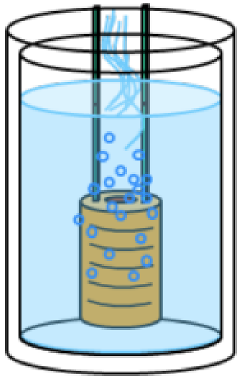
PHY524 Cryogenics-4

Liquefication, Helium & Dilution Refrigerator

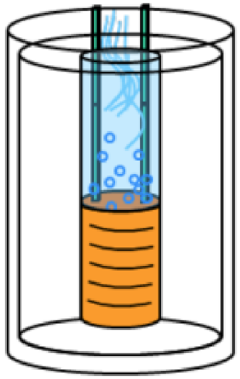
11/18/2025

Material adopted from lectures by Henri Godfrin, Grenoble

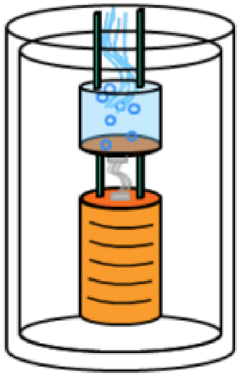
Various Cooling Methods



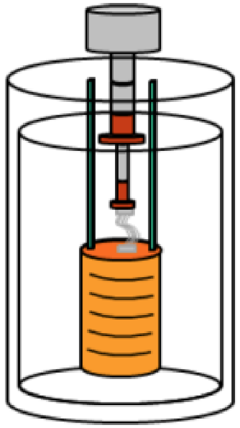
Direct cooling
Bath cooling



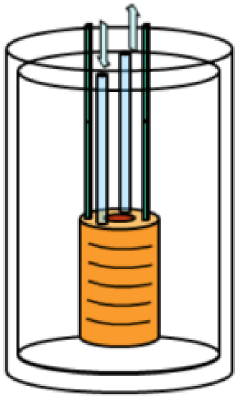
Indirect cooling
Bath cooling



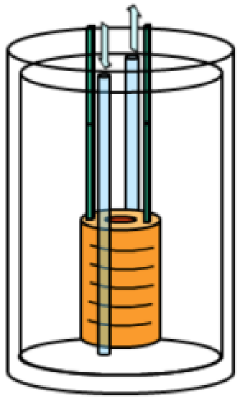
Indirect cooling
Bath as cold source



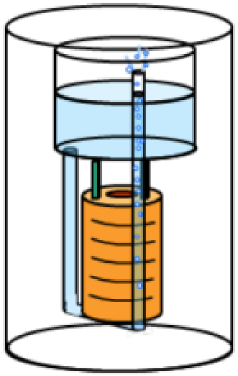
Indirect cooling
Cryocooler as cold source
Thermal link



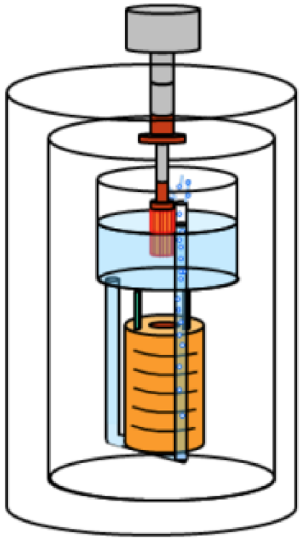
Direct cooling
Forced flow



Indirect cooling
Forced flow



Indirect cooling
Two-phase thermosiphon



Indirect cooling
Two-phase thermosiphon
Coupled with a cryocooler

Cooling down

- Remove the heat "stored" in material
  - → Enthalpy of the material H
  - Between 2 temperatures, we have : $\Delta H = \int_{T_1}^{T_2} C_P dT = H_2 - H_1$
    - H expressed in J/g
    - Enthalpy H(T) of some materials (J/g)

T(°K)	1	4	15	20	60	80	300
Aluminum	$2.5.10^{-5}$	$4.63.10^{-4}$	$1.8.10^{-2}$	$4.8.10^{-2}$	3.64	9.37	170.4
Chromium	$1.42.10^{-5}$	$2.37.10^{-4}$	$0.53.10^{-2}$	$1.28.10^{-2}$	0.904	2.77	78.9
Copper	$0.6.10^{-5}$	$1.3.10^{-4}$	$1.07.10^{-2}$	$3.4.10^{-2}$	2.58	6.02	79.6
Iron	$4.5.10^{-5}$	$7.42.10^{-4}$	$1.45.10^{-2}$	$3.16.10^{-2}$	1.43	3.84	81.1
Nickel	6.10^{-5}	$9.8.10^{-4}$	$1.85.10^{-2}$	$4.1.10^{-2}$	1.79	4.56	82.1
Niobium	4.10^{-5}	$7.3.10^{-4}$	$2.6.10^{-2}$	$6.6.10^{-2}$	2.76	5.8	59.2
Titanium	$3.5.10^{-5}$	$5.99.10^{-4}$	$1.56.10^{-2}$	$4.0.10^{-2}$	2.59	6.37	101.4
Zinc	5.10^{-5}	$1.4.10^{-4}$	$3.4.10^{-2}$	$12.5.10^{-2}$	5.01	9.70	87.1
Teflon		10.10^{-4}	21.10^{-2}	52.10^{-2}	7.02	12.52	144.6

Cooling by evaporation

- Starting from these data.
  - Calculation of the quantity of fluid which will be evaporated to cool a given mass
  - Exemple for 1 kg of Copper
    - Cooled by helium from 300 to 4 K
    - $\Delta H = (79.6\text{-}13.10^{-5})*1000 = 79600$ Joules
    - Helium evaporated = 79600/2562 = **30.3 Liters!**

$$\frac{\Delta H * m}{L} = V$$

ΔH in j/g

m in g

L in J/l

V in l

Starting temperature	^4He	H_2	N_2
300 K	30.3 liters	2.5 liters	0.49 liters
77 K	2.1 liters	0.17 liters	

