Lecture 8

Introduction to Biophysics

Diffusion

How fast are small molecules moving in a cell?

How often do things come in contact?

Are chemical reactions rates limited by availability of food (ATP)?

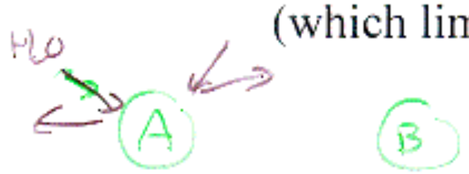
Movement by random motion: diffusion.
   Limits to cell size based on oxygen diffusion/availability.

What limits how fast a cell can reproduce?
(<1 hrs for bacteria; ~day for humans)

Inertia does not matter for bacteria or anything that is small / microscopic levels.

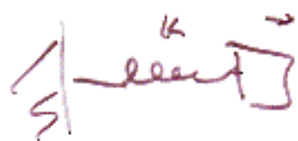
Translation & Equipartition Theorem

For two things to react, need to come in contact.
What is average speed (and distance between) molecules in cell?
Time between collisions?
How long oxygen to take to go across cell (which limits cell size)?



m_A, T

Thermal Energy $= \frac{3}{2}kT$



P.E. $= \frac{1}{2}kx^2$

free

K.E. $= \frac{1}{2}mv^2$

Equipartition Theorem

For each degree of freedom ~~that dep~~ where energy depends on (deg. of freed)^2 $\equiv$ $\frac{1}{2}kT$ of energy

$\frac{1}{2}kT$ P.E. $\frac{1}{2}kx^2$

$\frac{1}{2}kT$ K.E. $\frac{1}{2}mv^2$

$K_B = 1.4 \times 10^{-23}\ J/{}^\circ K$

$T = 300\ {}^\circ K$

$K_B T = 4.2 \times 10^{-21}\ J$

$(1\ J = 1\ N\text{-}m)$

$1\ N \sim 1/4\ lb$

$1\ N \approx 1/5\ lb)$

What is velocity of water molecule at room temperature?

5

$$\tau_{collision} = \frac{d}{v}$$ → mean free path
→ dist. H_2O mol. will go before hitting another H_2O molecule.

Density of water = 1 g/cm³

(55 M H_2O)

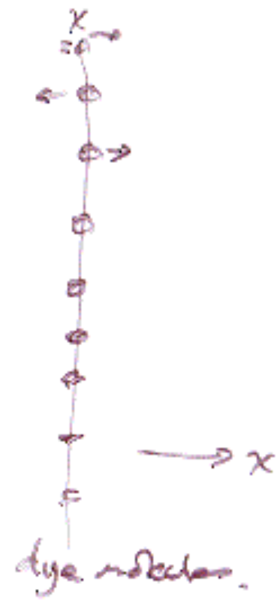
water molecules / cm³

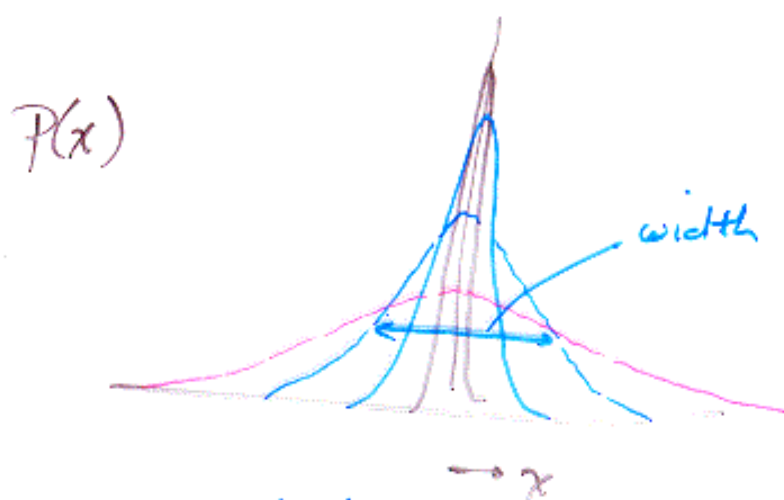
av. dist = $(\ )^{1/3}$ → cm

$\tau_{collision}$ = ??

