PHY524 Cryogenics-2

Thermal conduction w/ convection, radiation;

Thermal contact

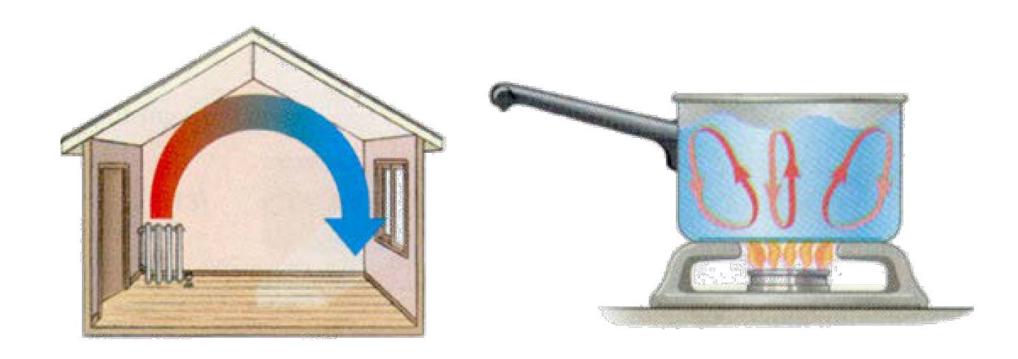
11/11/2025

Material adopted from

- 1. https://www.stonybrook.edu/commcms/case/courses/PHY695
- 2. http://cryocourse2016.aalto.fi/#program
- 3. http://cryocourse2011.grenoble.cnrs.fr/spip.php%3Frubrique13.html
- 4. USPAS | Materials | by Year | 21onlineSBU | Cryogenic Engineering
- 5. <u>USPAS | Materials | by Year | 10MIT | Cryo Engineering</u>

Convection

• Convective heat transfer through gases and liquids



Mechanism of Convection

ideal gas at constant pressure

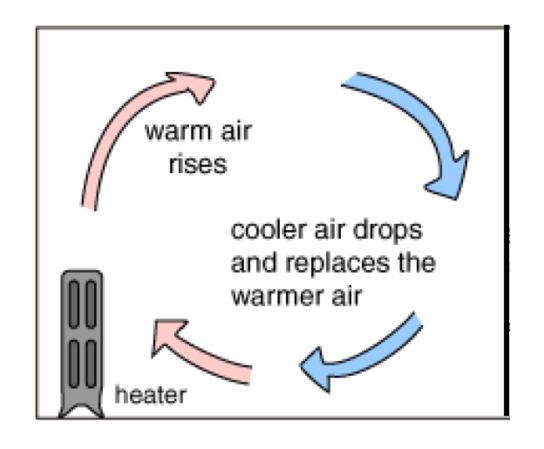
$$\frac{V}{T} = \text{const}$$

$$\varrho = \frac{m}{V}$$

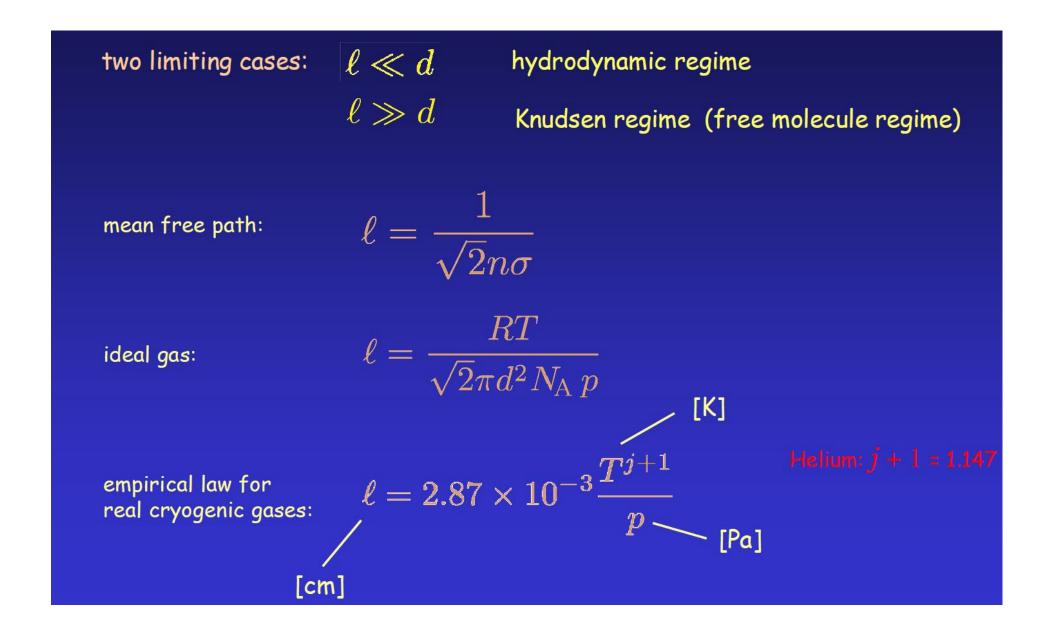
Temperature rise

→ volume rises

→ density decreases



Convective Heat Transfer through Gases and Liquids



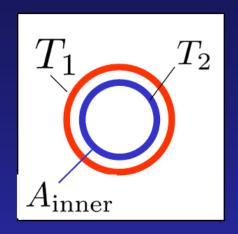
Mean Free Path depends on pressure (Ideal Gas Law)

Vacuum range	p[mbar]	Molecules / cm ³	mean free path
Ambient pressure	1013	2.7×10^{19}	68 nm
Low vacuum	3001	10 ¹⁹ 10 ¹⁶	0.1 100 μm
Medium vacuum	110-3	10161013	0.1 100 mm
High vacuum	10-310-7	10 ¹³ 10 ⁹	10 cm1 km
Ultra high vacuum	10-710-12	10 ⁹ 10 ⁴	1 km 10 ⁵ km
Extremely high vacuum	<10 ⁻¹²	<10 ⁴	> 10 ⁵ km

Mean free path
$$\sim \frac{1}{n\sigma}$$

Low density

Knudsen Regime $\ell\gg d$



$$\dot{Q}_{
m gas} = k\,a_0\,p\,A_{
m inner}\Delta T[W]$$
 [10-2 mBar] [cm2] [K]

$$a_0 = \frac{a_1 a_2}{[a_2 + (A_1/A_2)(1 - a_2)a_1]}$$

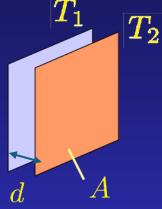
$$a_1 \approx a_2 \approx 0.5 \quad \rightarrow \quad a_0 = \frac{0.5}{1 + (A_1/A_2)0.5}$$

parallel plates
$$A_1 = A_2 \quad o \quad a_0 = 0.33$$

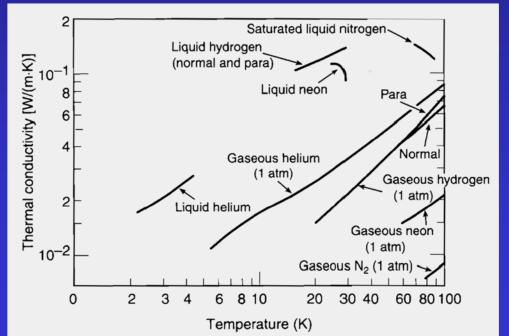
High density

Hydrodynamic Regime $\ell \ll d$

thermal conductivity independent of pressure: $\lambda \propto \ell n$, $\ell \propto rac{1}{n}$



