Return to the entropic arrow of time

<u>Next time</u>: (and beyond)

QM + SR -> Field theory: are there "building blocks"?

- A new conception of "empty"space
- Cosmology – the beginning and end of time

Entropy and QM

- QM gets rid of some ambiguity about the meaning of entropy. Classically, entropy had something to do with the number of micro-arrangements consistent with the macroscopic appearance. This is ambiguous in several respects- one being that there is no obvious classical way of counting arrangements, which have a continuum infinity of possibilities. QM avoids this, because we can write a discrete list of quantum states and their probabilities, given some macroscopic knowledge of the system. Entropy is the log of the number of possible quantum states.
  - Technical details: some quantum states will in general be more likely than others, given known facts about the system. There's still a precise formula for the entropy in terms of the states' probabilities.
  - Sometimes what's known about the system says not just how likely it is to be in the various states in your list, but also what particular superpositions of those states are most likely. (e.g. two-well system, in one well) There's a precise generalization of the entropy formula which deals with those cases too: $-\mathrm{Tr}(\rho\ln(\rho))$.
  - The entropy comes out *independent of what list of quantum states you chose*.
    - e.g. eigenstates of energy, or eigenstates of some other variable

Entropy and Subjectivity

So QM has also allowed us to clean up the definition of entropy

<u>but the definition of entropy still has a somewhat subjective ring</u>

- **<u>If you specify the quantum state, the entropy is zero, because log[1]=0</u>**
- Therefore the entropy is still a function of the set of variables specified (energy, etc.) and of the accuracy of their specification, not just of the state of the system.

Restate the second law

Left to itself, any nearly closed system is equally likely to end up in any of the allowed quantum states consistent with the known energy, number of particles, etc.

- **Therefore you will find the system with values of various measurable quantities which give maximum entropy,**
  - **i.e. are consistent with the maximum possible number of states.**
- **This remains highly directional in time. It says what will be, but not what used to be.**

Order, disorder, etc.?

Do NOT be misled by various statements about "order".

- The net entropy of water and its surroundings *increase*
  - when an ice-cube melts in warm weather,
  - *and* when an ice-cube freezes in cold weather.
  - Energy is released by the freezing, and that energy goes off to make entropy in the surroundings.
- The temperature is a measure of how much energy is needed to make some entropy.
  - If T is high, entropy is maximized by having the water in the apparently disordered state.
  - But if T is low, the released energy can make lots of entropy, so total entropy is maximized by the ice forming.
- **The ancient tradition (e.g. Aristotle) was full of explanations about why things tended toward some final state, such as heavy stuff down, absence of motion of things on the Earth, etc. (teleology) The <u>various teleological principles are now replaced with this blank "equal probabilities of states"</u>, coupled with statistical mechanics, the techniques for counting states.**

Boltzmann's statistical argument for the 2nd law

1. **You don't know the actual microstate.**
2. **So your best guess about the future will be one consistent with as many microstates as possible, so long as they are consistent with the current observation.**
3. **Which can be shown to lead to constant or increasing entropy.**

What's wrong with that reasoning as an explanation for the time asymmetry?

Assume that Boltzmann's mathematical argument for point (3) is ok.

Boltzmann in reverse

Pointed out by Boltzmann's friend Loschmidt

1. You don't know the actual microstate.
2. So your best guess about the past will be one consistent with as many microstates as possible, so long as they are consistent with the current observation.
3. Which can be shown to lead to constant or *decreasing* entropy.
   By just the same math as used before.

A false conclusion!

- So Boltzmann must have snuck in an assumption:
  - essentially that the past was ordered in a special way which precludes the use of simple statistical arguments about it.
  - But that is precisely the asymmetry which we wanted to explain.
  - It's now clarified, but not at all explained.

Why does S increase, not decrease?

- Boltzmann also argues that it is meaningless to argue about why the past is the low-entropy direction in time, that given an asymmetry we are bound to use different names for that direction and the other one.
- The question for Boltzmann is *why there's an asymmetry*, not why the "past" has a particular property.
- The implication is that if somehow a low-entropy configuration happened by very rare accident, the times around that, in which entropy is changing monotonically in time, will be suitable for experiencing one direction as past and the other as future.
- The explanation for why we are observing one of these rare stretches of time would then be that no observers could evolve in more ordinary, equilibrium times.
  - An early anthropic argument!
- This still does nothing to explain the *homogeneity* of time's entropic arrow: why would it be changing the same way throughout the known universe? If the original low-entropy condition is an accident, we need an explanation for why the whole universe is part of the same accident. That will require another look at cosmology.