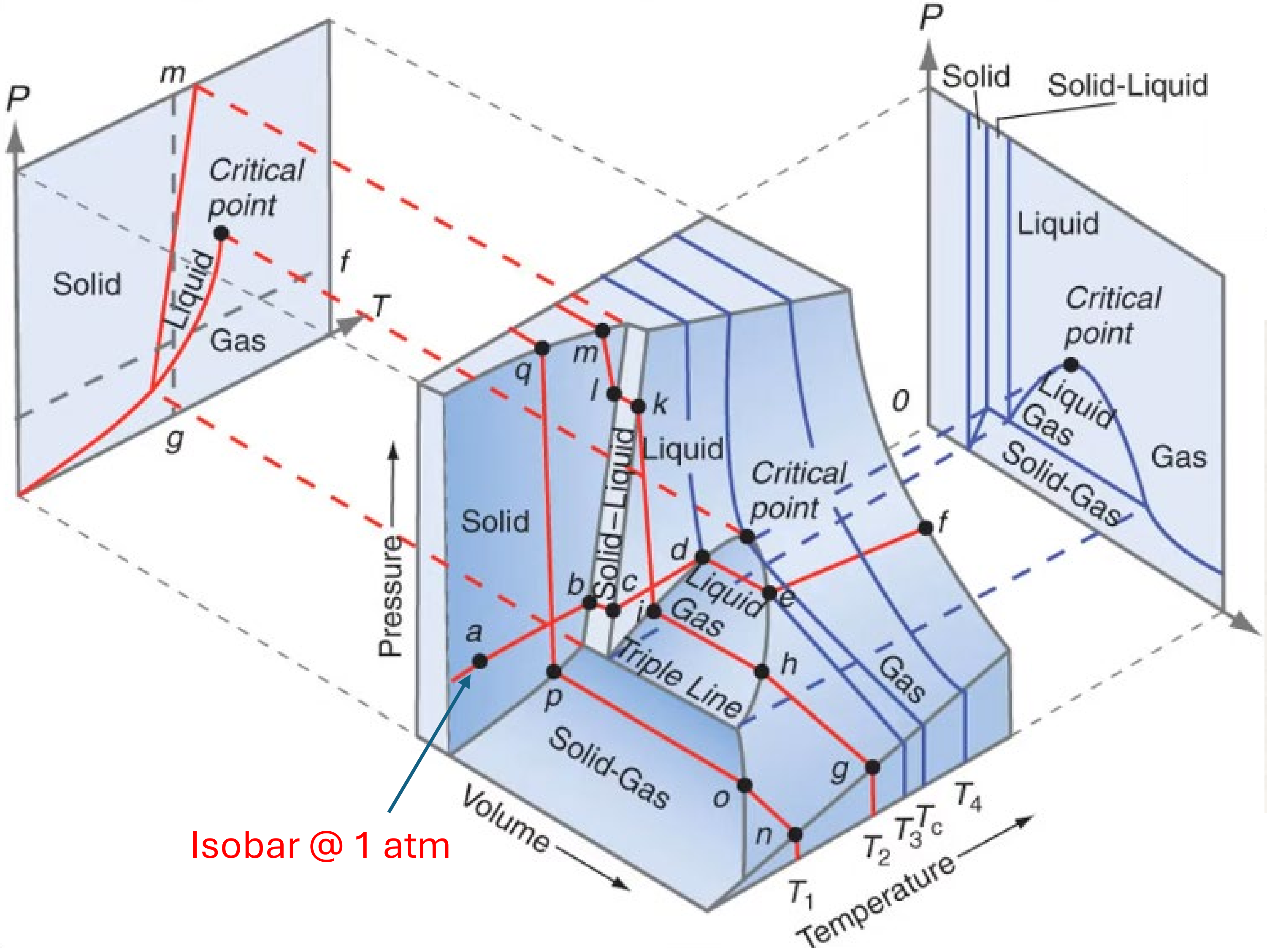
- *Interest of pre-cooling with liquid nitrogen! !*

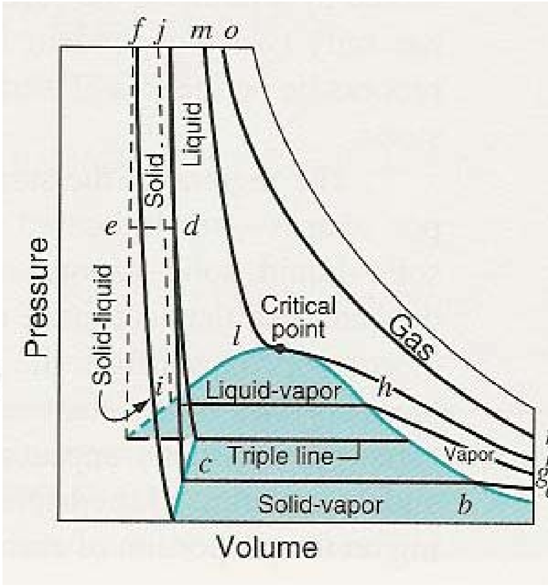
Phase diagram

Phase diagram is a 3D map of the Equation of State for an element or compound.

Regions and points of interest in Cryogenics.