Longer Distances (>> mean free path)

Random Walk: Diffusion (more later lectures)





$t=0 \quad P(x=0)=1$
$\qquad\;\; P(x\neq 0)=0$

$\langle x \rangle = 0$ average pos. of dye = 0 → as likely to ~~drift~~ randomly move left (−) as right (0)

$\langle |x| \rangle = \langle \sqrt{x^2} \rangle$

$\langle x^2 \rangle$ at $t=0$ $=0$

$\langle x^2 \rangle$ at later time t $\neq 0$

$=$ pos. #.

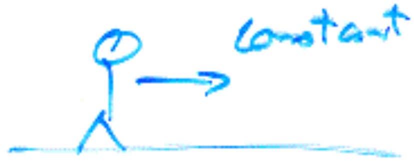
$\langle x^2 \rangle$ is a measure of width of distribution

$\langle x^2 \rangle$ gets bigger in time.

if linear.

$x = \cancel{\frac{\Delta x}{t}}\ vt$

constant speed with no bump.



go! $x = \text{linear}(t)$

In a given period of time
X is smaller in random diffusion
than if molecule just went
at constant speed

$$X \sim \sqrt{t}$$

$$\langle x^2 \rangle = 2Dt$$ one dim

D = constant, diffusion const.
property of molecule

depends on dimension

1-D: # = 2

2-D: # = 4

3-D: # = 6

2 = #
t = time

3-D.
$$\langle r^2 \rangle = 6Dt$$

$$|x| \sim \sqrt{2D\underline{\underline{t}}}$$

if in 1 second it's gone a distance x_1
" 2 sec " " " $\sqrt{2}\,x_1$

Diffusion: x^2 = # Dt

Diffusion as a Random Walk

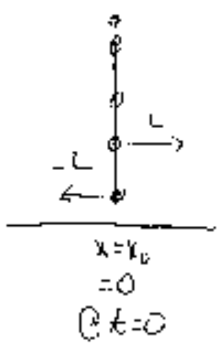
1-D case (first)

Particle at $x=0$ $t=0$.

1) Assume equally likely to step to right as step to left.

2) Take steps of length L every τ seconds.

moving with velocity $\pm v$ between collisions ($L = \pm v\tau$)
R steps/sec ; total of N steps
(For now take v, τ as constants - they actually depend on size of particle, nature of fluid, Temp)



Of course in reality, distribution of step sizes but this model works amazingly well.

Thermal Motion: Move ±L

How far do particles move due to thermal motion?

We cannot predict motion of individual molecules, but can make statistical (probabilistic) arguments about average properties, as well as distribution (standard deviation) of those properties

Position after N steps $= x_N$

" " $N+1$ " $= x_{N+1}$

$x_{N+1} = x_N \pm L$

$\langle x_N \rangle = x_0 = 0$ ← for convenience we'll call starting position $x_0 = 0$

$\langle\ \rangle$ means average if we looked at many molecules (or 1 molecule many times, ~~at~~ each time, after N steps)

$\langle x_N \rangle = 0$ by symmetry - equally likely to step left as right

$\langle x_N^2 \rangle =$

$=$

$\langle x_N^2 \rangle =$

$$\langle X_1^2 \rangle = \langle X_0^2 \rangle + L^2$$

$$\langle X_2^2 \rangle = \langle X_1^2 \rangle + L^2 = \langle X_0^2 \rangle + 2L^2$$

$$\langle X_3^2 \rangle = \langle X_2^2 \rangle + L^2 = \langle X_0^2 \rangle + 3L^2$$

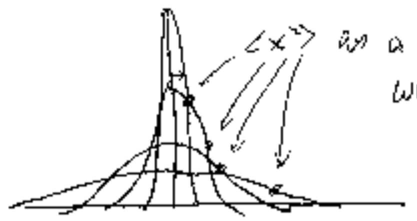
$$\vdots$$

$$\langle X_N^2 \rangle = \langle X_0^2 \rangle + NL^2$$

$$\sqrt{\Delta X_N^2} = \sqrt{\langle X_N^2 \rangle - \langle X_0^2 \rangle} = NL^2$$

$$\boxed{\Delta X_N^2 = NL^2}$$

The average distance moved $\sim \sqrt{N}\, L$



$\langle x^2 \rangle$ as a function of time
width increases as the sqrt (time)

In order for a particle to wander twice as far takes 4 times longer

Plug in some #'s for $D \sim \cancel{300}\ 250\ \mu m^2/sec$ (small molecule in water)

To diffuse ~

$0.1\mu m \Rightarrow \quad 0.01\mu m^2 = (2)(250\mu m^2/sec)\, t$

$t \sim 20\mu sec$ (fast)

$\langle x^2 \rangle = 6Dt$

D - diffussion const.

if molecule gets bigger

D↑ or D↓

$$D = \frac{k_B T}{6\pi\eta R}$$

Size of cell?

