

Gravity and accelerating frames

implications of equivalence

Is curvature necessary?

Gravitational waves - space is real

Singularities

Global properties of GR: intro to cosmology

Topology, time travel, and other oddities

Next time

Cosmology

Philosophical implications

Einstein proposed another modest generalization

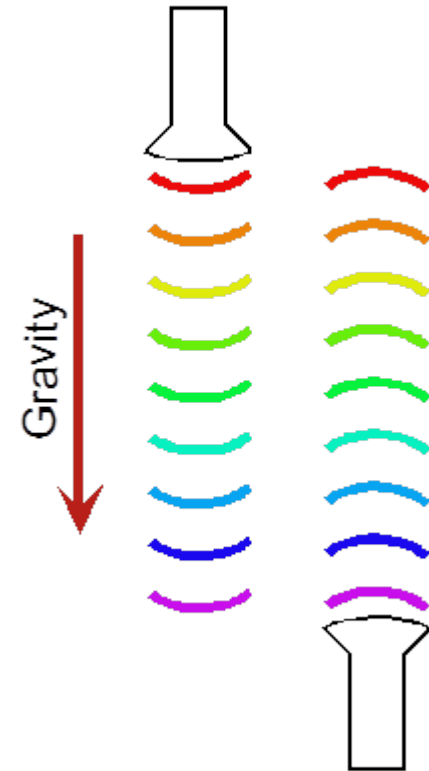
- No measurement of any sort can detect a uniform gravitational field.
 - And, by the way:
- No *local* measurement can detect *any* gravitational field.
- So no sane person can reject a universe with gravity: you can't get rid of gravity without getting rid of everything. The gravity isn't uniform, so our actual reference frames are like the ones with non-uniform accelerations.
- If Einstein is right, then a world with gravity has all those bizarre effects we found for accelerating frames, whether you like them or not.

Observational Consequences

- But if you can't tell whether balls fall to the floor because of g or because you have chosen an accelerated reference frame, then g must make those weird effects that we found for accelerated reference frames. Otherwise you could use them to test whether you were accelerating. So g should give the strange clock effects found in accelerating frames.
 - One consequence is the Gravitational red shift
 - The clocks at the bottom run slower than those at the top.

One can use the frequency of light as a clock (count the crests as they pass by). The most accurate clocks work this way). So, do an experiment – drop light from the top of a tower:

- Light becomes redder as it goes up and bluer as it goes down. (This can also be understood in terms of the energy of "photons".)
- The effect can be seen in starlight and also in buildings. (Pound & Rebka, 1960)



space must be non-Euclidean

in a *non-uniform* \mathbf{g} , just as with non-uniform \mathbf{a} .

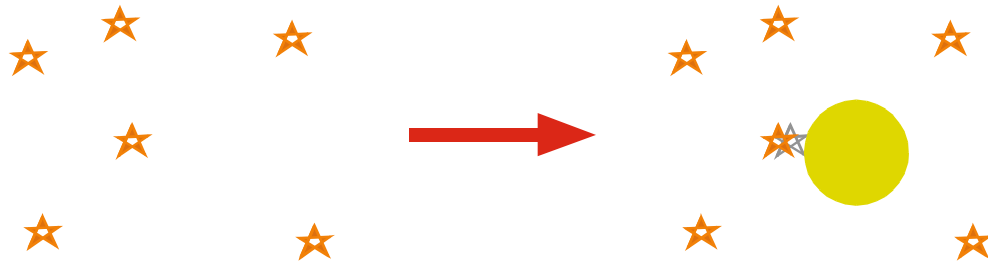
On the merry-go-round, where things would fly outward,

$C > 2pR$. Near a star, where things fly inward, there must be the opposite effect,

$C < 2pR$. (Here each length is measured with a meter-stick with some uniform construction, say same number of nickel atoms.)