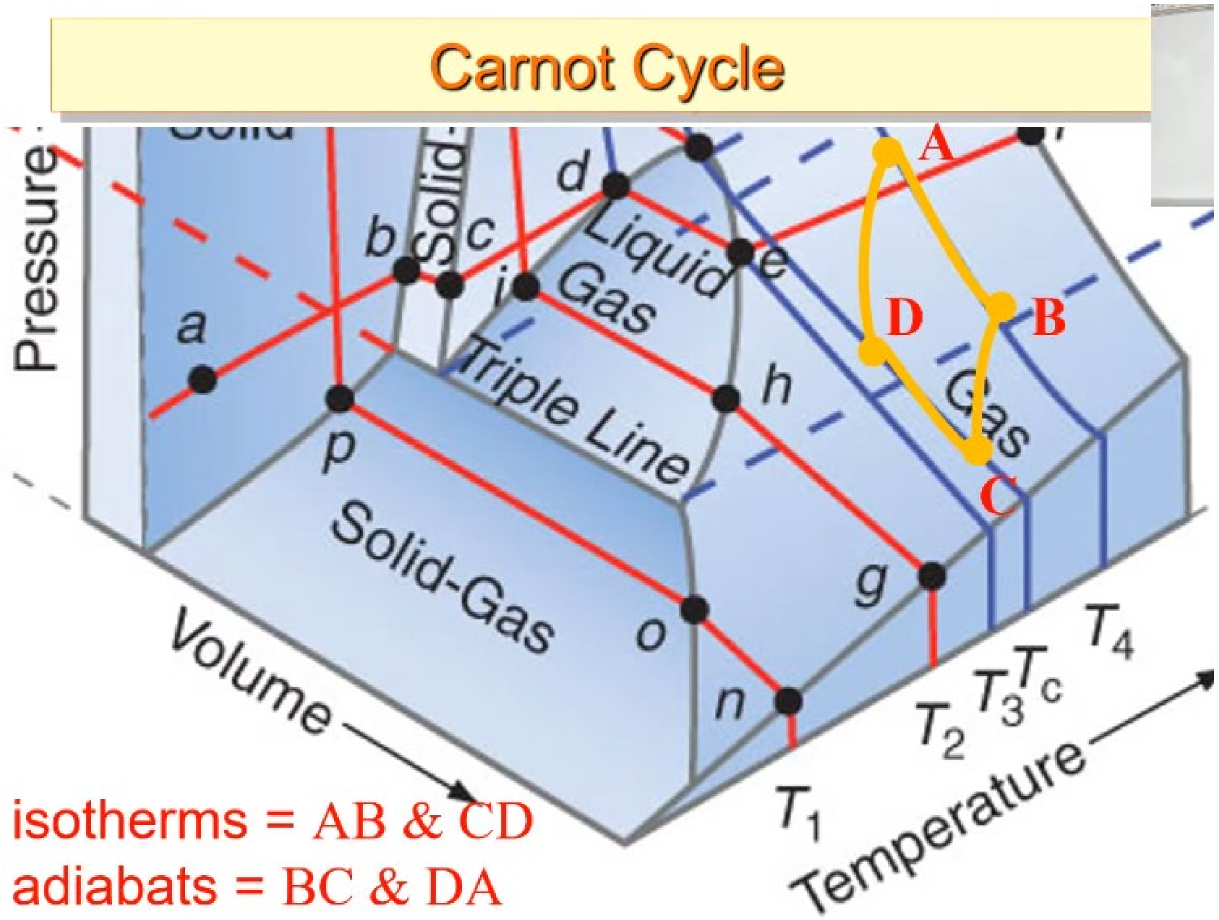
- Critical point (Tc, pc, vc)
- Phase co-existence
- Normal boiling point (NBP)
- Triple point (TP)
- Subcooled liquid
- Supercritical gas
- Solid state





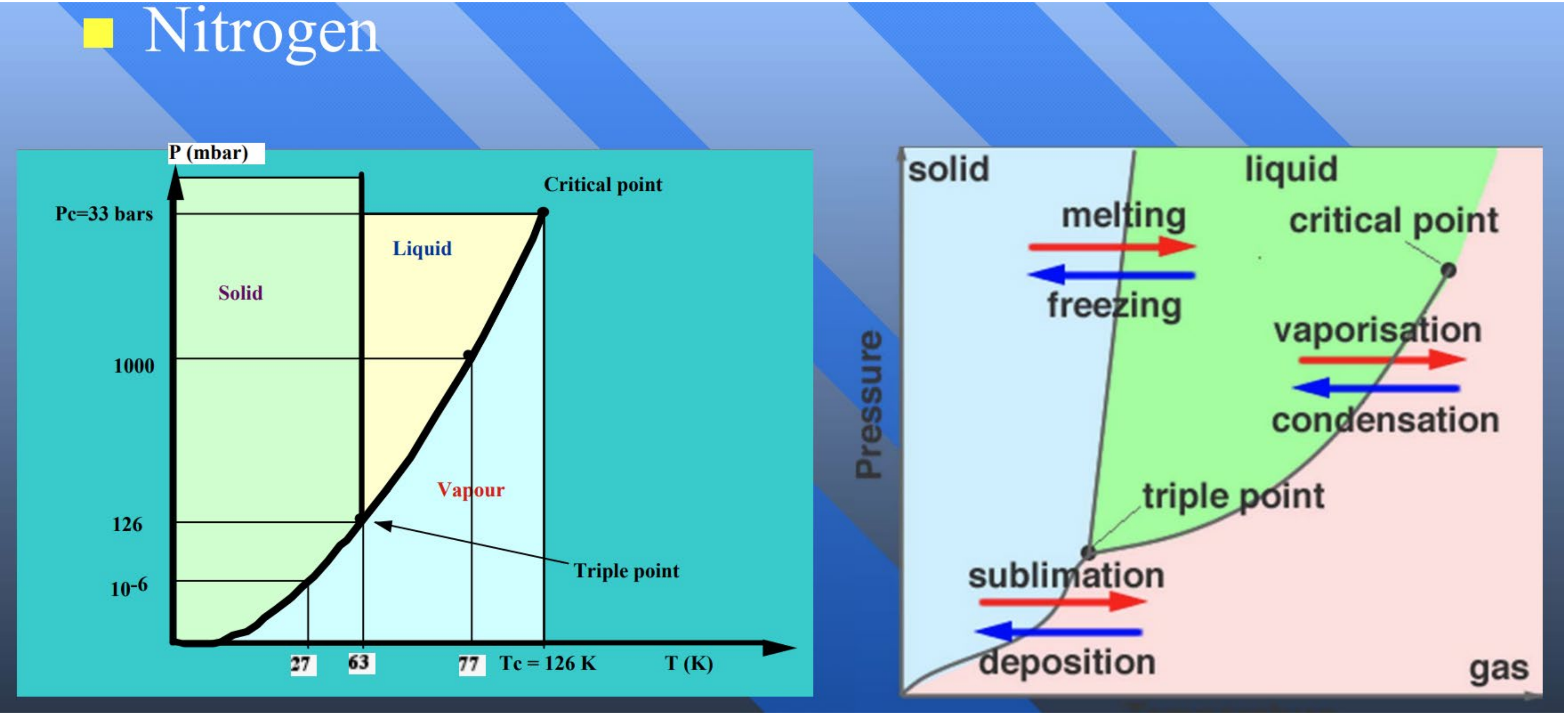
Constant Temp slices (isotherm)

Oscillating 'gas' flow cryocoolers



Stirling, GM & PTR all use cycles involving the 'gas' phase only.

