Beginning Modern Quantum

Born's probability interpretation

The indeterminacy ("uncertainty") principle

the Schroedinger equation

<u>Next time</u>:

We rule out any comfortable explanations

Particle Waves

- Light
  - is a wave. It exhibits interference (Young, 1814).
  - now it is seen to have some particle properties: photoelectric effect & Compton scattering
- Electrons
  - Appear at fluorescent screen (CRT) at a point, like particles.
  - H wave properties: Interference (Davisson, ~1922).
- <u>Our old particles </u>have frequency, wavelength…
- <u>Our old waves </u>have discrete lumps of energy, momentum….
  - **<u>The old dualism</u>** (world made of particles interacting by fields) **<u>is gone-</u> everything consists of quantum objects** which have both wave-like and particle-like aspects, which become relevant in different situations.
- The common claim that these objects are *both* waves and particles is false- they're just something else, with a resemblance to both classical waves and classical particles, but **also with properties of *neither***.
- We *seem* to be saying something very incoherent. A wave cannot have a wavelength, even approximately, unless it is spread out over distances large compared with the wavelength. A particle is supposed to have a particular position. How can we say "the momentum of the particle is given by its wavelength?"

The wave and its equation

- The electron is described by a wave function, $\Psi(\mathbf{r},t)$, which obeys a differential equation. The non-relativistic version is called Schrödinger's equation.

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(r)\psi = i\hbar\frac{\partial\psi}{\partial t}$$

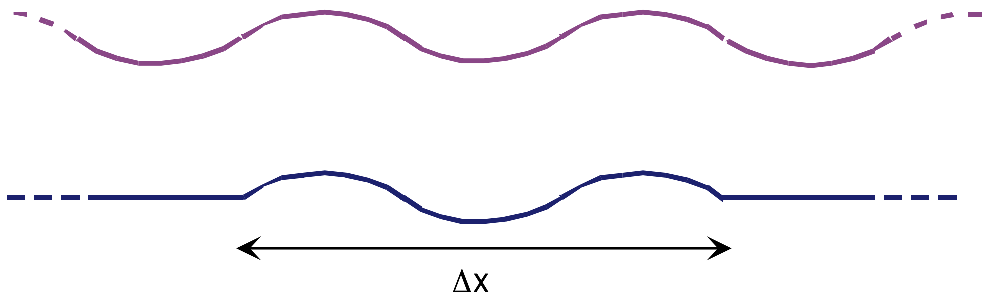
- First term, (squared momentum), depends on how y wiggles in space. Like 1/wavelength squared, $p^2/2m$
  - Second term, (potential energy), due to various neighbors (whose positions are presumed fixed in our reference frame).
  - Third term (total energy) is how fast y changes in time: frequency. E=hf.
- This equation is linear, which means that the principle of superposition works:
  - **Adding any two solutions produces another solution.**

Born's probability interpretation

- Recall that the intensity (energy density) of a wave goes as the *square* of the amplitude (for light the magnitude of the electric field, for water ripples the height change).
- Quantum mechanics says that if we consider an ensemble (collection) of identically prepared electrons, each described by similar wave functions, y(x,t),(obviously with starting t shifted)
- $|\mathbf{\Psi}(x,t)|^2 \Delta V$ is the probability that an electron would be found in the little volume DV near point x at time t, *if* an experiment is done that could locate it that accurately.
- Because $|\mathbf{\Psi}^2|$ gives a *probability density*, when we have a large ensemble it tells us the rate at which electrons arrive at the spot of interest on the screen. In the places where the two waves interfere destructively, the probability is less than the sum of the two individual probabilities, and may even be zero.
- There will be a fundamental loss of determinism unless there's something else *beyond* the wave function (i.e. not in the theory) which guides the outcome.
- This recipe does not claim to tell us what $\mathbf{\Psi}$ "is".

