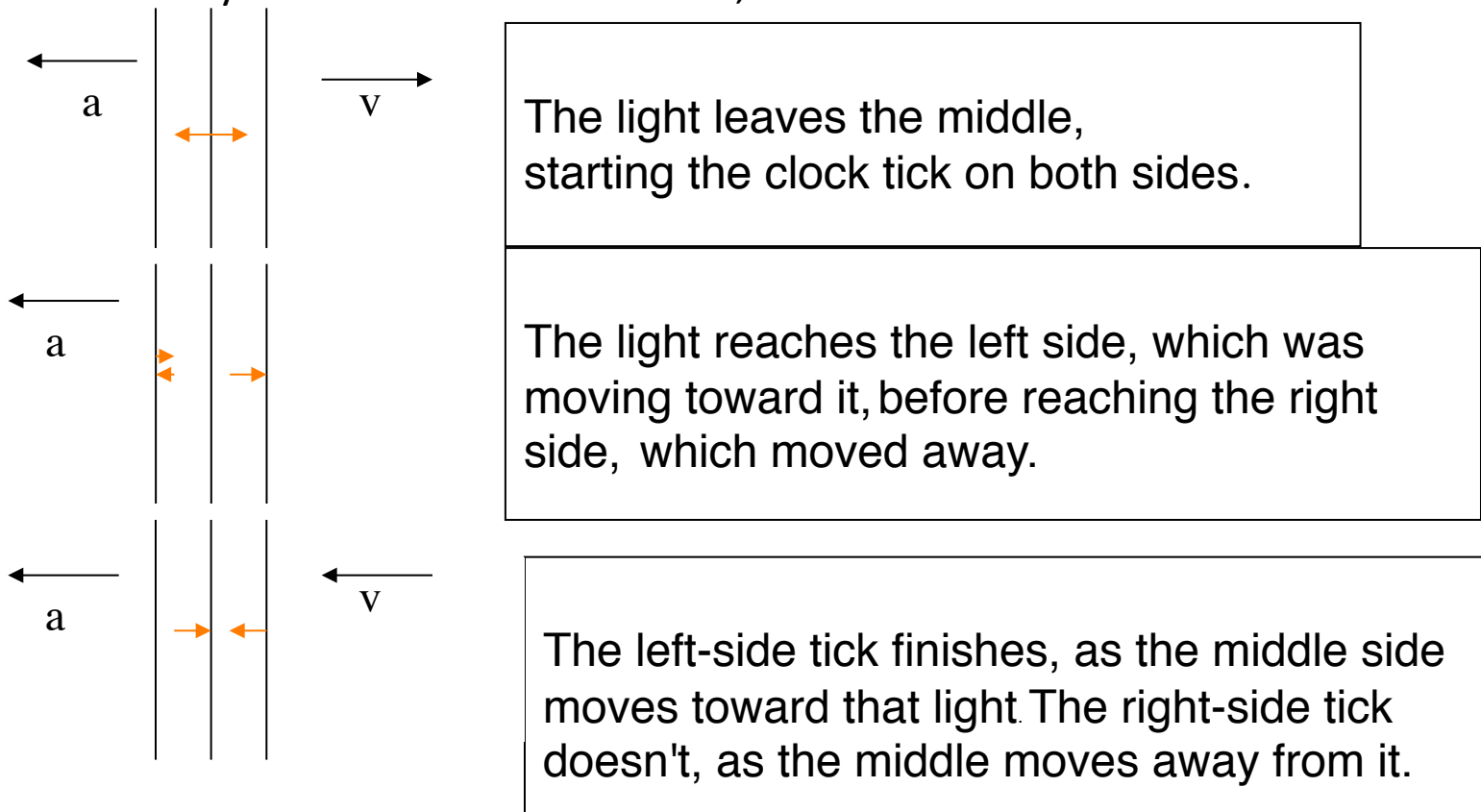
Can we follow Einstein's Recipe?

- Rigid bodies cannot exist if Special Relativity is correct.
 - Otherwise you could wiggle one end of a stick and transmit a signal to the other end infinitely fast.
- Each atom only feels forces from its neighbors after a delay- the forces are transmitted at the speed of light. So if a force acts on any part, it always distorts the object at least temporarily.

We also don't know if the clocks will stay synchronized.
So the existence of inertial frames is not simply given.

Accelerating Clocks

- We saw that clock rates must appear different to an accelerated observer. Let's investigate further. Here's a pair of our simple two-mirror clocks viewed over a brief interval during which they accelerate toward our left, at rest in the middle of the interval.