$$\dot{Q}_{\mathrm{gas}} = \frac{\overline{\lambda} A (T_1 - T_2)}{d} = /\frac{\overline{\lambda} A \Delta T}{d}$$



mean thermal conductivity

for gases: $\lambda = c\,C_V$ constant 1.5 ... 2.5

Ekin 2005

Fluid dynamics

Continuity
$$\nabla . \mathbf{v} = 0$$
 $\nabla . \mathbf{v}^* = 0$

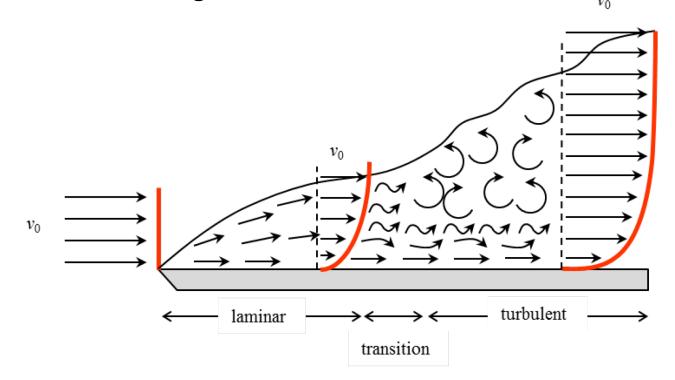
Navier–Stokes $\rho \mathbf{v} . \nabla \mathbf{v} = -\nabla p + \mu \nabla^2 \mathbf{v} + \mathbf{f}'$ Dimensionless $\mathbf{v}^* . \nabla \mathbf{v}^* = -\nabla p^* + \operatorname{Re}^{-1} \nabla^2 \mathbf{v}^*$

Energy $\rho C \mathbf{v} . \nabla T = k \nabla^2 T + Q$ $\mathbf{v}^* . \nabla T^* = \operatorname{Re}^{-1} \operatorname{Pr}^{-1} \nabla^2 T^*$

- Reynolds number, $Re = \frac{\rho L v}{\mu}$
 - ratio of the inertia to the viscous forces
- Prandtl number, $Pr = \frac{\mu C}{k}$
 - ratio of the momentum to the thermal diffusivity

Laminar vs turbulent flow

- Re < 2300: laminar regime
 - the **viscous forces dominate**, creating an 'ordered' flow with streamlines. In this regime, the surface heat transfer is low and so is the surface friction.
- $Re_L > 5 \times 10^5$: turbulent regime
 - Re_D > 4000 for a plate and a tube-shaped geometry, respectively.
 - **the inertia forces dominate**, and the flow becomes highly irregular (velocity fluctuation), as shown below. The surface heat transfer and the friction are higher than those in the laminar regime.



Newton's law

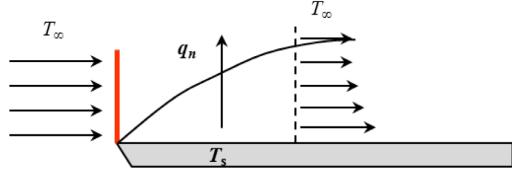
 When there is an interaction with solid surfaces in convection, the heat transferred in (or out) of the fluid to solids is evaluated by the Newton's law:

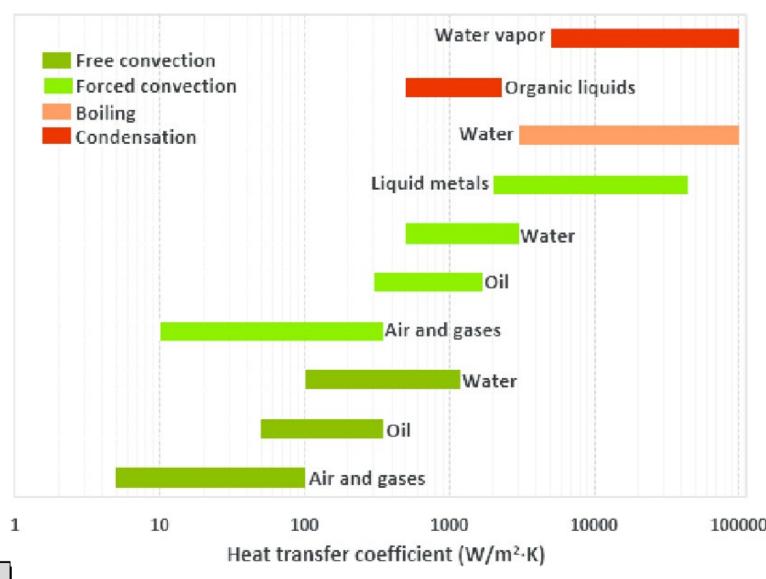
$$q = h(T_S - T_{\infty})$$

where h is the heat transfer coefficient [W·m⁻ 2 ·K⁻¹], T_s the temperature of the solid and T_∞ is the temperature far from the solid.

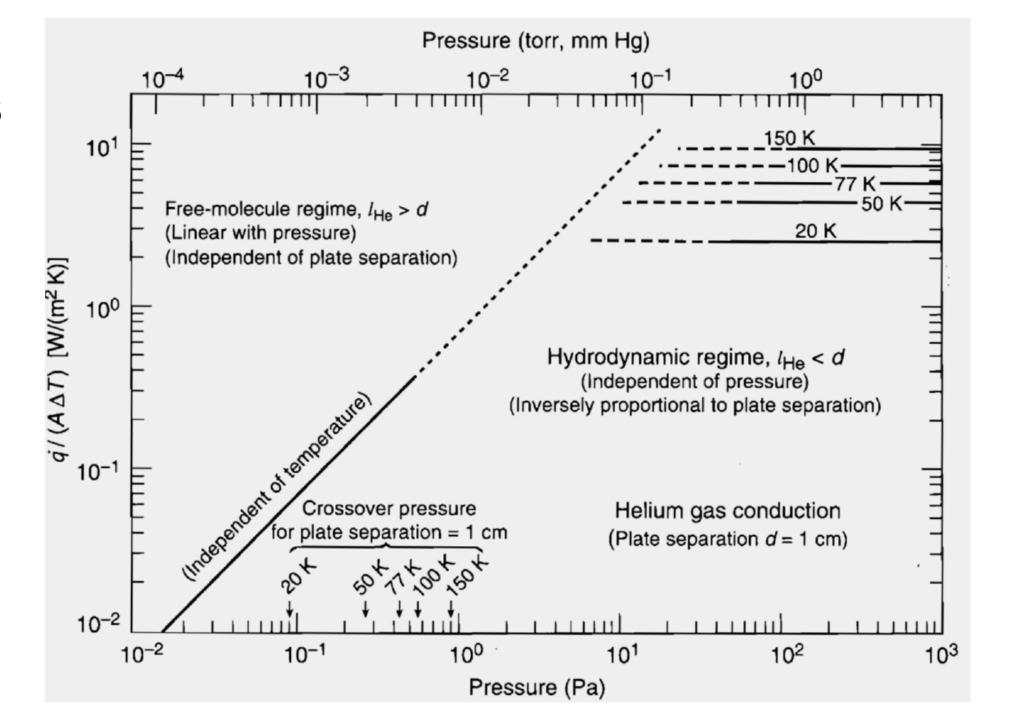
At the boundary, the local heat flux through the conductive layer of the fluid is