Entropy: the Liouville problem

- There's a classical theorem that the volume in classical "phase space" that some system might be in *does not change in time* as it follows the classical equations of motion. But the classical entropy is just the log of the accessible volume in phase space.
- So how can it increase in time?

- Although the net volume in phase pace doesn't change, the possible results change from being a solid trunk of nearby values (all positions and momenta approximately known) to a set of small fibers (all the positions and momenta might be very near any of a large number of very different possibilities).
- If your description is forced to be "coarse grained" that means that you lose all predictability about the results. I.e. you get some useless information about exactly what the coordinates are if they are approximately known, but you lose the basic information about what the approximate values will be.
- How can a basic law of nature depend on our "coarse graining" of the description?

Quantum coarse-graining

- There's a quantum analog of the coarse-graining problem.
- There is no linear operator (i.e. linear function of φ like the operators representing *all* the other physical variables) whose expectation value always increases in time. Entropy is not like the other physical variables.
- Given some initial probabilities to be in a collection of energy eigenstates, the linear time dependence says that those probabilities never change in a closed system. (quantum Liouville)
- Hence the entropy (defined in terms of those probabilities) never changes.
- Only if you choose some cruder description than the formal probabilities of the states can you get an entropy increasing in time in a closed system.
- Again, how can a universal law of nature depend on the crudeness of our description?
- The second law is not philosophy, but the key guiding principle in chemistry, thermal physics, engine design, materials science, etc.
- How can they escape the philosophical problems?

Quantum coarse graining

- The second law says that all the allowed quantum states become equally probable in a closed system. But we ran into problems, because if you're in some state of a closed system, you stay in that state.

- The density of states of real systems becomes exponentially huge as you consider larger systems. The states are extremely close to each other in energy.

- Although external perturbations become small and unsystematic, there is no way to prevent them from causing transitions to nearby states. They serve to randomly stir the system. **Thus for large systems, the claim that the system actual is in one particular quantum state can have no practical consequences.**

- Also, for large systems, the dependence of the entropy on the details of what's specified (e.g. whether you specify energy or temperature) becomes very small compared to the total entropy.

- So in practice, for large systems you can just speak of an entropy, without worrying about the details of the knowledge of the system.

Entropy’s philosophical problem

- The inevitable tiny interactions with the outside world stir any large system among its many quantum states. So it's fine to use the concept of entropy in practice.
- But what are we to make of the claim "The entropy of the universe always increases"?
- **Is the universe in a quantum state or not?** If it is, what is the definition of its entropy? Remember, log[1]=0, always.
- Is there some physics, currently unknown to us, which sets an *intrinsic* coarse-graining scale, i.e. some scale on which the wave-function is not an appropriate description?
- Is that related to the quantum measurement question of whether there is some natural scale on which the wave-equation no longer gives the complete description of the time-dependence?
- Is it related to quantum gravity?

Entropy and Quantum Entanglement

- No system (except possibly the whole universe) is completely closed.
- Interactions cause local systems to become entangled with increasingly remote systems.
- Start with two weakly interacting remote systems, un-entangled.
  Any possible state of A can go with any state of B, so:
  - $S_{tot}=S_A+S_B$
- The Liouville theorem says that S_{tot} doesn't change assuming QM is correct.
- If they each evolved *independently* the Liouville theorem would say that each of S_A and S_B would stay constant.
- The interaction makes A and B entangled. Some states of A coexist only with some states of B and vice versa.

Second Law and Entanglement

- Say there were 10^{10} initial states possible for each A and B, or 10^{20} total for the whole un-entangled combo.
- After a little entanglement you have 10^{11} possible states for each. But each can only coexist with 10^{9} states of the other.
- Still 10^{20} total.
- Since not every state in A pairs up with every state in B, $S_{tot} < S_A + S_B$.
- So we started with $S_{tot} = S_A + S_B$.
- And now have $S_{tot} < S_A + S_B$.
- But S_{tot} didn't change (by theorem).
- So $S_A + S_B$ must have grown!
- Increasing entanglement is the same as increasing entropy of the various local parts. There's a *negative entropy of remote entanglement*.
- Can this growth of local entropies go on forever?
- That depends on cosmology!
- But before we can really discuss cosmology, we need to know a little more about the ingredients: how to combine SR+QM?