Bacterial cell ~1 μm
10x less distance than eukaryotic cell
100x less time

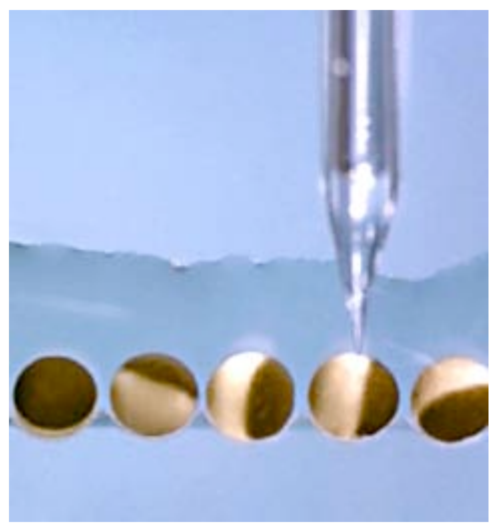
Metabolism of bacterial can (and is) much higher than eukaryotic cell
↳ 10-100μm.

bacterial cell ~1 x 3μm
eukaryotic cell → 10-100μm

Size of eukaryotes limited by size (diffusion time of O_2). As size gets bigger, everything happens more slowly.

Large cell: frog oocytes– basically everything happens slowly.

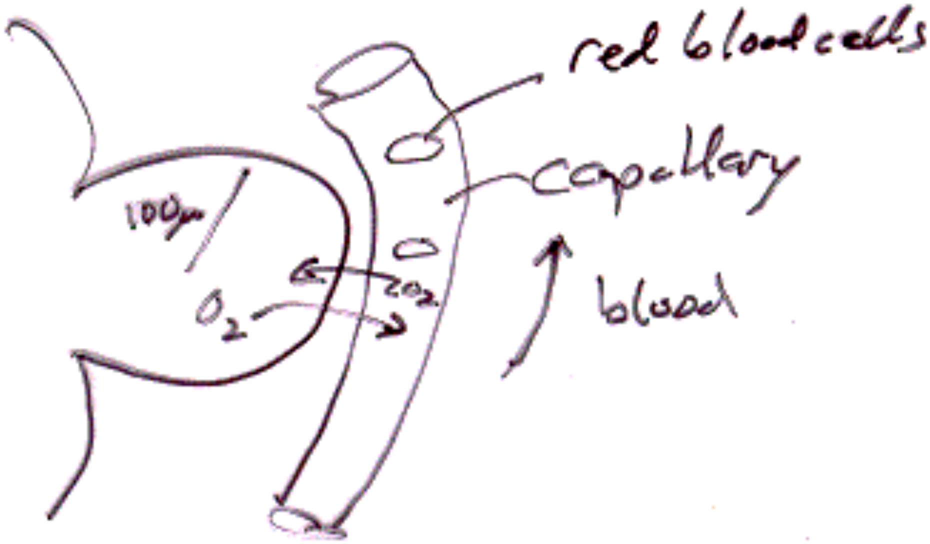
<u>Every cell needs to be within 50-100 μm of blood supply!</u>



Oocyte:1-2 mm!

Lung + Diffusion of O_2/CO_2

- Billions of air sacks (alveoli)



Can diffusion move O_2, CO_2 + enough?

Efficiency of Diffusion

Diffusion moves things short distances very fast!

$$\langle x^2(t)\rangle = 2Dt$$

$$\sqrt{\langle x^2(t)\rangle} = \sqrt{2Dt}$$

The mean squared displacement increases linearly with time

The mean (average) absolute distance increases as square-root of time.