An important mathematical property of waves

- The wavelength of a wave, describes a sinusoidal function of position, sin(kx), where k = 2π/l. The sine function oscillates for all x between ± ∞. Thus, if we limit the spatial extent of the wave, it is no longer a simple sine wave and is not described by a single wavelength.



  Superposition lets us write a spatially limited wave as the sum of a many sine waves of various wavelengths. ("Fourier decomposition") Fourier analysis shows that if the wave is limited to a spatial region Δx, the spread of k values in the sum is approximately 1/Δx. One can prove that $\Delta x\, \Delta k \geq 1/2$. Δ means "the spread of," or "the uncertainty of."

- Classically, position and momentum were specified by separate vectors. In QM they are both specified by Ψ(**r**,t), butΨ(**r**,t) cannot both wiggle at a steady rate in space and be confined to one point in space, so *it cannot specify precisely both **r** and **p***.
- **A quantum state cannot have both precise p_x and precise x.**

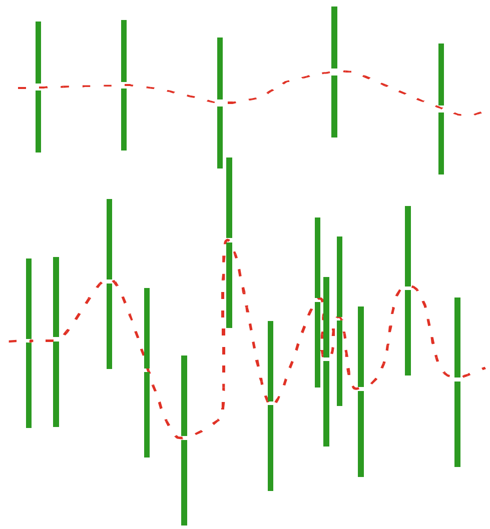
Heisenberg uncertainty principle

- Our Fourier analysis of the wave, gives: $\Delta x \, \Delta p \geq \frac{\hbar}{2}$.
- One often reads that the uncertainty principle is merely a statement about our lack of knowledge of the electron's position and momentum, but that's false. Exact position and momentum are not attributes that any object ever has at the same time. Assuming otherwise leads to incorrect predictions.
- Why call that an "uncertainty"? A water wave also has a spread in positions and directions, but we don't blather about its "uncertainty." This is uncertainty, not just classical spread, because various measurements (e.g. letting the electron hit a screen) don't give results with the whole spread. We only see part of the spread, and are uncertain which part it will be. For the water wave we are certain to see the whole spread.

- Note the difference between this QM unmeasurability and previous unobservables, such as the ether. If we assume that the ether exists, we open up the possibility of making various hypotheses about how to find it, none of which work. So, for simplicity, we say it doesn't exist. If we say that precise x and p simultaneously exist (at least in the usual meaning of those words) we will directly run into predictions which violate both QM and experience, since interference is found between parts of the wave at different x's and p's, leaving it very hard to see how those variables could have had only single values.

Trajectories?

- <u>The uncertainty principle means there are no classical trajectories</u>
- Suppose we try to make an accurate measurement of the path of our electron by passing it through a bunch of slits or otherwise determining the position to some accuracy, e.g. by looking via light