$$q_n = k \cdot \nabla T_n$$

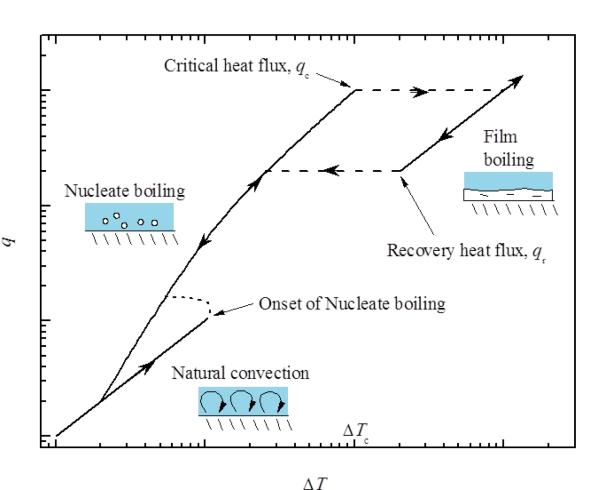




He Gas



Types of convection in liquids



Natural convection

• When the fluid movement is created internally, by a decrease or increase of the fluid density or by the buoyancy effect.

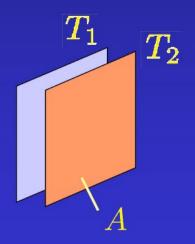
Forced convection

- Mostly turbulent flows. A laminar flow in pipes is very rare, except in porous media
- Boiling convection
 - Heat transfer combines
 - natural convection in the liquid,
 - latent heat to be absorbed for the bubble formation, and
 - the bubble hydrodynamics.
 - The heat transfer process depends on the bubble growth rate, the detachment frequency, the number of nucleation sites, and the surface conditions.

Radiative Heat Transfer

radiative heat flow from a body with surface A

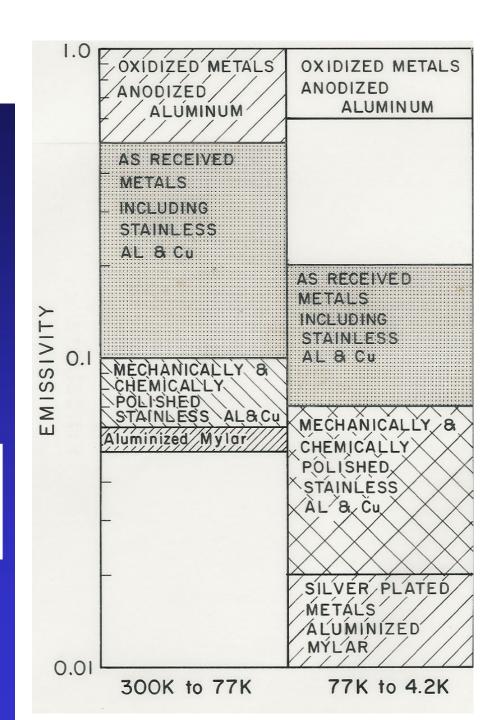
$$\dot{Q}_{
m rad} = \sigma \, arepsilon \, A \, T^4$$
 Stefan-Boltzmann Law emissivity 0.01 ... 1
$$\sigma = 5.67 \times 10^{-8} \, [{
m W/(m^2 K^4)}]$$



$$\dot{Q}_{\rm rad} = \sigma \, \mathcal{E} \, A \, \left(T_2^4 - T_1^4 \right)$$

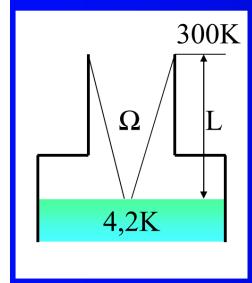
Wien's law $\lambda T = 2900 \text{ [}\mu\text{m K]}$

wave lenght 300 K 0.01 mm 3 K 1 mm



Radiation heat load

Radiation of neck: Q_{Rcol}



We have :
$$Q_{Rneck} = S\varepsilon_e \sigma T^4 \cdot \Omega$$

with $T^4 = 300^4$

with
$$T^4 = 300^4$$

$$\Omega = solid \ angle = \frac{R^2}{R^2 + L^2}$$

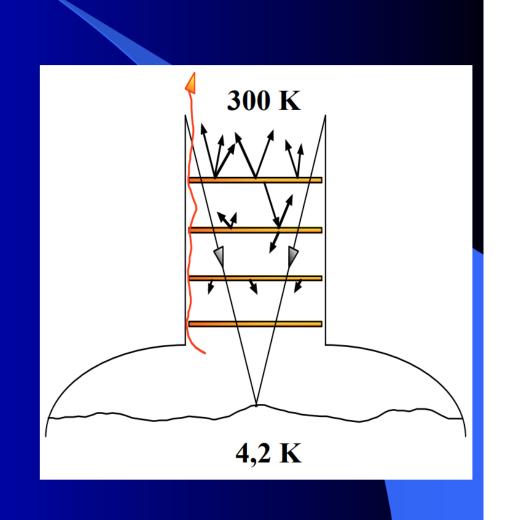
- Example
 - $\phi O = 150 \text{ mm then, } S = 180 \text{ cm}^2$
 - L = 300 mm
 - \bullet εΩ ~ 0.05 (typical value)
 - ←A large value!

Homework:

1. Estimate the heat load due to thermal radiation from the top assembly

Baffles

- The cover plate radiates directly on the bath
 - ◆ Important losses!
- Solutions
 - Thermal shields distributed along the neck
 - Low emissivity ($\varepsilon < 0.1$)
 - The heat recovered by the shields is evacuated by the vapors



Heat Load Calculation

A few Numbers

twisted pair of copper wire: diameter 125 µm, 1,5 m long

heat transfer between 300 K to 4 K: 0.14 mW

stainless steel tube: diameter 2 cm, wall thickness 0.4 mm, 1,5 m long

heat transfer between 300 K to 4 K: 51 mW

radiation: two plates with 30 cm² surface area

heat transfer between 300 K to 4 K: 26 W

exchange gas: two plates with 30 cm² surface area, 1 mBar, 1 cm

heat transfer between 300 K to 4 K: 45 W

Homework:

 Estimate the heat load due to thermal radiation from the top assembly

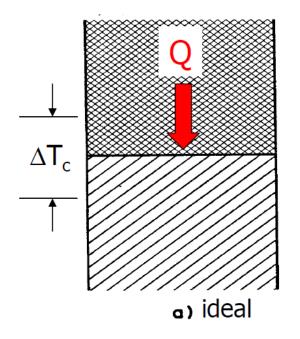
Thermal anchoring

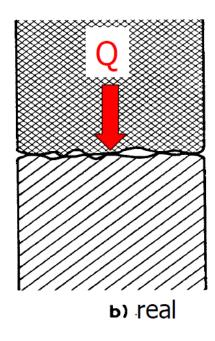
Samples, Sample Holders, Wires and Cables