Distance moved via diffusion $\sim \sqrt{t}$

$$V_{ave} = \frac{d\langle\sqrt{x^2_{ave}}\rangle}{dt}$$

$$\frac{d}{dt}\left(x^2 = 2Dt\right)$$

$$x\frac{dx}{dt} = D$$

$$\frac{dx}{dt} = \frac{D}{x} \sim \sqrt{\frac{D}{t}}$$

$\boxed{\text{very fast spreading for short times}}$

$t \to 0 \quad v \to \infty!$

What's wrong? Special Relativity doesn't allow this!

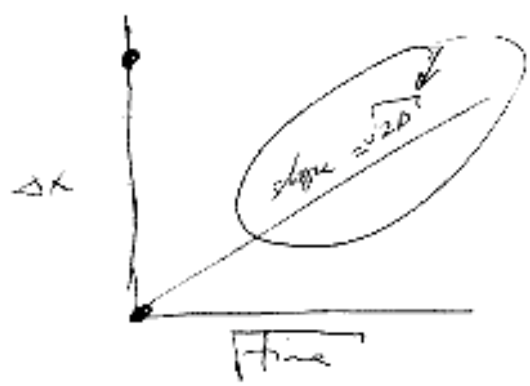
Experimentally: How do you measure D?

Brownian Motion

Robert (?) Brown – botanist saw diffusion
i.e. thermal jiggling of pollen
with reg. light μscope
– res. ~ λ ~ 500nm
+ By eye t ~ 0.1 sec.
How could he have seen this?

Ans: you see only very rare events that
lead to big v, slow fluctuations
Most of jiggling you don't see by eye.

Label/look at pollen grain – note $x(t=0)$
Wait time t, measure Δx, repeat many
times or measure at diff times

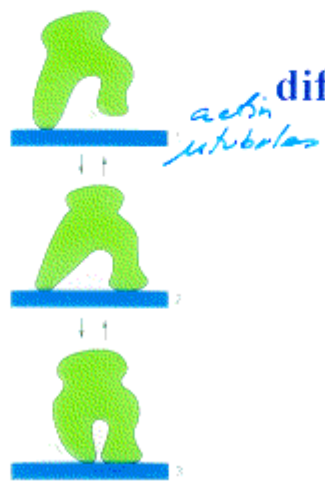


Diffusion good for small distance:

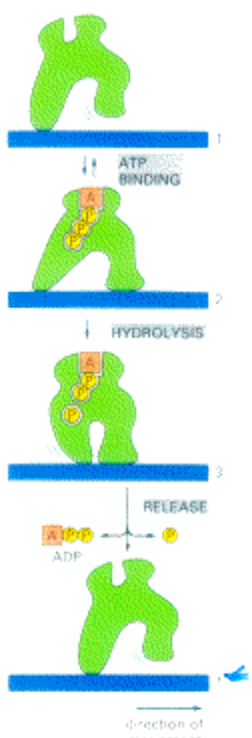
But not good for bringing you from A to B.

➔ Molecular Motors

Directed vs. Random Motion



**No energy input,
diffusion leads to up and back motion,
spread in position.**



**Energy input, leads to
unidirectional motion.**

Where bacteria live

A single teaspoon of topsoil contains about a billion bacterial cells (and about 120,000 fungal cells and some 25,000 algal cells). The human mouth is home to more than 500 species of bacteria. Each square centimeter of your skin averages about 100,000 bacteria. Bacteria live on or in just about every material and environment on Earth from soil to water to air, and from your house to arctic ice to volcanic vents.

Some bacteria (along with archaea) thrive in the most forbidding, uninviting places on Earth, from nearly-boiling hot springs to super-chilled Antarctic lakes buried under sheets of ice. Microbes that dwell in these extreme habitats are aptly called extremophiles.

How Bacteria move

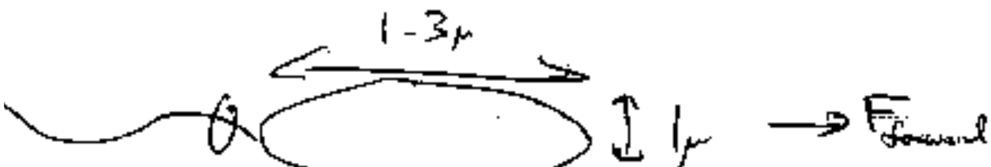
Inertia doesn't matter for microscopic world

Life at low Reynold's number

Why study?