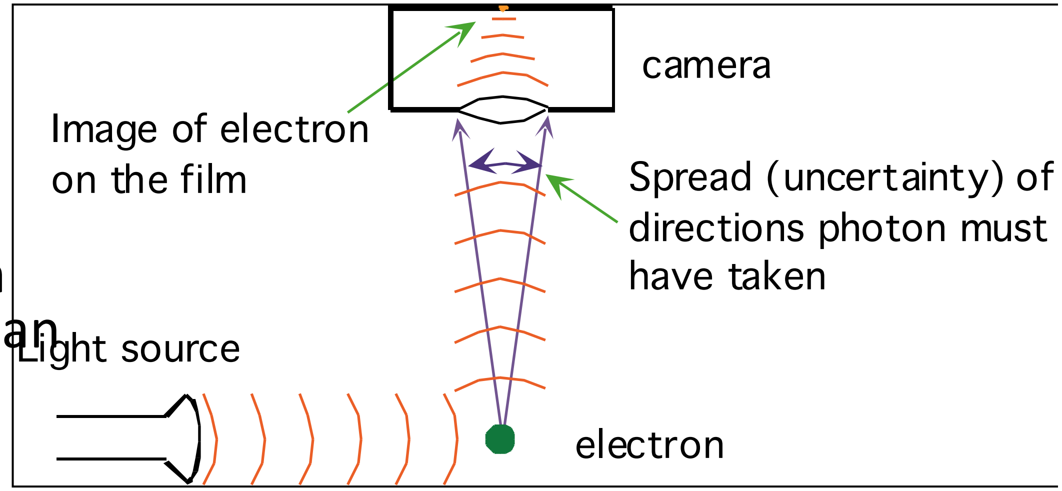


- What happens if we try to improve the accurately by narrowing the slits? The uncertainty principle foils us.
- Any attempt at increased accuracy merely yields a more scattered set of measurements. You can't do something to measure the trajectories without ending up with a different arrival pattern- meaning that <u>you haven't found the trajectories of the initial problem.</u>
- For classical <u>waves</u> there are also no trajectories- but that's not a problem because the wave is *certainly* spread out. If you try to measure where the wave is, you don't get the strange result that it's just at one spot, you see that it is spread out.

The Heisenberg microscope

How the quantum nature of the photon prevents our violating the uncertainty principle.

- Suppose we try to measure both the position and momentum of an electron by looking at it with a camera:

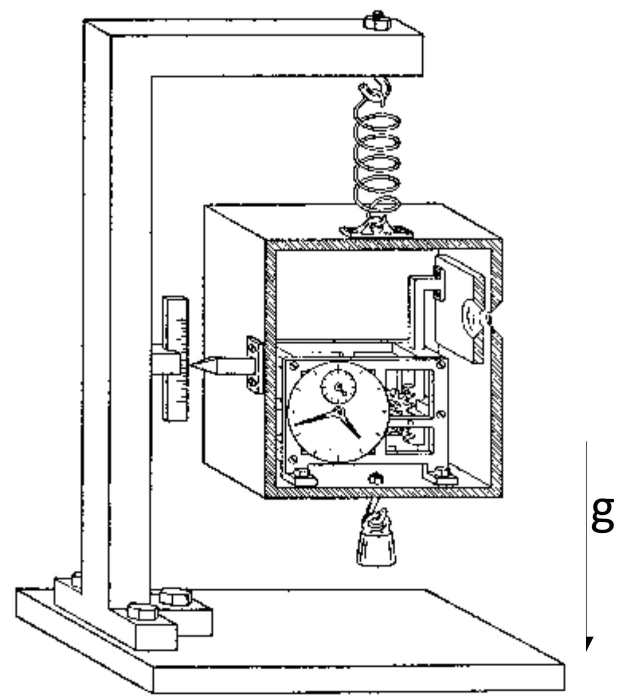


We are limited by the quantum nature of the photon. The spot on the camera film is limited by diffraction as the light passes through the lens. If we make the lens bigger, then we don't know which direction the photon was going after it bounced off the electron. Similarly, reducing the wavelength reduces **Δ**x, but the photon now has more momentum, and thus a larger Δp.
Thus, we can have small Δx or Δp, but not both.

Every type of object, or none, must share the uncertainty rule, otherwise you could unravel the uncertainty of one type with microscopes using a less uncertain type.