- Welding
- Soldering
- Varnish and glue joints
- Press contacts
- Exchange gas

Contact Resistance

- Joints or contacts can lead to considerable resistance in a thermal (electrical) circuit.
- Contact resistance can vary considerably depending on a number of factors
 - Bulk material properties (insulators, metals)
 - Surface condition (pressure, bonding agents)





Heat transfer coefficient

$$Q = h_c A \Delta T$$

Thermal contact conductance (conductive)

Contact point between two materials can produce significant thermal resistance

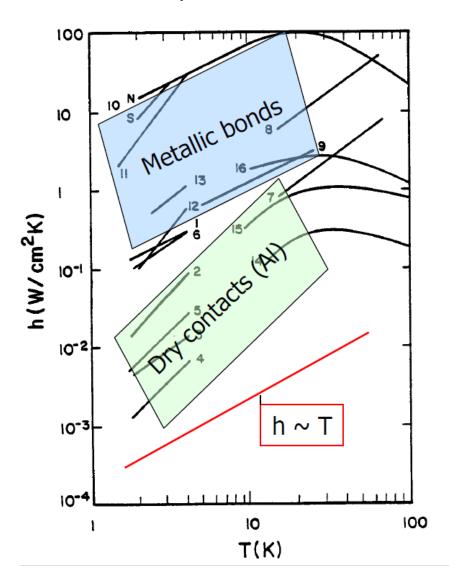
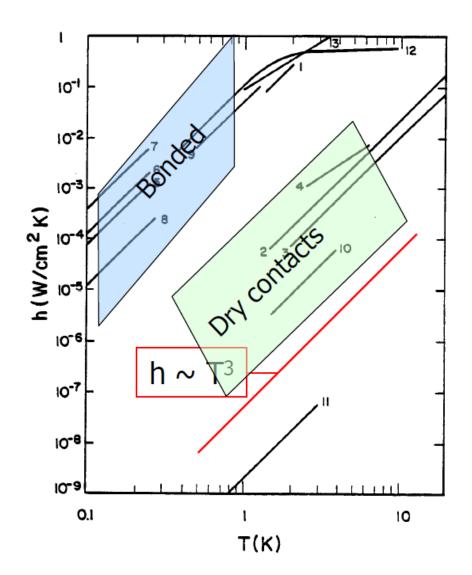


Table II. Thermal Conductance of Metallic Contacts

	Material	Contact	Pressure (MPa)	$h(W/cm^2 K)$	Temp. Range (K)	Ref
1.	Al-Al (alloy)	machined	torque=20 Nm	0.075 T	1.8-4.2	Wanner /13/
2.	Al-Al alioy	electropolished	torque=20 Nm	3.6x10 ⁻³ T ^{2.3}	1.8-4.2	
3.	Al-Al alloy	Au plated	torque-20 Nm	1.9x10 ⁻³ T ^{1.4}	1.8-4.2	
4.	Cu-Cu	machined	2.8	$4x10^{-4}$ T^2	1.8-4.2	Berman /8/
5.	Cu-Cu	machined	14	1.67x10 ⁻³ T ²	1.8-4.2	
6.	Au-Au		5.6	0.05 T1.3	2-4	Berman & Mate /7/
7.	ss-ss (302)	polished	21	0.014 T1.5*	15-300	Lyon & Parrish /14/
8.	.SS-SS (302)	polished	390	0.10 T1.5*	15-300	
9.	Cu-Cu	machined	7	0.13 T	5-25	Nilles & Van Sciver /15,
10.	Cu-Cu	in solder		7.5 T*	2-150	Radebaugh /16/
11.	Cu-Cu	Pb solder		0.64 T ^{2.8}	1.5-4	Challis & Cheeke /17/
12.	Cu-Cu	woods metal		0.018 T ^{2.5}	2-4	
13.	Cu-Cu	PbSn solder		0.13 T1.6	2.5-4	Poster /18/
14.	Al-Al	SnPb foil	26	0.02 T ^{0.8} *	10-300	Friedman & Gasser /19/
15.	Cu-Al	SnPb foil	9	0.04 T*	10-300	
16.	Cu-Cu	SnPb Foil	7	0.17 T*	10-300	

Contact conductance (Insulating)



	Material	Contact	Pressure (MPA)	h(W/cm ² K)	Temp range (K)	Ref.
1.	In-sapphire	bonded		0.03 T ³	1.4-2.1	Neeper & Dillinger /6/
2.	sapphire-sapphire	dry	4	9 x 10 ⁻⁶ T ³	2-20	Berman & Mate /7/
з.	Cu-diamond	dry	4	2 x 10 ⁻⁵ T ³	1.5-20	
4.	Cu-Teflon-Cu	12 mil foil	4	$1.8 \times 10^{-4} \text{ T}^2$	2-5	Berman /8/
5.	Cu-epoxy-Cu	bonded		0.09 T ³	0.05-0.25	Peterson & Anderson /2/
6.	Al-epoxy-Al	bonded		0.13 T ³	0.05-0.25	
7.	Pb-epoxy-Pb	bonded		0.40 T ³	0.05-0.25	
8.	Ве-ероху-Ве	bonded		0.013 T ³	0.05-0.25	
9.	Cu-LiF	Ge-7031		0.05 T ³	0.4-1.3	Ackerman & Anderson /9/
10.	Cu-sapphire-Cu	dry	0.1	$1 \times 10^{-6} \text{ T}^3$	1.5-4	Yoo & Anderson /10/
11.	Cu-sapphire-Cu	A1 ₂ 0 ₃	0.1	2 x 10 ⁻⁹ T ³	0.8-3	
12.	Cu-epoxy-Cu	bonded		0.16 T ^{0.5}	2-8 Mat	sumoto et al./11
13.	Cu-epoxy-Cu	bonded		0.089 T1.9	1-4	Schmidt /12/

Welding: metal to metal contact

Oxygen-acetylene welding very hot flame, porous joint, surface oxidation Large surface area for trapping gas

Electric-arc welding conducts electric current to heat, same problems as oxygen-acetylene welding \rightarrow inert gas welding

Laser-beam welding similar to electric are welding, heat sourse laser beam

Electric resistance welding Joule heating melts the metal locally usually restricted to small areas: spot welding

Pressure welding soft metals, very cleans surfaces, very high pressures, cold welding

Friction welding used for joining dissimilar metals: Steel and Al, ...