1) Simple Ex of $F = ma$
2) Don't need much bio
3) Results are broadly applicable to bio @ μ level

Bacteria



Go ~ 25 μm/sec ~ $\boxed{\text{10 body lengths/sec}}$

Compare to you: ~~running~~ Walk: 4 miles/hr ~ 6ft/sec

~ $\boxed{\text{1 body length/sec}}$

Swimming: pool ~~length~~ lap ~ (50 yards) ~ min.

~ yard/sec ~ $\frac{1}{2}$ body length/sec

Olympic swimmer ~ body length/sec

$\boxed{\text{Bacteria: good swimmers}}$

If turn off “propeller,” how far Bacteria coast?

$F = ma$

Solve eq'n of motion ~~mder~~

(ii) $$\frac{m\,dv}{dt} = -\gamma v$$

$$\int \frac{m\,dv}{v} = \int -\gamma\,dt$$

$$m \ln v = -\frac{\gamma}{m} t$$

$$v = v_0 e^{-\frac{\gamma t}{m}}$$

$$= v_0 e^{-t/\tau} \qquad \tau = m/\gamma$$

What is mass of bacteria?

$$m \sim \cancel{\tfrac{4}{3}} \tfrac{4}{3}\pi r^3 \rho \qquad r \sim 1\mu m \quad \rho = 1 g/cm^3$$

$$\boxed{\sim 4 \times 10^{-15}\, kg = m}$$

$$\gamma = 6\pi\eta r \qquad \eta = 0.001 \qquad r = 10^{-6} \text{ meters}$$

$$\gamma \sim 20 \times 10^{-9} \frac{N\text{-}s}{m} \sim \boxed{\frac{20\, nN\text{-}s}{m} = \gamma}$$

Plugging in #'s

$m = 4 \times 10^{-15}$ kg

$\gamma = 20 \frac{nN\text{-}s}{m}$

$\tau = \frac{m}{\gamma} = 0.2\ \mu sec$

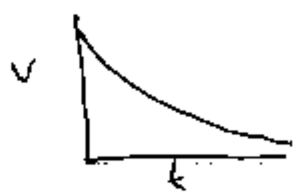
So bacteria stops in 200 nsec! Very fast.

So once forces are turned off, bacteria forgets about history very quickly.

$\boxed{\text{History does not matter to bacteria}}$

How far does bacteria coast in 0.2 sec?

$$x = \int v\,dt = \int_0^{\infty} v_0 e^{-t/\tau_0}\,dt$$



$v_0 \sim 25\ \mu m/sec$

$x = 0.05$ Å ! < diameter of H-atom

$\boxed{\text{Inertia is irrelevant to bacteria}}$

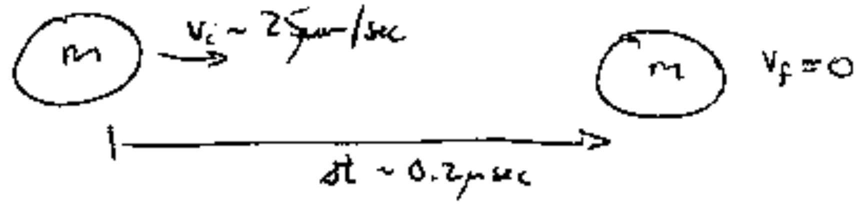
Once force is over, no forward motion!

Person swimming

- a good swimmer coasts ~ 1 body length.

$\boxed{\text{Inertia is much more important to bigger organism}}$

Size of drag force on Bacteria



$$F = \frac{\Delta p}{\Delta t} = \frac{(4\times10^{-15}\,kg)(25\,\mu m/sec)}{0.2\,\mu sec}$$

$$= 5\times10^{-13}\,N$$

$$= 0.5\,pN \qquad (pico = 10^{-12})$$

How compare to it's weight?

$$w = mg = (4\times10^{-15}\,kg)(10\,m/s/s) = 0.04\,pN$$

So:

> Bacteria swim ~~drag~~ as if dragging 10× their own weight!

Power Consumed by Bacteria

What is size of force needed to keep bacteria moving?

ans: when $F_{prop} = F_{drag}$, steady motion $= \gamma v$

$F_{prop} = 0.5\,pN$

(25μm/sec)

$\gamma = 6\pi\eta r$

What is power produced by bacteria?