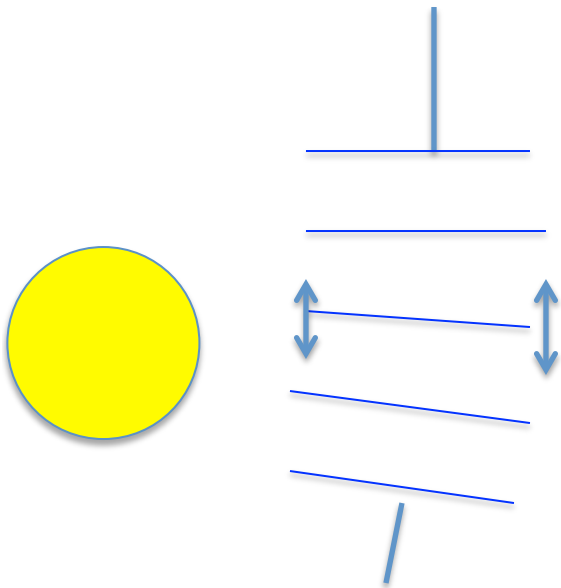
The result is a curvature of the paths of anything, including light, *beyond that given by treating gravity as a force.* (Doubles the curvature of light.)

CONFIRMED, 1919 solar eclipse



Gravitational time-warp

Classically, energy goes up as something falls. We'll see that corresponds to frequency. Clocks run faster away from the star, so synchronized processes like the wavefronts passing seem faster, measured by local clocks, below.

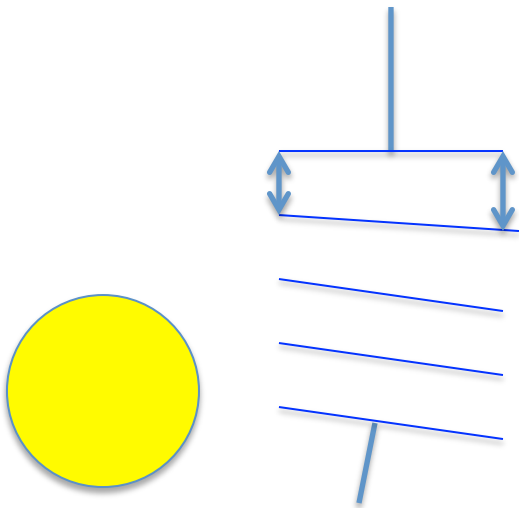


The (locally measured) frequency of the wave is higher near the star, so the wavefronts are closer near the star. The wave must bend.

This time-warp term is essentially just the classical force term, to a good approximation.

Gravitational space-warp

- On the merry-go-round, there was missing space inside the circle. Near a star, there should be the opposite effect. A wave passing by the star has to traverse more distance on the part near the star than on the part farther away. The wavefronts farther away get ahead, which turns the wave as if it were being pulled toward the star. That's the extra curvature of light observed in 1919.



It looks like the wavefronts are closer together near the star, but that's only because I've forced the picture onto this nearly Euclidean page. Really there's *extra space* there and all the fronts are equally spaced, as measured locally. In a standard coordinate system (Schwarzschild) it's the radial lengths that are larger than you'd think from the circumferential ones, giving extra curvature on the way toward and away from the star, not on closest approach.

Note the necessity for an operational definition of distance (and time).

More Implications

The new equations to describe the gravitational interactions are not the same as Newton's. (Obviously not, since *we've now found experimental predictions which differ.*) The new equations used to describe how things look from accelerated frames and ones with gravity (i.e. all the frames which exist in the actual world) are called General Relativity.

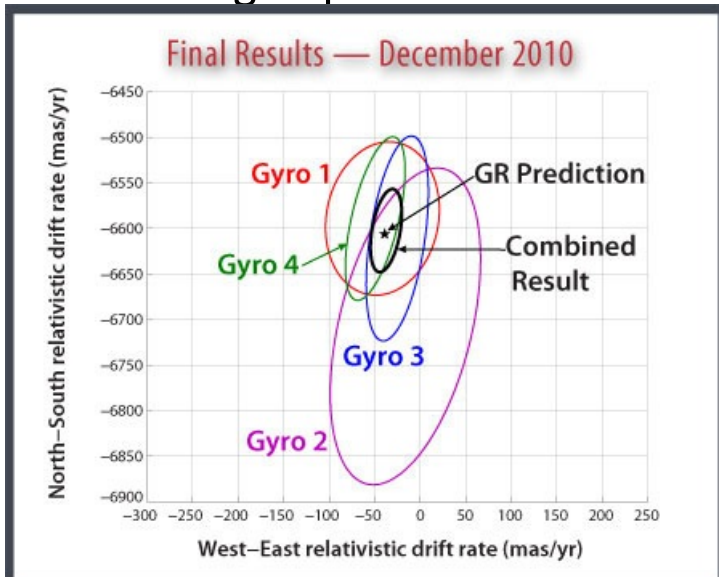
- GR also solved a long-standing problem. Remember way back when we were describing Newton's triumphs?
- The orbit of Mercury had been known since 1859 to deviate from its Newtonian prediction. (The axis of the ellipse rotated, *i.e.*, precessed.) Many unsuccessful searches had been made for objects, such as other planets, that would explain the motion. GR correctly predicts the orbit.
- Notice how small the original problems were in the data. GR, unlike SR, was not the result of the attempt to resolve a burning issue. GR was driven by philosophical motivation.