Cryogenic fluids (nitrogen, hydrogen, helium, etc…)



A cryogenic fluid at atmospheric pressure is always boiling

Properties of Cryogenic Liquids

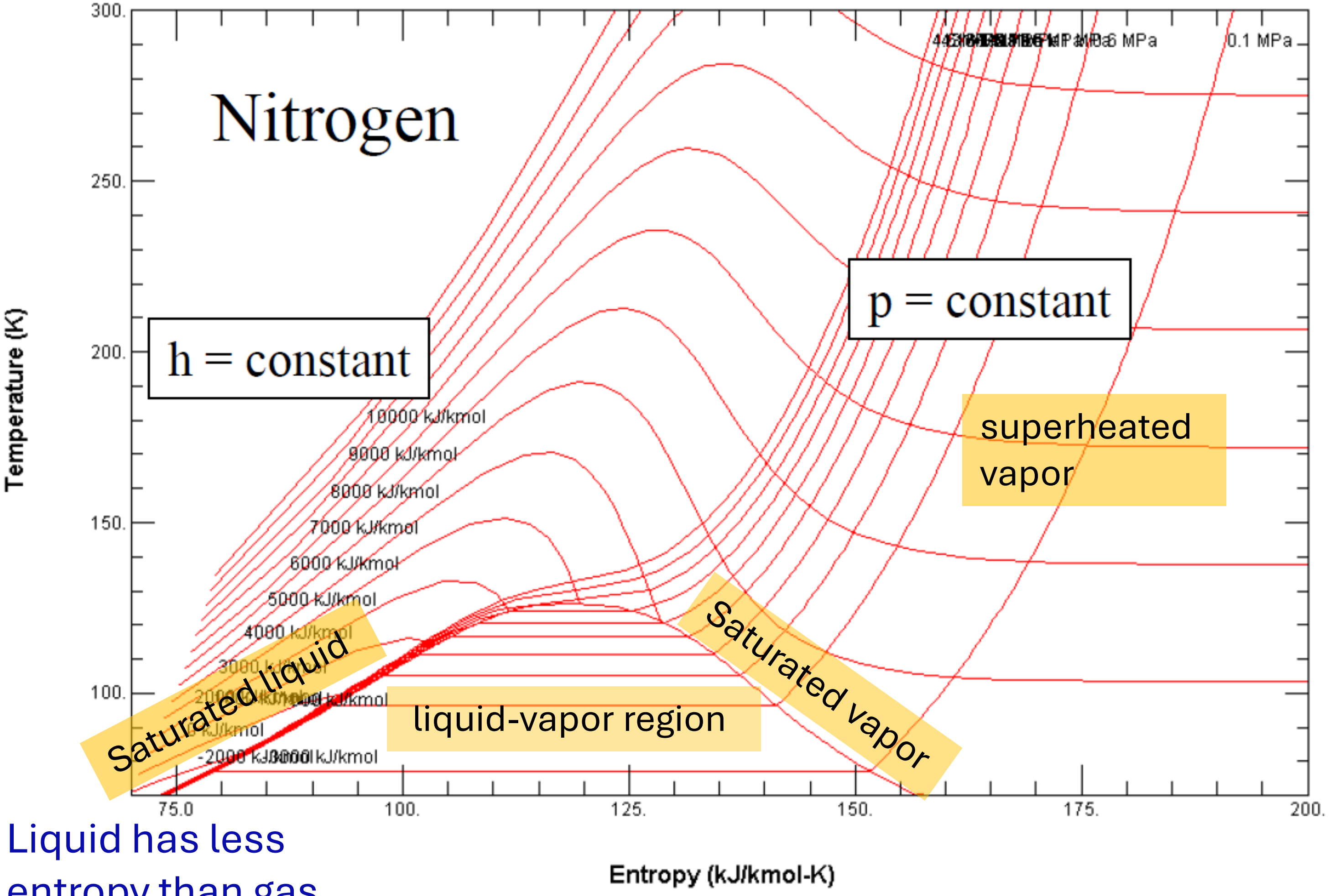
Table 2.1. Selected Properties of Cryogenic Liquids at Normal Boiling Point

	Liquid helium-4	Liquid e-hydrogen	Liquid neon	Liquid nitrogen[a]	Liquid air	Liquid fluorine	Liquid argon	Liquid oxygen[b]	Liquid methane
Normal boiling point (K)	4.224	20.268	27.09	77.347	78.9	85.24	87.28	90.18	111.7
Density (kg/m^3)	124.96	70.78	1204	808.9	874	1506.8	1403	1141	425.0
Heat of vaporization (kJ/kg)	20.73	445.6	86.6	198.3	205.1	166.3	161.6	212.9	511.5
Specific heat (kJ/kg K)	4.56	9.78	1.84	2.04	1.97	1.536	1.14	1.70	3.45
Viscosity ($kg/m \cdot s \times 10^6$)	3.57	13.06	124.0	157.9	168	244.7	252.1	188.0	118.6
Thermal conductivity ($mW/m \cdot K$)	27.2	118.5	113	139.6	141	148.0	123.2	151.4	193.1
Dielectric constant	1.0492	1.226	1.188	1.434	1.445	1.43	1.52	1.4837	1.6758
Critical temperature (K)	5.201	32.976	44.4	126.20	133.3	144.0	150.7	154.576	190.7
Critical pressure (MPa)	0.227	1.293	2.71	3.399	3.90	5.57	4.87	5.04	4.63
Temperature at triple point (K)	—	13.803	24.56	63.148	—	53.5	83.8	54.35	88.7
Pressure at triple point ($MPa \times 10^3$)	—	7.042	43.0	12.53	—	0.22	68.6	0.151	10.1

[a] Reference 3.
[b] Reference 1.

Timmerhaus and Flynn, Cryogenic Process Engineering

State Properties Summary (T-S Diagram)



Liquid has less entropy than gas

Lines of constant h are called "isenthalps"
Lines of constant p are "isobars"

Enthalpy, h, is a useful thermodynamic property for quantifying heat content of a fluid.

$h = E + pv$ and in differential form, $dh = Tds + vdp$

so the specific heat is also given as $C_p = \left.\frac{\partial h}{\partial T}\right|_p$

- **isenthalpic** process is one where a system's enthalpy remains constant (Δh = 0). This typically occurs in throttling processes, such as *a fluid passing through a valve or porous plug*, where there is no significant change in enthalpy despite changes in pressure and sometimes temperature.

- **isentropic** process involves constant entropy (ΔS = 0). It is often mistaken for being adiabatic, though it may or may not be.