Bohr and Einstein

- Had an extended debate over whether there was some way around the uncertainty relations. One uncertainty relation (between the frequency of a wave and the time at which it comes by) translates to an uncertainty relation between the energy of a particle and the time at which it is emitted.
- Einstein proposed putting some particle emitter on a scale, e.g. a spring-held platform in the Earth's gravitational field. If a shutter is opened briefly, with the time of opening set by a timer, you know just when the particle got out. If the scale is initially in balance, you see the weight change by watching the rate at which the box picks up upward momentum after it is lightened by emitting a particle. So you know the change of weight of the box, so you know m (and $E = mc^2$). These are classical measurements, so it seems that you should be able to know E and t to arbitrary accuracy, contrary to the uncertainty principle.



Bohr Wins

- Bohr pointed out that the position and momentum of the scale had to obey an uncertainty relation. It's true that to weigh the emitted particle, all you care about is the change of the *scale's* momentum, so Einstein had assumed that the momentum was well defined. But Bohr reminded him that General Relativity implied that <u>the rate at which the clock ran depended on how high up it was in the Earth's gravitational field</u>! Trying to get a well-defined momentum gives a very uncertain position, and that IS relevant because it affects what the time of the event is (in our frame). If the scale position/ momentum initially obey the uncertainty relation, so will the measurements of the particle energy and time.

$$mg = \frac{p}{t}$$

$$\Delta E = c^2 \Delta m = \frac{c^2 \Delta p}{gt}$$

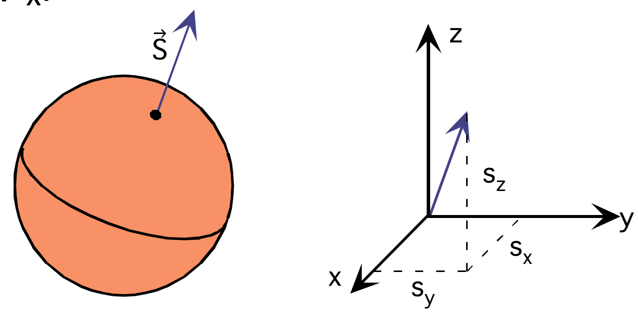
$$\Delta t = t \Delta x \left(\frac{g}{c^2} \right) \quad from \quad G.R.$$

$$\Delta t \Delta E = \Delta x \Delta p \geq \hbar / 2$$

Notice the astounding unity of physics:
the self-consistency of QM was saved by GR!
Einstein temporarily gave up.

Uncertainty relations for Spin

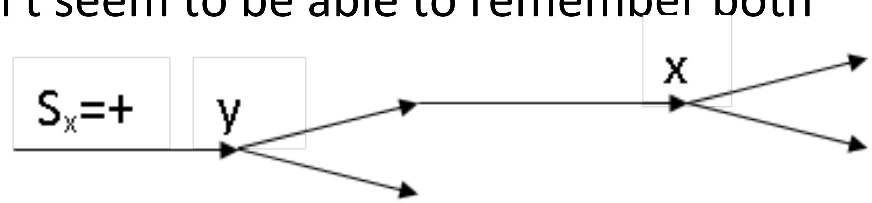
- In QM, many physical systems have complementary pairs of observables which cannot be measured at the same time. E.g. the product of the uncertainties in position (x) and momentum (p_x) must exceed .
- Another physical quantity, spin, will be important in arguments to follow. Think of a classical spinning ball. Its spin angular momentum points along the axis of rotation and has a length equal to the rate of rotation times the moment of inertia. It is a vector, , and all three components can be specified.



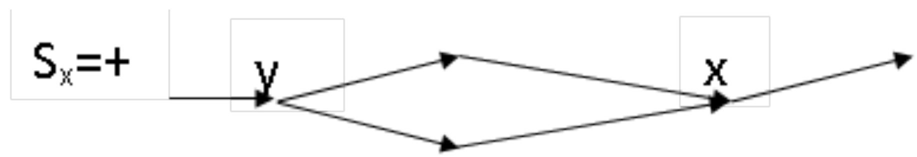
- In QM, pairs of spin components satisfy uncertainty relations $\Delta s_j \Delta s_k \geq h$. At most one component of the spin can have a definite value. Results of spin measurements are quantized. When one measures s_x, one always finds a multiple of $\hbar / 2$.