Philosophical comments

- Sociologist of science Steven Fuller, in the famous "Social Text", praises Japanese physicists ca. 1900 for using terminology which
 - "...managed to avoid protracted discussions of the ontological status of gravity and inertia that was continuing to haunt philosophically oriented Western physicists, such as Ernst Mach and Albert Einstein..."
 - So "...to discover the essence of tomorrow's science we should look to the ways in which recently enfranchised citizens of the republic of science-... from all over the world- separate the wheat from the chaff in the West's scientific legacy."
 - In other words, those philosophical worries of Einstein's were somehow deeply culture-bound "chaff" to be discarded in a broader context.
- Comments?
- Let Fuller try to use a GPS with its GR corrections turned off!

Some confirmations of General Relativity

- SR background: Repeats of Michelson-Morley, good to one part in 10^{20} .
- GPS devices positioning would drift off by about 7 miles a day, without GR corrections!
- Modern versions of the light-deflection experiment: repeatedly confirmed, now to better than 1% accuracy.
- Gravitational slowing of clocks: repeatedly confirmed, in one case to 0.03% accuracy.
- Time-delays affected by travel near sun: repeatedly confirmed to within 5% accuracy.
- Existence of gravitational lenses
- Probable existence of black holes
- Slowing of pulsar rotation due to gravitational radiation (quantitative)



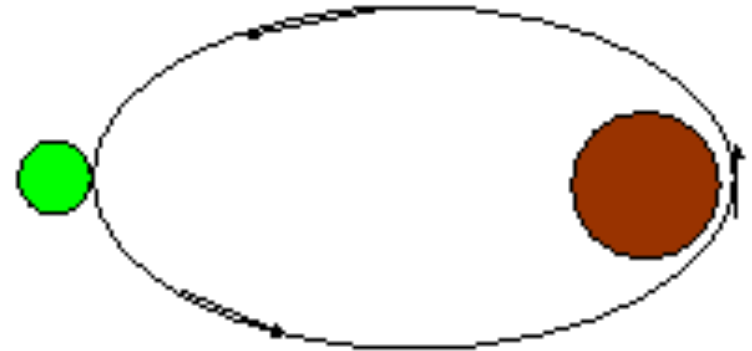
Gravity Probe B

“Frame-dragging” around Earth (to ~20%)

Earth’s geodetic effect (reduced C/R) to 0.2%

Back to the twins

- What if Alice doesn't accelerate back toward Earth using rockets, but instead just slings around using gravity from some big planet?



She feels no accelerations, at least not like she would feel a non-gravitational force. So we can't argue anymore that the one who's younger is the one who "feels no accelerations"- neither twin *feels* any accelerations. Nonetheless, Alice is younger, because she spent time DOWN in the gravitational field of that big planet, so her clocks were all running *slow*. The only evidence she would "feel" would be that clocks on one side of her ship would run faster than the other- an effect down by a factor of ship size/orbit size from the twin effect.