- **Isobaric** process: constant pressure

Joule-Thomson (JT) cooler

(case of a nitrogen liquefier)

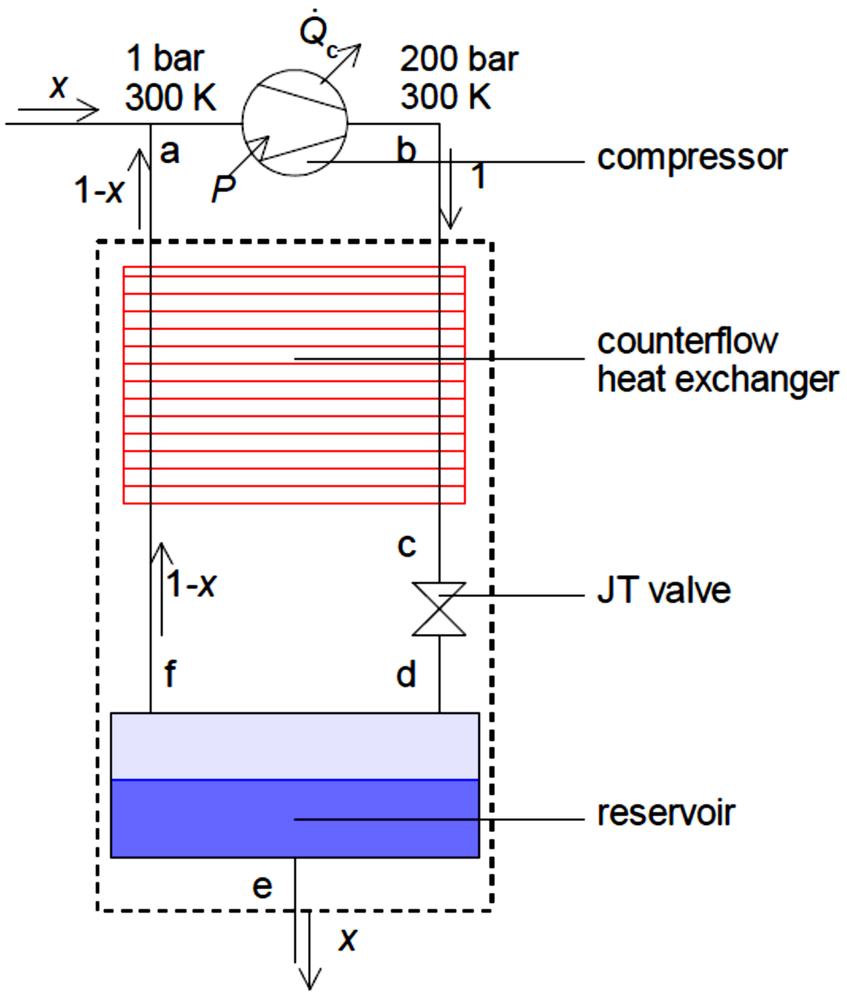
Invented by **Carl von Linde** and **William Hampson**, it is sometimes named after them.

Tt is a very simple type of cooler which is widely applied as the (final stage) of liquefaction machines.

It can easily be miniaturized, but it is also used on a very large scale in the liquefaction of natural gas.

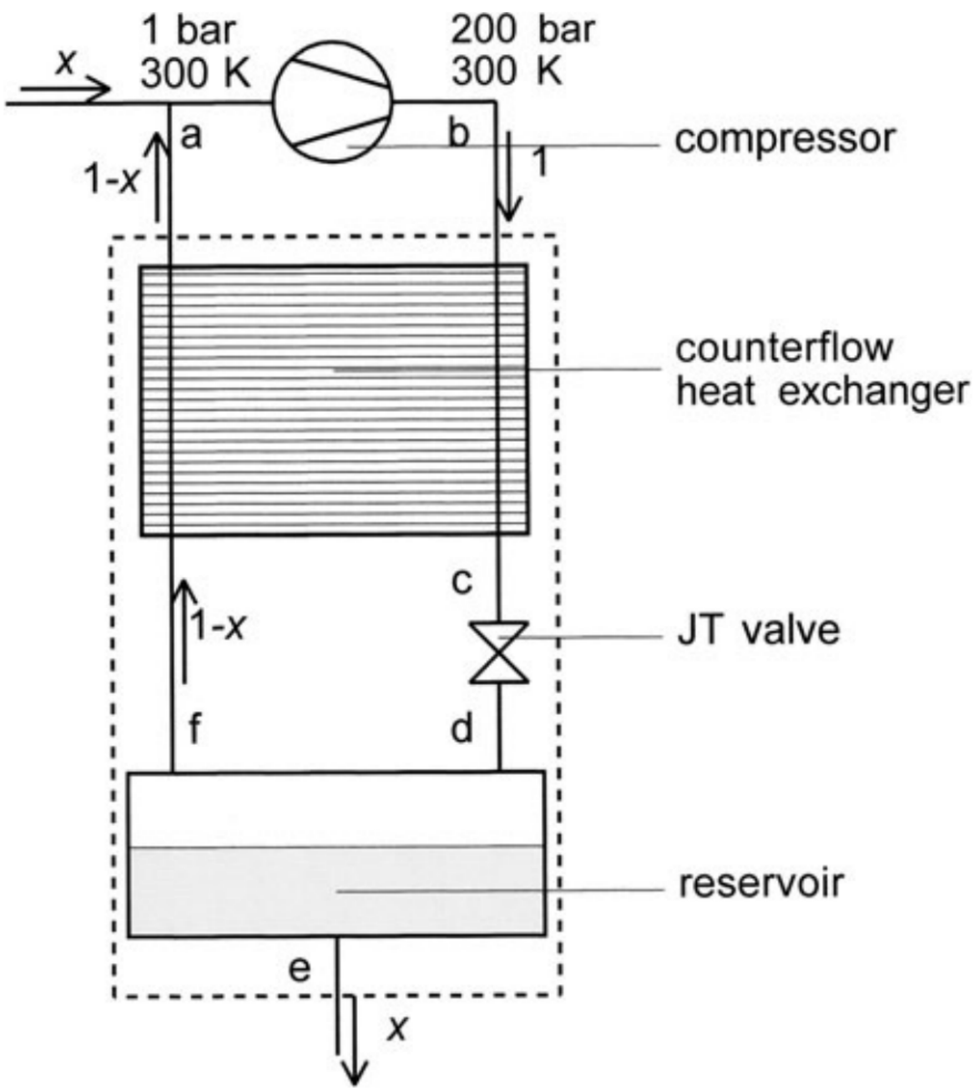
At the liquid side a fraction x of the compressed gas is removed as liquid.