Solder Joints

Hard solders: silver solder ~ 700 °C

Soft solders: lead and tin based solders 200 to 400 °C

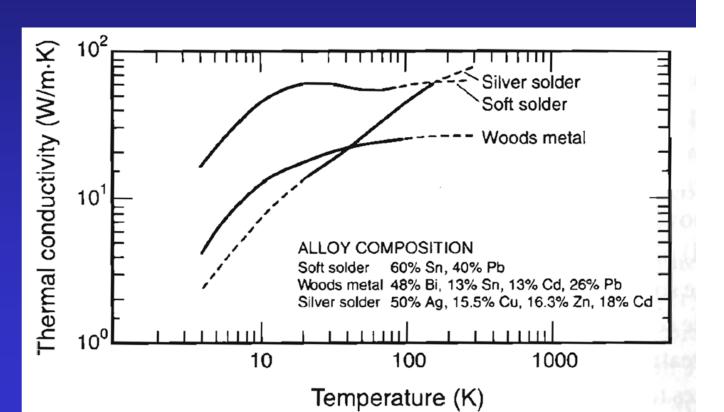
Very low temperature solders: Bismuth based below 100 °C

general problem at very low temperatures:

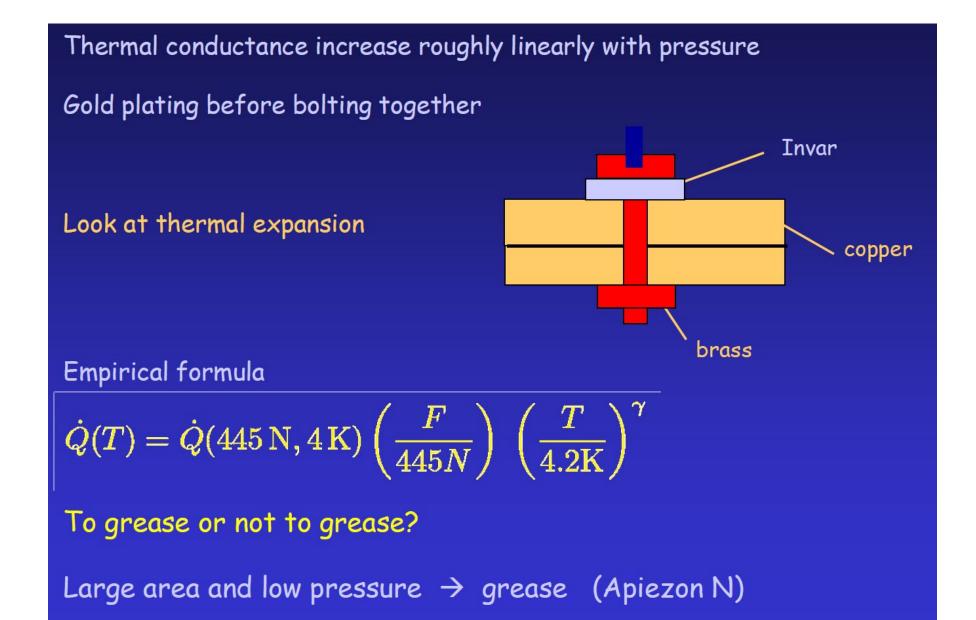
solder Joints become Superconductive

use the right flux

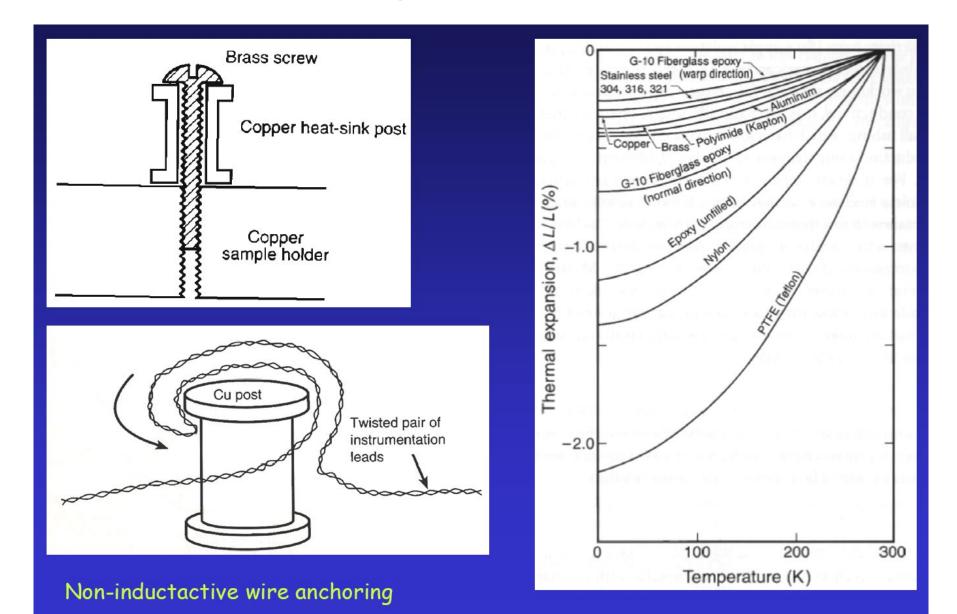
acid flux can be dangerous by creating pin holes