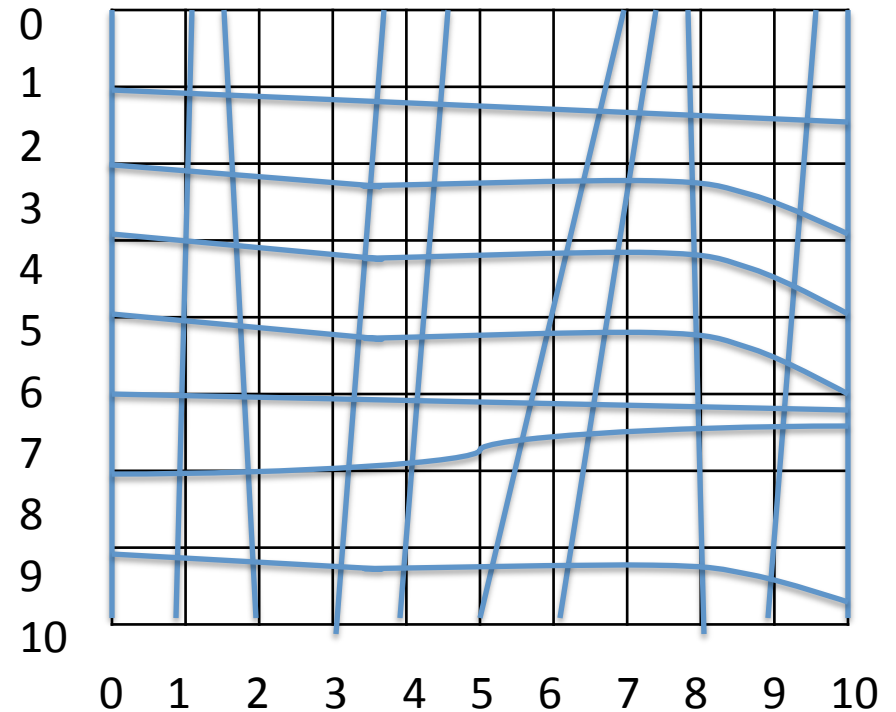
You can see G.R. outgrow some of the arguments which led into it. Now we are abandoning the deconstructive phase of asking what traditional constructs are not required by experience. We've got a new theory, and are cranking out results to see if the predictions agree with measurements, not worrying about the philosophy.

Choices of Coordinates

Either of these grids (black or blue) is OK!

How can you keep the same laws of physics?

Distances are a function not just of the difference in the coordinate labels but also of a “metric tensor” field defined at each coordinate point. That’s not an increase in complication because gravity already required some sort of field like that.

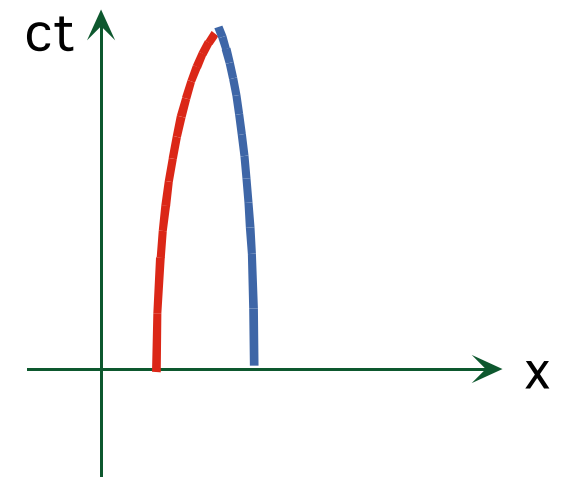
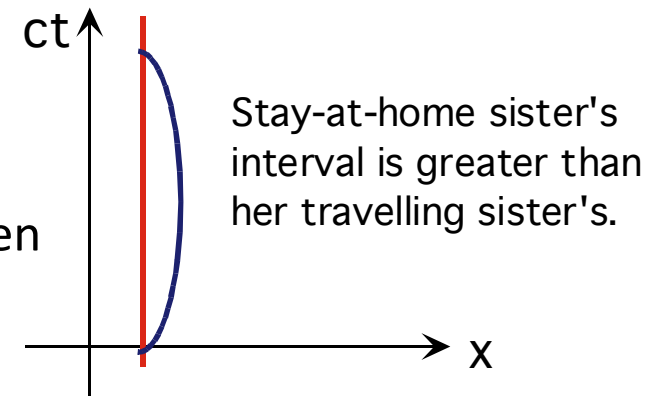


What will our straight lines be?