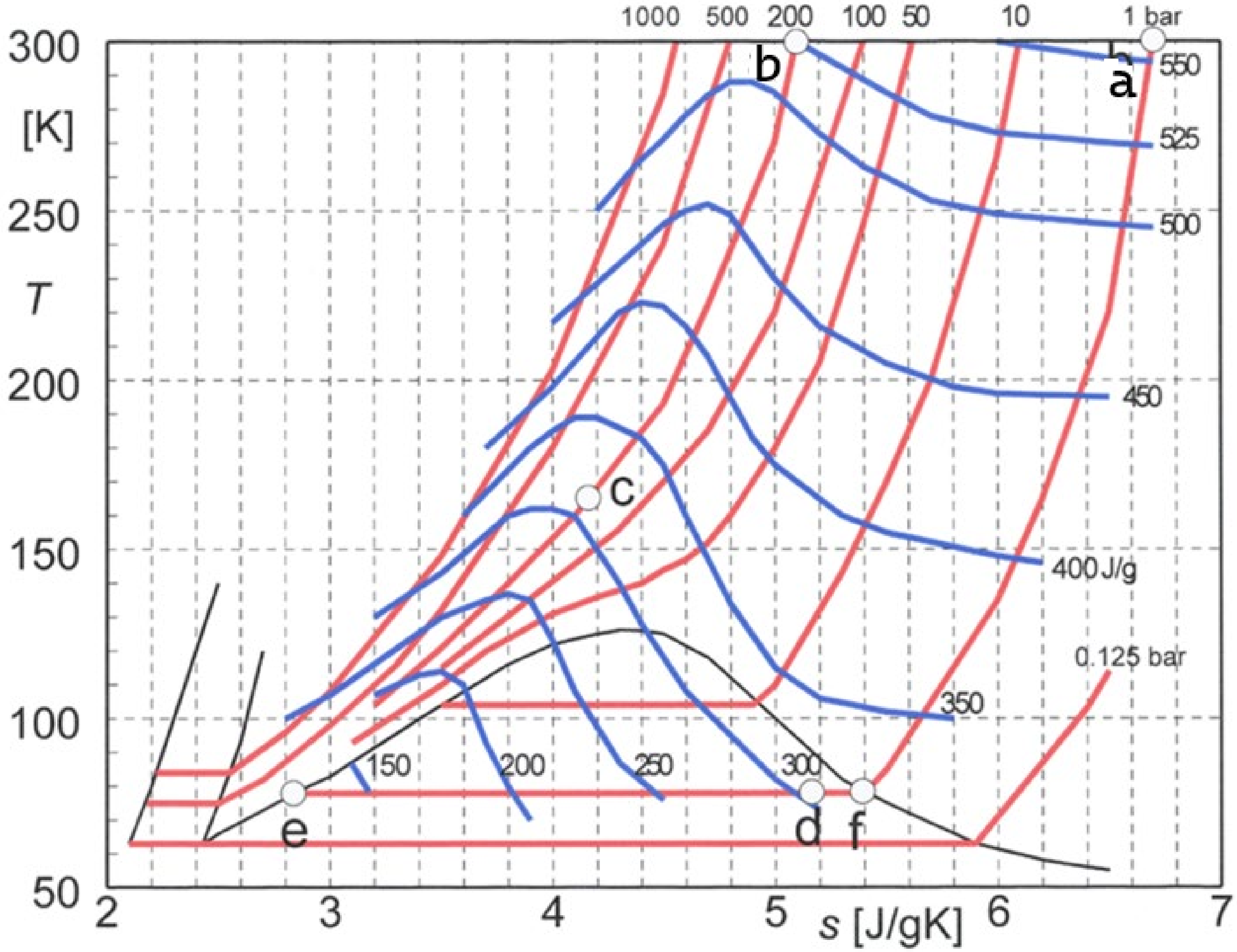
At room temperature it is supplied, so that the system is in the steady state.



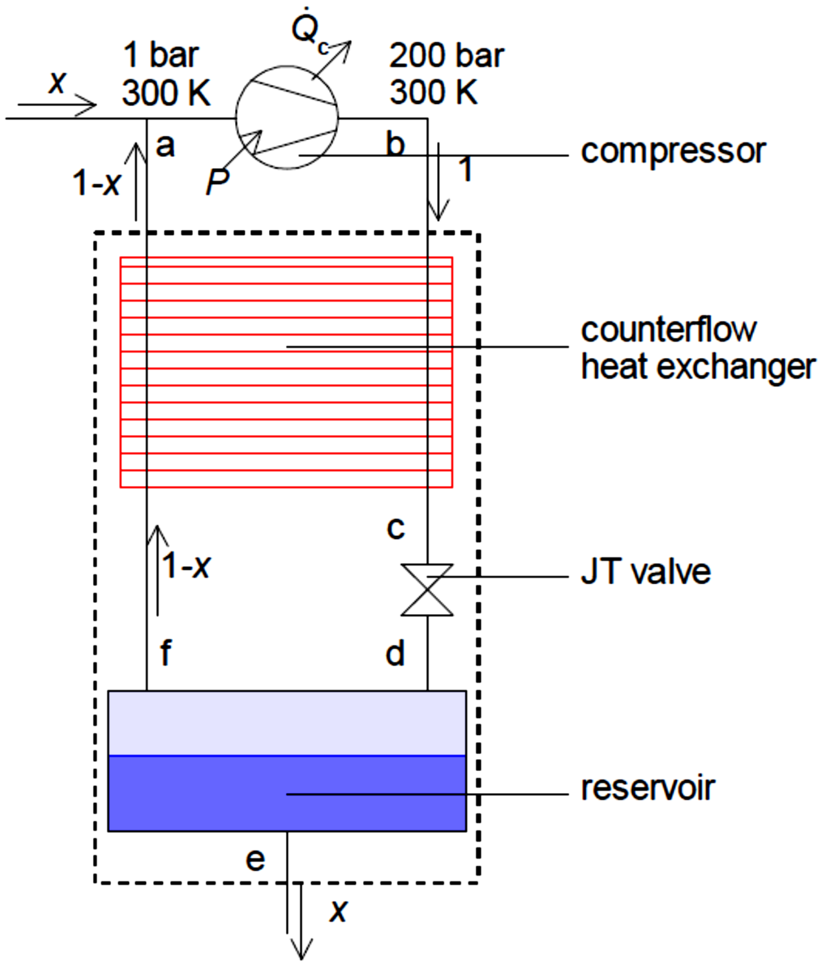
Schematic diagram of a JT liquefier.
The symbols a…f refer to points in the S-T diagram.