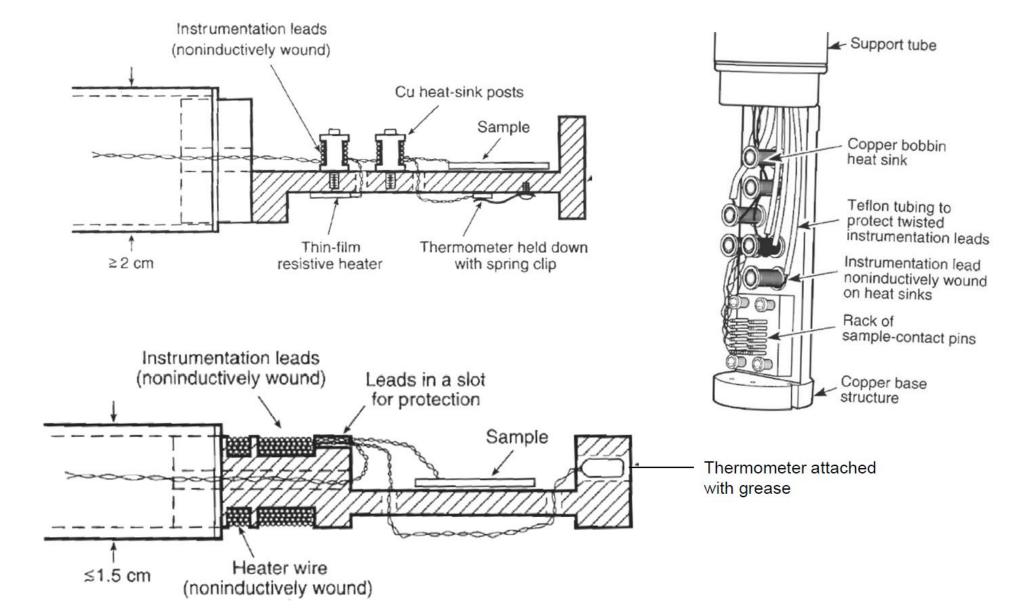
Press Contact



Thermal Anchoring of Wires and Cables



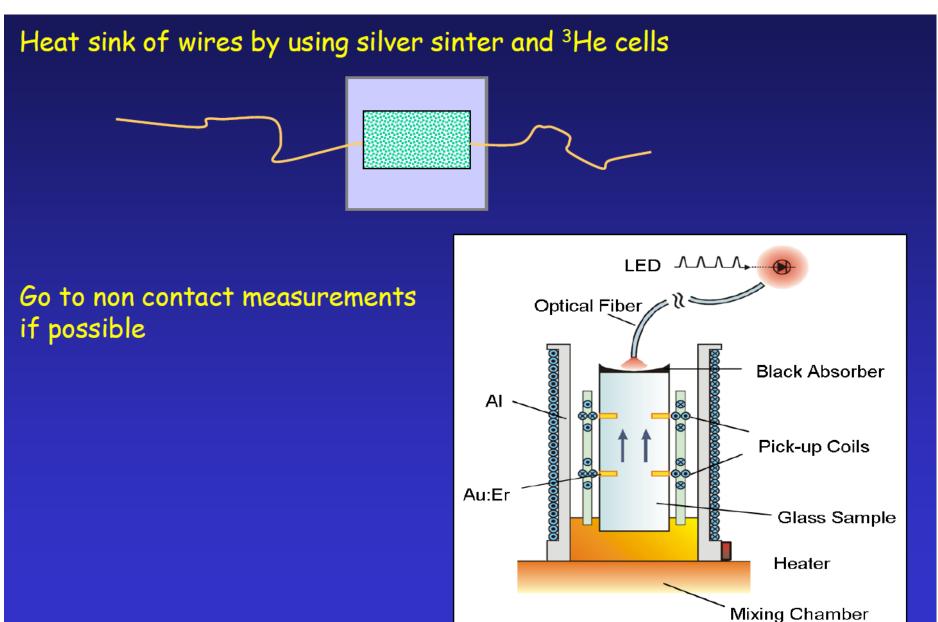
Thermal Anchoring of Wires and Cables



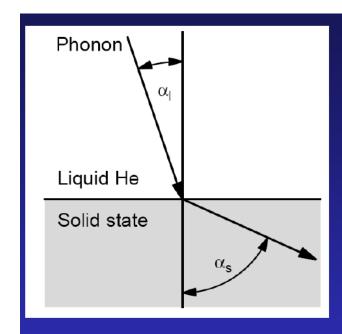
Thermal Anchoring of Wires and Cables

One instrumentation Beryllium oxide lead soldered to Soldered Heat sink of wires each terminal strip lead Magnetic-field direction Demountable Cu platform ~3 mm Screw to attach Cu platform to sample holder Chip soldered or glued to Cu platform Heat sinking coaxial cables for high frequencies: Mircostripline gold strip Top view copper Sapphire Side view

Ultra low temperature (T < 10 mK)



Thermal Boundary or Kapitza-Resistance



Kaptiza-Resistance occurs at any solid-solid, liquid-solid interface

Partciular problematic for liquid helium because of the low sound velocity

Snell's law of refraction

$$\frac{\sin \alpha_{\ell}}{\sin \alpha_{\rm s}} = \frac{v_{\ell}}{v_{\rm s}}$$

critical angle of total reflection

$$\alpha_{\ell}^{\rm c} = \arcsin\left(\frac{v_{\ell}}{v_{\rm s}}\right)$$

for Iq. helium and copper $~\alpha_\ell^{
m c}=$ 4°

fraction of phonons incident within critical angle

$$f = \frac{1}{2}\sin^2\alpha_\ell^{\rm c} = \frac{1}{2}\left(\frac{v_\ell}{v_{\rm s}}\right)^2 < 10^{-2}$$

Theory on the Kapitza resistance

Transmission coefficient

$$t = \frac{4Z_{\ell}Z_{\rm s}}{\left(Z_{\ell} + Z_{\rm s}\right)^2} \simeq \frac{4Z_{\ell}}{Z_{\rm s}} = \frac{4\varrho_{\ell}v_{\ell}}{\varrho_{\rm s}v_{\rm s}}$$

fraction of phonons crossing the interface

Heat transfer (using Debye model)

$$\dot{Q} = \frac{\mathrm{d}\dot{\mathcal{Q}}}{\mathrm{d}T}\Delta T = \frac{2\pi^2 k_{\mathrm{B}}^4 \varrho_{\ell} v_{\ell}}{15\hbar^3 \varrho_{\mathrm{s}} v_{\mathrm{s}}^3} A T^3 \Delta T \qquad \dot{\mathcal{Q}} = \frac{1}{2} ftuv_{\ell} A = \frac{\pi^2 k_{\mathrm{B}}^4 \varrho_{\ell} v_{\ell}}{30\hbar^3 \varrho_{\mathrm{s}} v_{\mathrm{s}}^3} A T^4$$

Kapitza Resistance

Acoustic impedances

$$Z_{\ell} = \varrho_{\ell} v_{\ell}$$
$$Z_{s} = \varrho_{s} v_{s}$$

$$ft = \frac{2\varrho_\ell v_\ell^3}{\varrho_{\rm s} v_{\rm s}^3} \qquad \begin{array}{c} \text{helium-copper} \\ ft < 10^{-5} \end{array}$$

helium-copper
$$ft < 10^{-5}$$

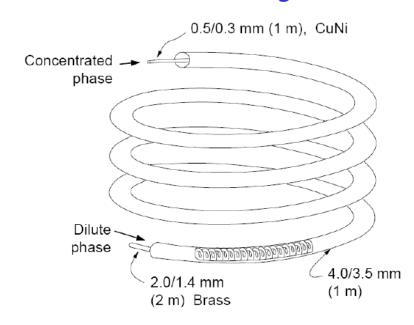
$$\dot{\mathcal{Q}} = \frac{1}{2} ftuv_{\ell} A = \frac{\pi^2 k_{\mathrm{B}}^4 \varrho_{\ell} v_{\ell}}{30 \hbar^3 \varrho_{\mathrm{s}} v_{\mathrm{s}}^3} A T^4$$

$$R_{\rm K} = \frac{A\Delta T}{\dot{Q}} = \frac{15\hbar^3 \varrho_{\rm s} v_{\rm s}^3}{2\pi^2 k_{\rm B}^4 \varrho_{\ell} v_{\ell}} \frac{1}{T^3}$$

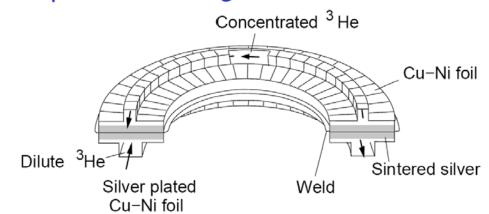
Solutions

continuous heat exchanger

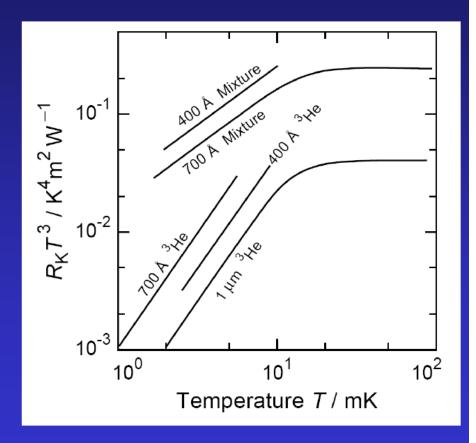
Used for dilution refrigerators



step heat exchanger



Data for Kapitza Resistance



Homework

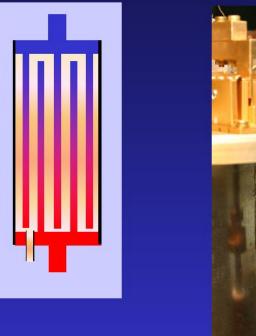
#1: Estimate the heat load due to thermal radiation from the top assembly