- If a good definition of a straight line is "the path followed by an object which experiences no forces" we must recognize:
- Gravity is not experienced: in principle a gravitational field cannot be felt locally.
- A straight line would then be a path followed by some small object in the presence of no influences other than gravity, i.e. in free fall.
 - These are called spacetime geodesics
 - This is the non-Euclidean generalization of Newton's 1st law.
- A geodesic is defined to be the shortest line between two points on a surface. In spacetime, this is generalized to be the world line which has the largest interval between two events (4-d points).
 - ("Largest" is an artifact of the - sign.)

On the sphere there are no parallel lines. All geodesics intersect.

Similarly, the interpretation of the trajectory of a dropped ball near the Earth is that its world line appears to start out parallel to the Earth's, but inevitably intersects it.



Do we need curved spacetime?

- Just to be sure it is clear:
The 2-d examples of curved surfaces allow us to look at things from “outside.” However, a 2-d person confined to the surface and with no knowledge of the 3rd dimension could still infer the *curvature from his own geometrical measurements* (e.g., the angles of triangles). That’s the situation we 4-d people find ourselves in. The curvature can be completely defined and all experimental predictions, etc. made with *no reference to some other dimensions*.
 - This claim does not mean that no other dimensions exist, just that they are irrelevant to the question of whether our familiar dimensions form a flat space.
- The immediate question is do we need curved spacetime, or can we get away with “flat” geometry? (Sklar, 55-67) We have already seen that if we want to put accelerated observers on a par with inertial ones, we need curvature. If we don’t do this, then we need to say that the forces accelerated observers experience are pseudoforces akin to the centrifugal force. Are these two views equivalent?

Global vs. Local

- They are equivalent locally (if no singularities), but not globally. Only special configurations of gravity (uniform gravity, for example) can be completely eliminated by going to an accelerated reference frame. In particular, nonuniform gravity gives tidal forces which can't be transformed away. One can maintain flat geometry if one attributes effects such as the slowing of clocks to dynamics (similar to the pre-SR Lorentz contraction, etc.), but that gains one nothing and is vulnerable to the same criticism (it's *ad hoc*) that Einstein made of the pre-SR theories.
- Imagine the universe really had the topology of a sphere, that has global consequences (*e.g.*, one might be able to go around) which can't happen in flat space. The large scale geometry of the universe is still not known. (More on this later.) However, it has been shown mathematically that the geometry must have at least one singularity- an infinite deviation from the simple flat-space picture- which would be extremely hard to mimic in some fancy flat-space model.
- Furthermore, GR makes many detailed predictions which have been beautifully confirmed, and which beat the predictions of a whole slew of rivals- most of which also have curved space, anyway. There is no competing theory with the same simplicity and predictive power, except theories whose predictions are already known to be wrong.

Some conceptual implications

- Spacetime is beginning to become more substantial than it was (contrary to Einstein's original motivation). The geometry of spacetime varies from place to place in a way that is observable. We'll see next time that it's even more real than this.
- Elevating unexplained "coincidences" (the equivalence principle) into general postulates succeeded again.
 - Will it work in general?
- Geometry is empirical.
 - Kant was wrong that it was only possible to conceive of the world in Euclidean terms.
 - The world violates Euclid's axioms.

Is space real?

- If GR forces us into non-Euclidean geometry, does this require that space is “real”?
 - A relationist would say that this only implies that the geometrical relations between objects is not what we thought. No substantive spacetime is required.
- However, there are two new substantialist arguments.
 - In GR the gravitational interaction between objects is mediated by the geometry. That is, geometry plays the same role as, *e.g.*, the electric field. An object distorts the geometry in its vicinity. This distortion affects the motion of other objects, because the geodesics are modified. Thus, *spacetime plays a more direct role in the dynamics than in Newton’s physics.*
 - Finite propagation speeds give fields more of a reality in SR (and GR) than they had before. We already saw quite dramatically in electromagnetism, in which wave motion of the fields was predicted and then observed. GR makes a similar prediction for the gravitational field. If the Sun were to move we would feel the changed gravity at the Earth 500 seconds later. As the Sun shakes back and forth, say as the planets orbit, GR says it would emit gravitational radiation (waves) with many of the same properties of EM waves.
 - If they exist, gravitational waves are as real as any other object. They carry energy and momentum. Only indirect but strong evidence (binary pulsar period changes) exists now, but detectors are being built to look for them.