Joule-Thompson (JT) refrigeration

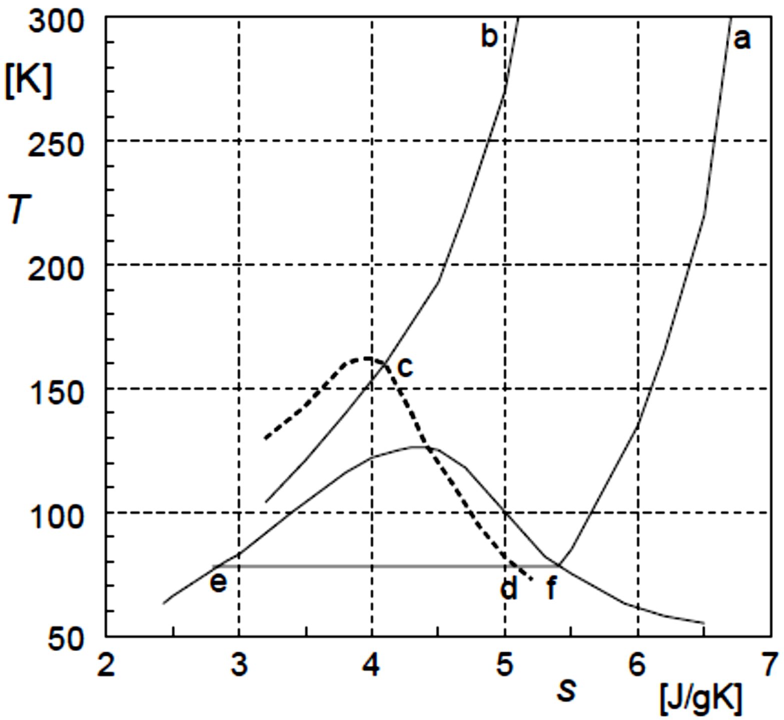




Ts-diagram of nitrogen with *isobars*, *isenthalps*, and the *lines of coexistence*. The pressures are given in bar, the specific enthalpy in J/g.



Ts-diagram of nitrogen with isobars at 1 and 200 bar, the coexistence line and the isenthalp of the JT-expansion indicated.



$$h_b = x h_e + (1-x)\, h_a$$

or

$$x = \frac{h_a - h_b}{h_a - h_e}$$

liquefaction if $x > 0$. As $h_a > h_e$ this means

$$h_a > h_b$$

$$x = \frac{555 - 525}{555 - 130} = 0.07$$

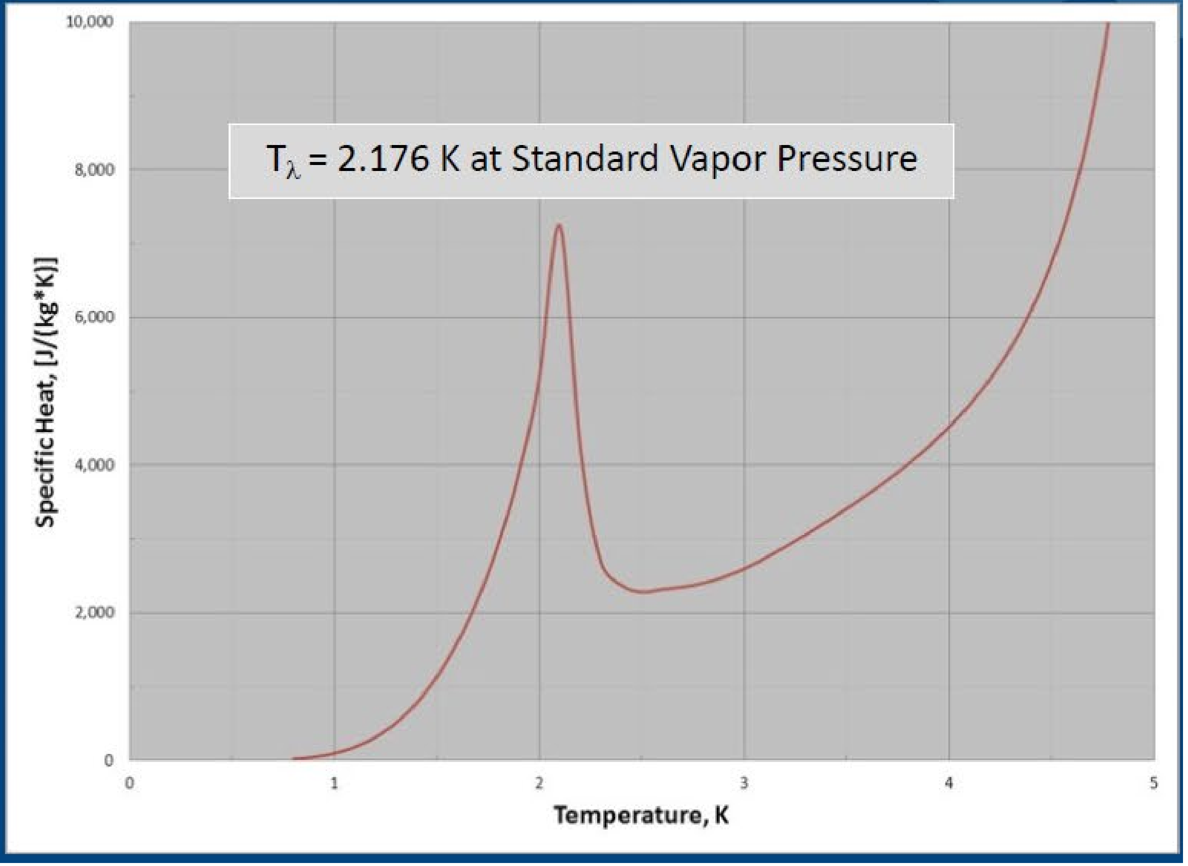
$$h_d = x h_e + (1-x)\, h_f = 307 \text{ J/g}$$

	p (bar)	T (K)	h (J/g)	s (J/gK)
a	1	300	555	6.7
b	200	300	525	5.1
c	200	(165)	(307)	(5.2)
d	1	78	(307)	(4.2)
e	1	78	130	2.8
f	1	78	320	5.4