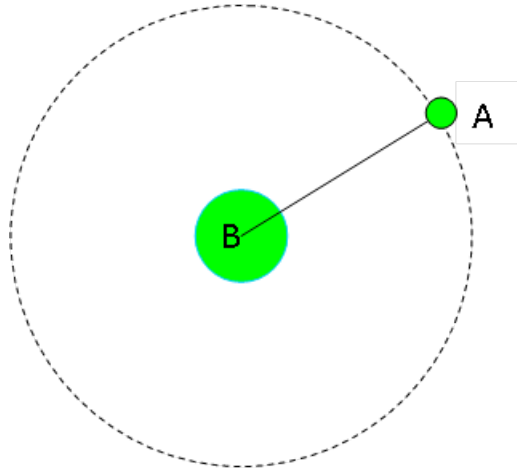
An observer can conclude *objectively* that the left clock is running faster than the right clock. (e.g. by film exposed on only the left side). The clock the acceleration is *toward* runs *fast*, the clock the acceleration is *from* runs *slow*.

Time and Position

- We could make a whole stack of these clocks, applying the same argument to each. The farther the clocks are away in the direction the acceleration is toward, the faster they run. The farther they are away in the opposite direction, the slower they run.
- Everybody agrees on this, even if they can't agree on which clock is "right". (*unlike* disagreement between inertial frames, in which they can't agree on who's faster.)
- Notice that this effect is just what we needed in the twin "paradox". As Alice accelerated toward Beth, (while turning back) she concluded that Beth's clock's were running fast.

How Big an Effect?

Let's calculate the effect of the acceleration is on clock rates:



B is a massive object,

A is less massive, held in orbit by a string.

B ~doesn't accelerate.

B says A's clock runs slow
by a factor of:

$$\frac{1}{\gamma} \equiv \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$$

What does A say about B?

If A weren't accelerating, she would see B as slowed by a factor of $1/\gamma$. But since A circles B repeatedly, they can compare times on each orbit, with no change in signal transmission times. So A and B must agree about who is faster and by what factor.

Thus A also thinks that B's time runs faster by a factor of γ . The net effect of A's acceleration in speeding the rate at which A sees B's clocks run is thus a factor of:

$$\gamma^2 = \frac{1}{1 - \frac{v^2}{c^2}} = \frac{1}{1 - \frac{\vec{a} \cdot \vec{R}}{c^2}} \approx 1 + \frac{\vec{a} \cdot \vec{R}}{c^2}$$

This agrees with our argument from

stacked light clocks: the clock rate increases with the distance in the direction of the acceleration, decreases the other way. This is also just the amount needed to get Beth and Alice to agree. (We won't try to make the argument accurate beyond the linear term in the acceleration- we would have to worry about who measures "a" and "R".)

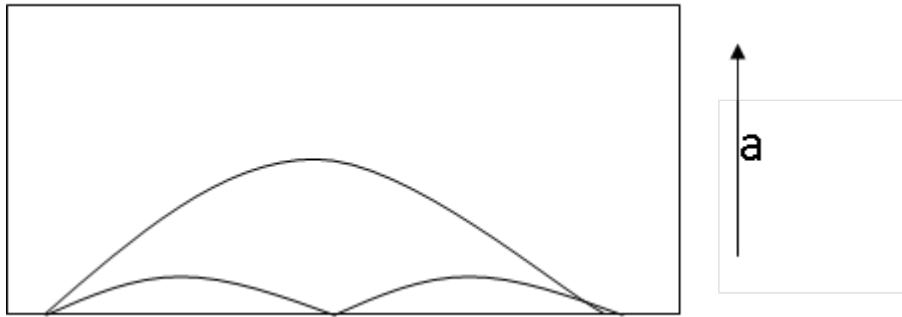
The Accelerating Observer

sees something very strange. In her reference frame, identically constructed clocks run at different rates, depending on where they're located!

- It would seem that a reasonable law of physics should be that identically constructed clocks run at the same rate. Thus this sort of gross effect should tell A that she is accelerated.
- In an accelerated frame, you cannot go through the exercise of building a lattice of identical synchronized clocks to define the coordinate system: the clocks don't stay synchronized.
- It certainly looks like accelerated frames are a curiosity, since in one we would have to abandon some simple laws of physics.
- At one historical point, the only reason to insist on looking at the laws that apply within accelerated frames was Einstein's Machian prejudice that the laws of physics should depend only on the relations among objects, not on absolute motion in *any* sense.