#2: Watch this video: <u>John Allen's Movie about Superfluid Helium</u> (https://www.youtube.com/watch?v=lEPc-rBMAuU&t=43s)

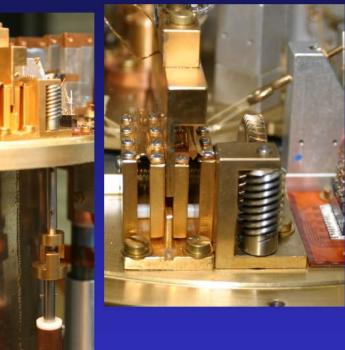
(42 mins) before Nov. 18

Heat Switches

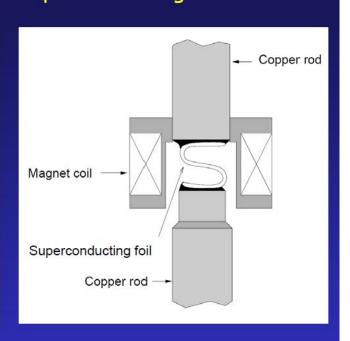
Gas heat switches



Mechanical heat switch



Superconducting heat switch



exchange gas ightarrow pumping

⁴He superfluid layer

→ creep

H₂ ortho-para

conversion

He no exothermic reation

no creep

high vapour pressure

large force needed typically 100 N

closed few mW/K ... 1 W/K @ 15K

heating on opening

only well below T_c

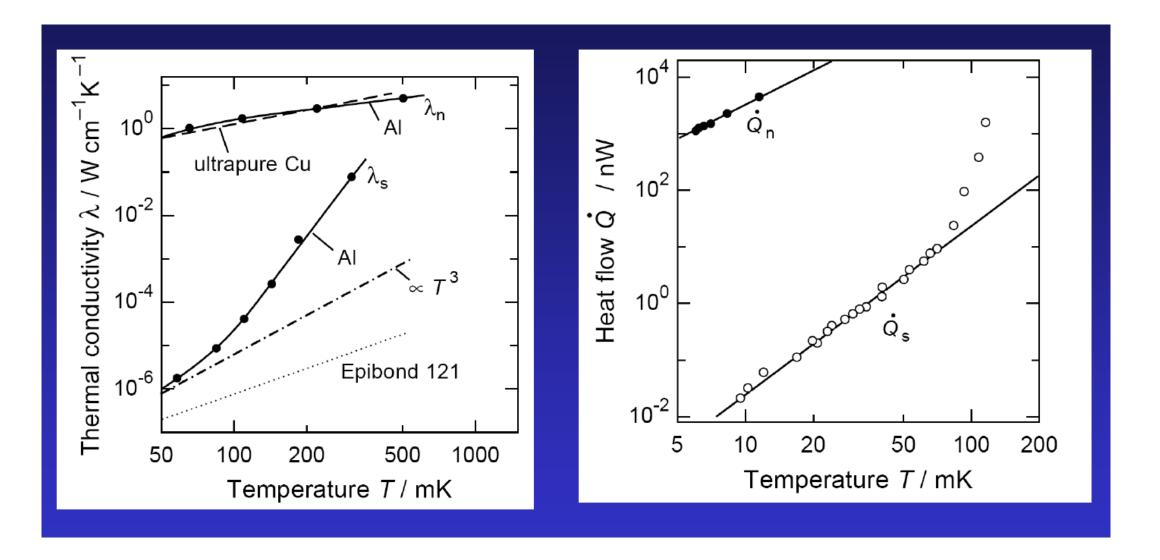
open means low conductivity

eddy currents

flux trapping

Grenoble 19.09.11 - 31

Superconducting Heat Switch



Thermal contraction

 All materials change dimension with temperature. The expansion coefficient is a measure of this effect. For most materials, the expansion coefficient > 0

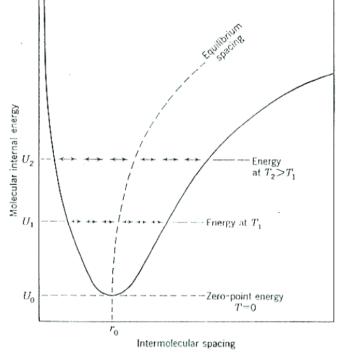
$$T_2 < T_1$$
 $D \rightarrow D - \Delta D$

Linear expansion coefficien $\rightarrow \Delta L \leftrightarrow \uparrow$

$$\alpha = \frac{1}{L} \frac{\partial L}{\partial T} \Big|_{p} = \frac{1}{3} \beta \quad \text{For isotropic materials}$$

Bulk expansivity (volume change):

$$\beta = \frac{1}{v} \frac{\partial v}{\partial T} \bigg|_{p} = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} \bigg|_{p}$$



Expansivity caused by anharmonic terms in the lattice potential

Temperature dependence of alpha and beta

- Most materials contract when cooled
 - The magnitude of the effect depends on materials:
 - Plastics > metals > glasses
- Coefficient (α) decreases with temperature
 - Thermodynamics:

$$C_{p} - C_{v} = \frac{Tv\beta^{2}}{\kappa} \underset{T \to 0}{\longrightarrow} 0$$

$$\frac{\partial v}{\partial T} \Big|_{p} = -\frac{\partial s}{\partial p} \Big|_{T \to 0} 0$$

(Third law: $s \rightarrow 0$)

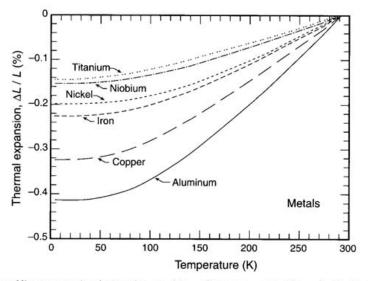


Fig. 6.6 Thermal linear expansion $\Delta L/L \equiv (L_T - L_{293})/L_{293}$ of common metals. (Compiled by Clark 1983 from data by Corruccini and Gniewek 1961, and Hahn 1970.) Tabulated values for these and other materials are given in Appendix A6.4.

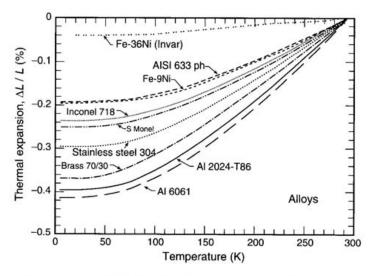


Fig. 6.7 Thermal linear expansion $\Delta L/L \equiv (L_{\rm T} - L_{293})/L_{293}$ of common alloys. (Compiled by Clark 1983 from data by Clark 1968 and Arp et al. 1962.) Tabulated values for these and other materials are given in Appendix A6.4.