Relationist vs. substantivalist: Split Decision

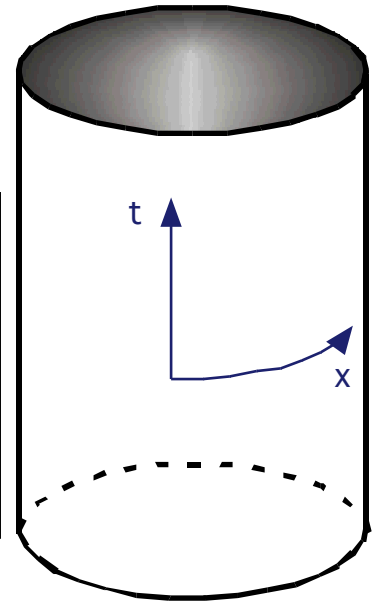
- Space-time seems to have observable properties in itself, like electromagnetic fields, etc.
- But: These properties are far from resembling those expected for Newton's space.
- Einstein's original motivation was to develop physics that followed Mach's principle, but GR does not follow that principle. (And Mach was unable to follow SR, much less GR.) It may be possible to add Mach's principle as a separate requirement, i.e. to rule out those GR spacetimes which do not obey Mach's principle, but nothing about the structure of GR itself tells you to do so.
- Newton said (in effect) that two masses tied together and spun would stretch a string taut, because they would need a force to keep them both accelerating toward the middle, regardless of the condition or existence of anything else. They have "absolute acceleration". Mach said the string could not go taut because "absolute acceleration" is meaningless- you need the other stuff in the universe to create the forces. Einstein abandons the phrase "absolute acceleration", but GR allows solutions in which the string is taut. Such a solution can either be described as two masses rotating in nearly flat space-time *or* as a strange twisty space-time exerting peculiar gravitational forces. But operationally, Newton and Einstein agree on what the possibilities are, and they *include* the possibilities excluded by Mach.

GR outgrew its philosophical ancestry.

Topology

- Curved geometry allows different connectedness, i.e. “topologies”, e.g. how many holes.
- Consider a cylindrical universe:

The **local geometry of a cylinder is flat**, but one coordinate is cyclic. If our universe had this topology, there would be **a preferred reference frame**, the one with the time axis along the cylinder, as shown. This is an example of the situation not manifesting the symmetry of the physical laws. We'll encounter this again later.



- Would this mean that Galilean relativity is wrong?
 - Is the geometry of the universe a “law” or just a “fact”?

Consider a donut universe (not valid):

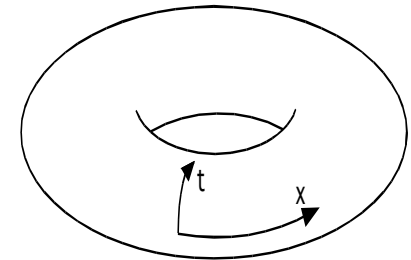
The local geometry is curved.

The time coordinate is cyclic.

How can one distinguish past from future here?

What about causality?

There may be *valid* solutions to GR with this problem.



Preferred Frames?

- Triplets A, B, and C are “not moving” together.
A and C give each other a shove and head opposite ways.
If the universe is finite, they’ll ultimately meet up again (and with B too). From B’s point of view, they should be the same age, by symmetry. (Assuming matter isn’t too lumpy, etc.)
Are they? Will A and C be younger than B?
- What if A is heavier than C? B says C travels faster, should age less.
- Is C younger than A when they meet again?
- But A’s motion as seen by C is just the exact reverse of C’s as seen by A!
- These “inertial” (free-fall) frames don’t agree whether A and C is older when the meet.
- But there’s an actual result. So only one frame (at most) can be right.
- This result is general for compact topologies.

The twin paradox in compact spaces

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Twins travelling at constant relative velocity will each see the other’s time dilate leading to the apparent paradox that each twin believes the other ages more slowly. In a finite space, the twins can both be on inertial, periodic orbits so that they have the opportunity to compare their ages