Oddities of Accelerating Frames

Other strange things happen in uniformly accelerated frames. A light ray travelling at right angles to the acceleration seems to bend, as if it were falling in the direction opposite to the acceleration.

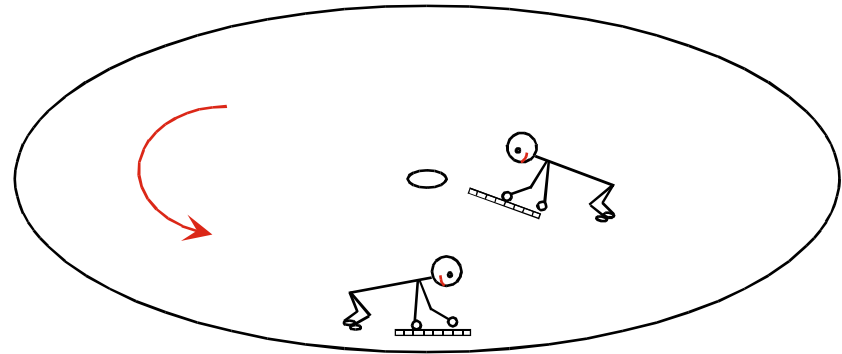


Straight lines are supposed to be the shortest paths between two points. Now the quicker path here *is* the path of the uninterrupted light ray (we can see that easily in the lab frame) but in the accelerated frame the spatial path length of that light ray seems *longer* than the path of the ray that makes a bounce off the wall. Isn't the speed of light supposed to be constant? If we tried to make our ordinary laws of physics work in such a frame, we couldn't identify light rays' paths with straight lines.

How would we define straight lines?

Non-uniform Acceleration

As long as we're looking at weird reference frames, let's see what would happen if you used a non-uniformly accelerating reference frame, e.g. a merry-go round.



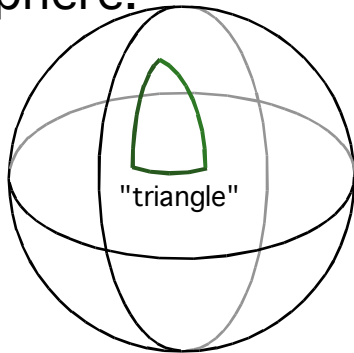
Before the merry go-round accelerates, you get a bunch of little meter sticks, all stacked together and the same length. If you measure the MGR circumference C and radius R by counting out meter sticks, you find $C = 2\pi R$, whether you do this on the MGR or the ground. Now set the MGR spinning. It will stretch, etc., but you tighten down any bolts needed to make its circumference still fall exactly above the previous circumference, traced out on the ground. If you measure on the ground, you get the same old C and R .

Due to the Lorentz contraction, the rulers measuring the circumference on the MGR have shrunk, but not the ones used for the radius. Therefore, in the MGR measurement, $C > 2\pi R$. Not only that, the ratio C/R depends on the radius of the circle. (It gets bigger for bigger R .) This is not Euclid's plane geometry, but rather resembles the sorts of geometries you get if you try to confine measurements to curved surfaces. No wonder it's hard to find straight lines with familiar properties!

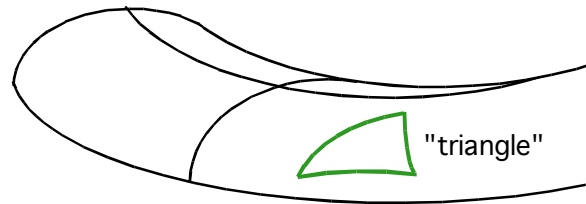
Curved Geometries

Here are 2-d surfaces in 3-d in which $C/2R \neq \pi$

Sphere:



Saddle:



If we want to accept non-uniformly accelerated frames as equally valid, we must accept such weird geometry as being the correct description of our 3-d space. In other words, we should be able to distinguish whether we are using a non-uniformly accelerated frame by whether measurements confirm Euclid's axioms.

Don't worry about whether the space looks like a piece of some hypothetical Euclidean higher dimensional space, which would have absolutely no physical significance.

Our Choice of Frames

We already saw that the outside observers in a rotating frame think that the clocks at the middle are running fast. So if we try to use a rotating frame, clock rates depend on position.