How much food does Bacteria need to burn for locomotion?

Burn food: (Glucose/sec)

Glucose + Oxygen → CO_2 + water + energy

Class evaluation

1. What was the most interesting thing you learned in class today?

2. What are you confused about?

3. Related to today's subject, what would you like to know more about?

4. Any helpful comments.

Answer, and turn in at the end of class.

Units of η (viscosity)

$$f_{drag} \sim \eta\, d\, v$$

f_{drag} → Newtons

d → Length (m)

v → Length/time m/s

$$\frac{f}{dv} \sim \eta$$

$$\frac{N}{m^2}\text{-sec} \sim \eta$$

Pressure × time

In c.g.s. $\frac{\text{dyne}}{cm^2}\text{-sec} = 0.01$ poise (1 centi-poise)

m.k.s. $\frac{N-\text{sec}}{m^2} = 0.001$ Pa-sec

↳ unit of Pressure
Pascal = N/m^2

(I just remember 0.01 poise + mks is 10x smaller #.

Introduction to Reynold's

When does inertia matter?

Inertia = mass

How big do you have to be
(or how low viscosity)

If you want to accelerate (or decelerate) something, is main thing you need to overcome the mass/inertia, or the friction?

Ex:

a) in outer space
resistance to acceleration is only mass

b) on ice
res. mostly mass

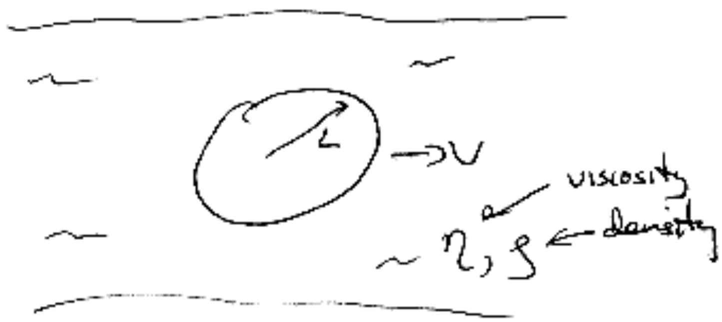
c) stuck in mud
resistance mostly friction

"ma" sometimes called inertial "force" (yuck!)

For bacteria, friction (viscous drag) dominates
For you? For ocean liner?

Reynold's # is ratio of "inertial to viscous" forces

viscous forces = drag - depend on properties of fluid ρ & pattern of fluid flow around object



$R = \frac{L v \rho}{\eta}$ (derived from $\frac{\text{inertial forces}}{\text{viscous forces}}$)

$R \ll 1$ friction dominates
Stir honey (big η) not too fast (reasonable v)
& motion stops immediately after stop stirring
i.e. inertia doesn't matter

$R > 1$ inertia matters
Stir coffee, motion continues.

$R \gtrsim 100$ Stir really fast → big disturbance
Turbulent flow

- if $R_e >> 1$ mass opposes Δ in motion
- " $R_e << 1$ viscous forces " " " "
- Note $R_e \propto L$

Calculating some Reynold's #'s

Bacteria: $L = 10^{-4}$ cm $\rho = 1$ g/cm $\eta = 0.01$ g/cm-sec
$v \sim 10^{-3}$ cm/sec

$\eta = .1$ centipoise $\nu = 10^{-2}$ cm²/sec

$\mathcal{R} = 3 \times 10^{-5}$

$$R_e \sim \frac{Lv\rho}{\eta} \sim \frac{(10^{-4}\text{ cm})(10^{-3}\frac{\text{cm}}{\text{sec}})(1\frac{\text{g}}{\text{cm}})}{0.01\text{ g/cm-sec}}$$

Units okay ✓ $R \sim 10^{-5} << 1$

$\boxed{\text{Bacteria knows nothing about inertia}}$

Fish: $L = 10$ cm $v = 100$ cm/sec

$$R_e = \frac{(10)(100)(1)}{0.01} \sim 10^6 >> 1$$

$\boxed{\text{Fish knows a lot about inertia}}$

Stop swimming: will coast

If you're Lauren in the pool, knows a lot about speed!