Expansion coefficient for materials

Table 2.5. Linear Thermal Contractions Relative to 293 Ka

Substance	T(K): 0	20	40	60	80	100	150	200	250
Aluminum	41.4	41.4	41.2	40.5	39.0	36.9	29.4	20.1	9.6
Copper	32.6	32.6	32.3	31.6	30.2	28.3	22.1	14.9	7.1
Germanium	9.3	9.3	9.3	9.4	9.3	8.9	7.3	5.0	2.4
Iron	20.4	20.4	20.3	19.9	19.5	18.4	14.9	10.2	4.9
Lead	70.8	70.0	66.7	62.4	57.7	52.8	39.9	26.3	12,4
Nickel	23.1	23.0	22.9	22.6	21.8	20.8	16.5	11.4	5.4
Silicon	2.10	2.16	2.17	2.23	2.32	2.40	2.38	1.90	1.01
Silver	41.0	41.0	40.3	38.7	36.5	33.7	25.9	17.2	8.2
Titanium	15.1	15.1	15.0	14.8	14.2	13.4	10.7	7.3	3.5
Tungsten	8.6	8.6	8.5	8.4	1.8	7.6	5.9	4.0	1.9
Brass (65 % Cu, 35 % Zn)	38.4	38.3	38.0	36.8	35.0	32.6	25.3	16.9	8.0
Cu + 2 Be	32.4	32.4	31.9	31.6	30.0	28.3	22.0	16.0	7.0
Constantan	944	: -	26.4	25.8	24.7	23.2	18.3	12.4	5.85
Invar ^b	4.5	4.6	4.8	4.9	4.8	4.5	3.0	2.0	1.0
304, 316 Stainless steel		29.7	29.6	29.0	27.8	26.0	20.3	13.8	6.6
Pyrex	5.6	5.6	5.7	5.6	5.4	5.0	. 3.95	2.7	0.8
Silica (1000° C) ^c	-0.1	-0.05	0.05	0.2	0.3	0.4	0.5	0.4	0.2
Silica (1400° C) ^c	-0.7	-0.65	-0.5	-0.3	-0.2	-0.05	0.2	0.2	0.1
Araldite	106	105	102	98	94	88	71	50	25
Nylon	139	138	135	131	125	117	95	67	34
Polystyrene	155	152	147	139	131	121	93	63	30
Teflon	214	211	206	200	193	185	160	124	75

^a Units are $10^4 \times (L_{293} - L_T)/L_{293}$. Sources of data include Thermophysical Properties of Matter (1977), Corruccini and Gniewek (1961), and American Institute of Physics Handbook (1972). Compiled by White.²

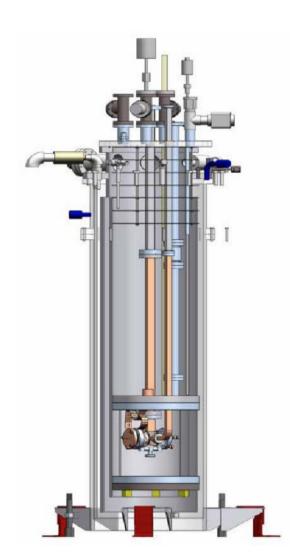
The expansion of Invar NiFe alloys containing ~36% Ni is very sensitive to composition and heat treatment.

' These silicas were aged at 1000° C and 1400° C.

Structural support

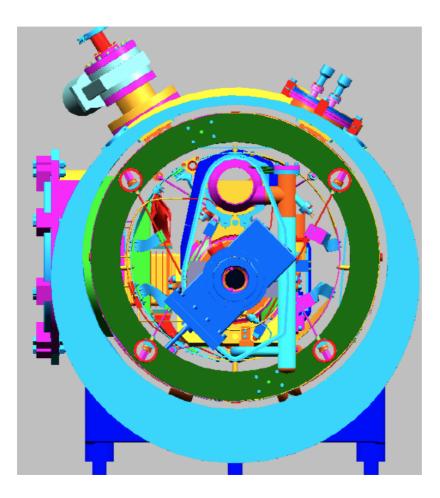
- Solution is highly dependent on cryostat requirements
- Choose materials carefully
 - Acceptable for cryogenic temperatures
 - Low heat leak
- Don't over constrain supports: allow for thermal contraction
- Does solution meet alignment and vibration requirements?
- Must alignment be changed while cold?

Structural support: Simple top load cryostat



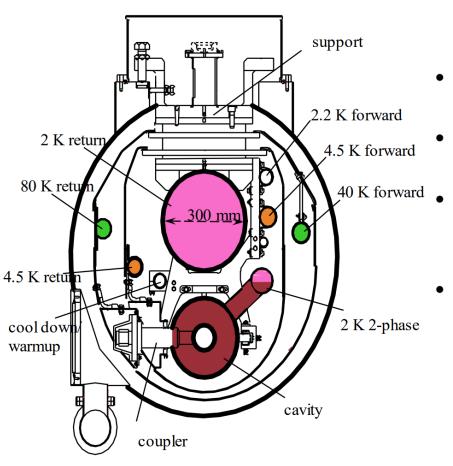
- Very common for test cryostats
- Everything hangs from 300 K top flange
- Connections made via low conductivity piping and supports
- Everything "contracts up"
- Allows easy removal and change of cryostat components
- Useful when precise alignment not an issue

Structural Support Example #2 JLab 12 GeV Upgrade Cryomodul



- All components are tied to space frame which rolls into vacuum vessel
- Connections to 300 K done via flex lines and bellows
- Same approach taken in ESS elliptical cavity cryomodules

Structural Support Example #3 ILC Cryostat



- FRP support between 300 K and Cryo temps
- Cavity assemblies tied to 300 mm pipe backbone
- All other connections to 300
 K have flex line or bellows
 in line
- Meets alignment specs

Tips on designing a cryo system

- 1. Define and prioritize requirements first.
- 2. Design in safety features from the start of the project
- 3. Only use materials shown to be appropriate for cryogenic temperatures.
- 4. Review literature & learn from previous efforts. Take advantage of existing codes and standards if possible.
- 5. Use tested commercial solutions whenever possible.
- 6. Intercept heat at higher intermediate temperatures (remember the 2nd law of thermodynamics)
- 7. Allow for the effect of thermal contraction on cryostat alignment and design. Do not over constrain the movement of cryostat components as they cool.
- 8. Avoid feed throughs & demountable seals at cryogenic temperatures.

More tips on designing a cryo system

- 8. Be sure to properly heat sink temperature sensor wires to ensure reduced heat leak and an accurate reading.
- 9. Install sensors such a pressure transducers and flow meters at room temperature when possible.
- 10. Analyze the design for possible thermoacoustic oscillations
- 11. Conduct design reviews. These should include experts not directly involved in the design under review. Ideally, there should be at least reviews at the preliminary or conceptual level and again once the detailed design is complete.
- 12. Conduct prototype tests when required. Leave enough time in the design process to benefit from the results of such tests.
- 13. In cases where a large number of cryostats are to be produced, carry out series testing of the production cryostats in addition to any prototype testing. Allow sufficient resources (time, facilities, funding) in the project plan to accomplish these tests.