We have a choice:

- **Reject accelerating reference frames, because they**
 - require clock rates to depend on position,
 - violate Euclidean geometry (with reasonable definitions of length),
 - generally make a mess out of familiar laws of nature.
- **Accept accelerating frames, and make new laws of nature that have all those weird effects.**
- **Anyone in his or her right mind would choose (1).**
- **Why then do physicists choose (2)?**

Why choose accelerating frames?

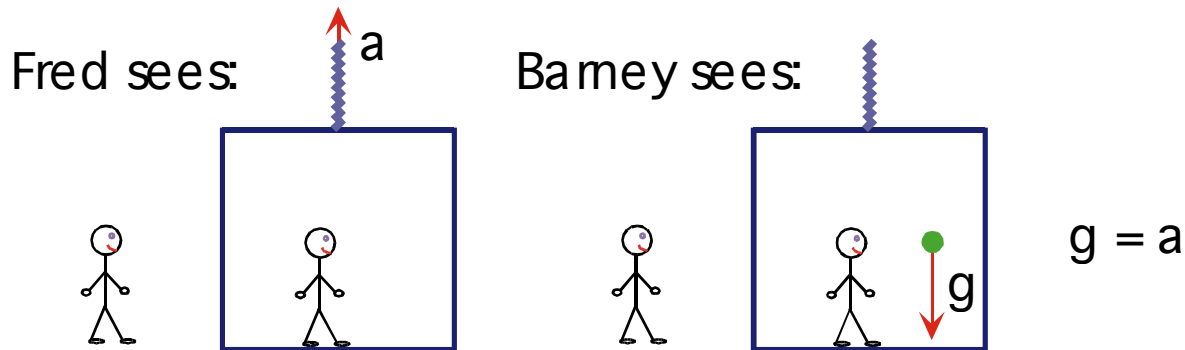
- Our previous argument suggests that physicists do so because of some mental defect.
- To confirm that:
- Where do you feel gravity?

The equivalence principle

- “The gravitational mass of a body is equal to its inertial mass.”
- Remember: inertial mass is the m that appears in $p = mv$ or $E = mc^2$, or (approximately) $F = ma$.
- Gravitational mass is the m that appears in Newton’s law of gravity: $F = GMm/r^2$. It tells us the strength of the gravitational force between two masses. It has been empirically determined that the two kinds of mass are exactly the same to within a part per trillion (10^{-12}). In other words, different types of mass all show the same acceleration (from a given starting velocity) in a gravitational field
- When you calculate the gravitational acceleration of an object using $a = F/m$ the object's own mass, *m drops out*,
 - $a = GM/r^2$.
- The result is that gravity makes every object accelerate together: the effect of gravity is completely describable classically by an acceleration field, as has been known since Galileo’s time.
- That means that you don't feel gravity in the same way that you feel other forces. Since all your parts are accelerating together, gravity creates no strains, tickles no nerves...
 - However, as Einstein put it, he was the first to “interpret” this fact.

The equivalence principle

- Consider the famous “elevator” gedanken experiment. We are somewhere in intergalactic space, with no planets or other junk nearby. Fred is resting at ease in his un-accelerated reference frame. Barney, on the other hand is inside a box and can't see out.
- Suppose there is a rope attached to the box, and some external agent pulls on the rope, accelerating the box at exactly 9.8 m/s^2 .



Fred says: “The box (and Barney) are accelerating. So what?”

Barney says, “I am not accelerating. I am in an elevator which is hanging from its cable in a gravitational field. It's the same field that's making Fred fall, because no cable supports him.” **Who is correct?**

Einstein insists that in the absence of a reason for preferring one point of view, one must accept both.

Einstein proposed another generalization

- No measurement of any sort can detect a uniform gravitational field.

And, by the way:

- No *local* measurement can detect *any* gravitational field.

If we accept Einstein's generalization...

We cannot distinguish an accelerated frame from a gravitational field. We said that no sane person would voluntarily accept accelerated frames, because they lead to all sorts of crazy effects.

- If gravity were completely uniform, you could get rid of it by transforming to another reference frame. Gravity can always be eliminated that way in a small region (*e.g.*, inside Barney's elevator), but not over a large spacetime domain, because there is an uneven distribution of matter.
- So no sane person can reject a universe with gravity: you can't get rid of gravity without getting rid of everything. The gravity isn't uniform, so our actual reference frames are like the ones with non-uniform accelerations.
- If Einstein is right, then a world with gravity has all those bizarre effects we found for accelerating frames, whether you like them or not.