Experimental implications

- If you separate a beam of neutrons into s_x= +1/2 and s_x =-1/2 beams (by running through magnet pole-faces), you can discard the (-1/2) part to get a beam of pure s_x =1/2 neutrons.
- Now try measuring s_y (just using a magnet turned 90°): you find that the measurements still give ±1/2, with a random pattern of + and - results.
- If you take either the s_y= 1/2 or the s_y= 1/2 beam, and again try measuring s_x, you also find random results. The neutrons don't seem to be able to remember both values at once. (the uncertainty relation)



- But if you recombine the s_y= 1/2 and the s_y= 1/2 beams without measuring, i.e. without letting them interact with some sort of detector, the resulting beam is still all s_x= +1/2.



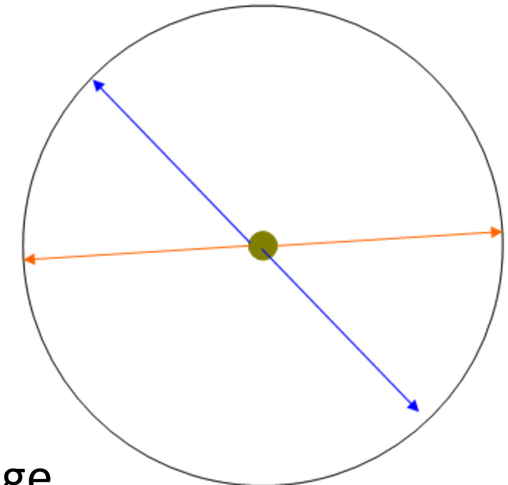
- Each s_x= +1/2 was BOTH s_y= +1/2 and s_y= -1/2, and follows BOTH pathways . Only a "measurement" makes it choose one or the other. Apparently s_y is not specified by a hidden variable, since each s_x= +1/2 neutron seems to have <u>both</u> values of s_y.

Into the Unknown

- Something major is going to have to change in our basic ideas about how we observe and describe reality, or even if that concept applies. This issue is the main focus of the next part of the course.
- Unlike the earlier parts of the course, where it was possible in a reasonable amount of time to put ourselves in the position of historical scientists wondering how to make sense of things, in this part the problems are too deep and unfamiliar to follow that procedure. We will try to present the main ideas of QM and the problems they raise in a logical fashion, rather than historically. In particular, experimental results from the 1980 on will be introduced fairly early, rather than allowing the sort of amorphous ideas about interpretation that prevailed from about 1930 until then.
- Before we discuss the many interpretations of "measurement", which range from the very troubling to the incredibly bizarre, we need to establish some ground rules, which will eliminate all the more comfortable possibilities. This is a departure from the historical discussion, in which the contradictions between QM and our ordinary sense of reality took some time to clarify.

Einstein-Podolsky-Rosen

- Einstein and collaborators (EPR) proposed that by using the conservation laws, one could show that QM was missing something. Consider a particle that flies apart into two particles, each detected somewhere on a sphere of detectors.
- The blue pair or the red pair might occur, but not a mixture, which would violate conservation of momentum
- Conservation of momentum says the particles have to go opposite directions.
  QM says they don't know which way they're going.



- Possible resolutions:
  - The particles don't have to be detected in opposite directions, the conservation laws only hold on the average. (Bohr thought this at one time, but it's completely wrong experimentally.)
  - The particles are always found in opposite directions, because there is some hidden variable which allows them to know which way they are going. QM is incomplete!
  - Even though it is predetermined that the particles go opposite directions, what those directions are is not determined until one is (randomly) detected. The other somehow knows which way to go, faster than the speed of light! (Einstein called this "spooky correlations at a distance")
- Einstein believed that this argument showed the incompleteness of QM. What we are about to show is that, even if you had never heard of QM, experiment shows that nature does indeed have "spooky correlations at a distance."