A fourth state in He: He II

He I → He II transition

- The transition is called *λ-transition* because the specific heat curve resembles shape of *λ*
- Second order transition with no latent heat
  - He I and He II do not exist in equilibrium
- Transition temperature is called T_{λ}
  - T_{λ} = 2.176 K at SVP
  - T_{λ} = 1.76 K at 25 bar

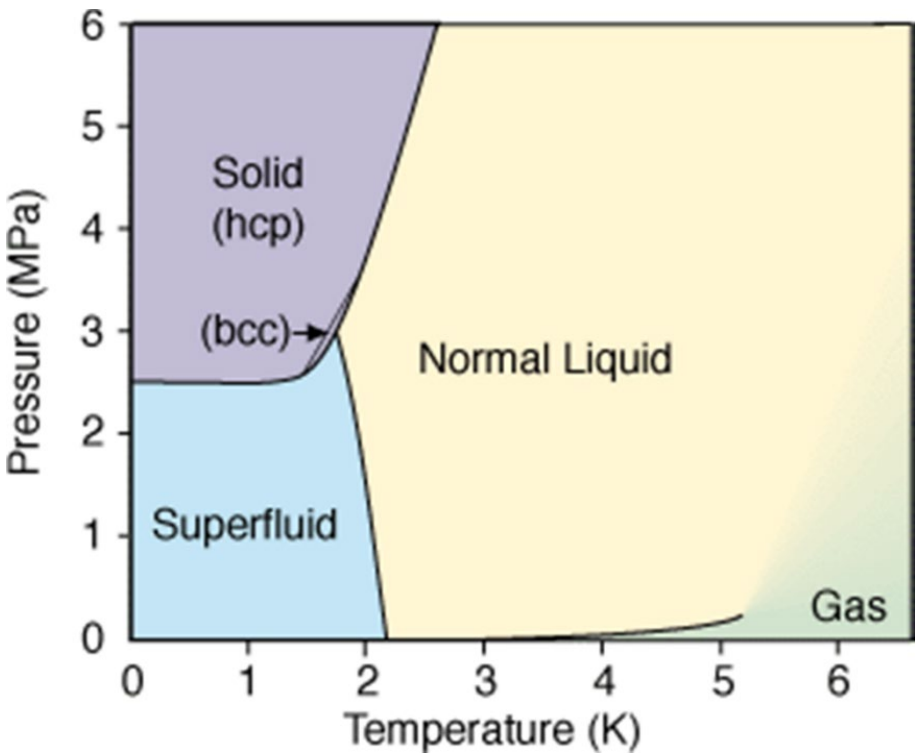


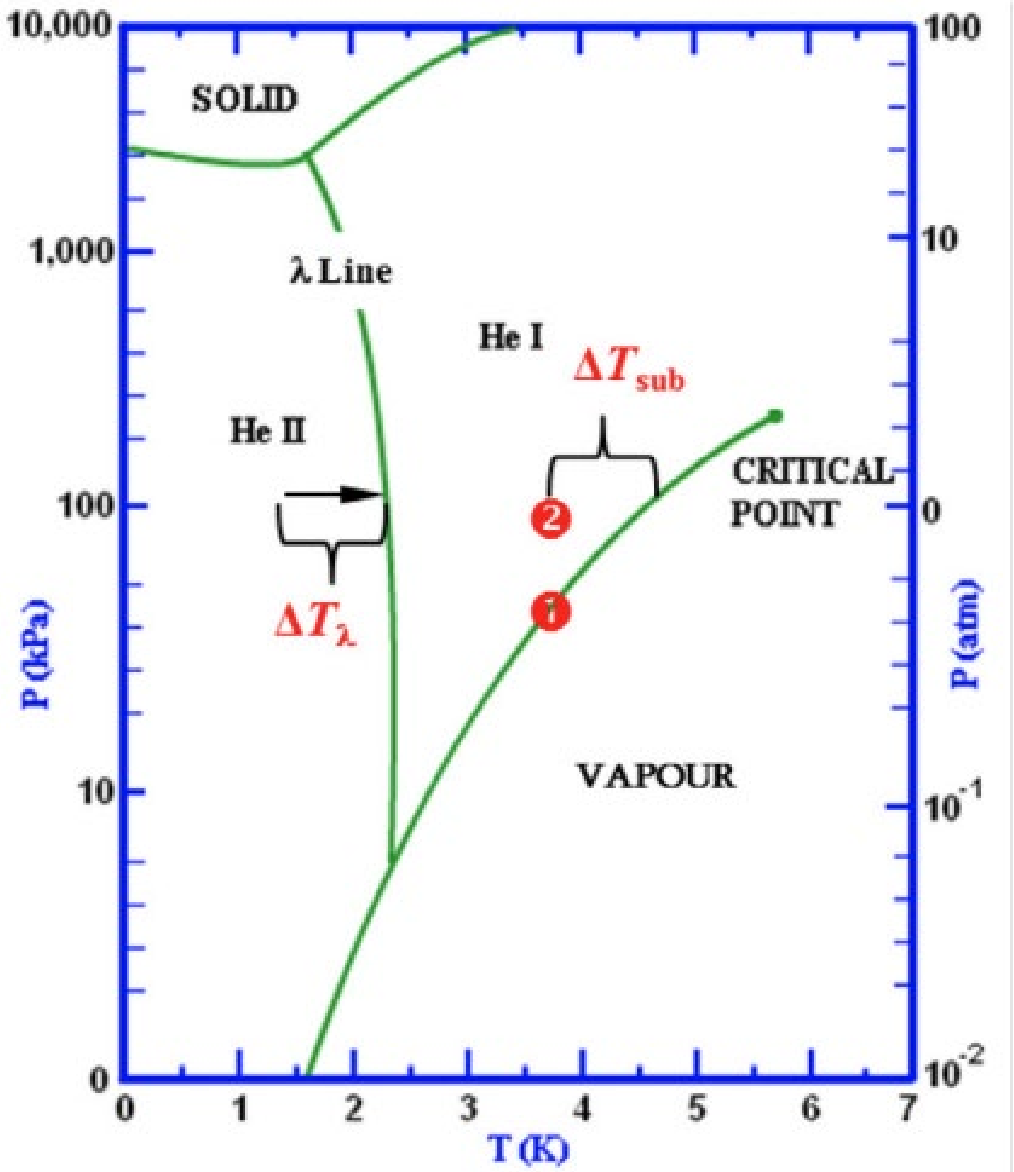
Helium

No solidification at low pressures.

Solid →increase P at low temperatures (T< 1K; P> 25bar)

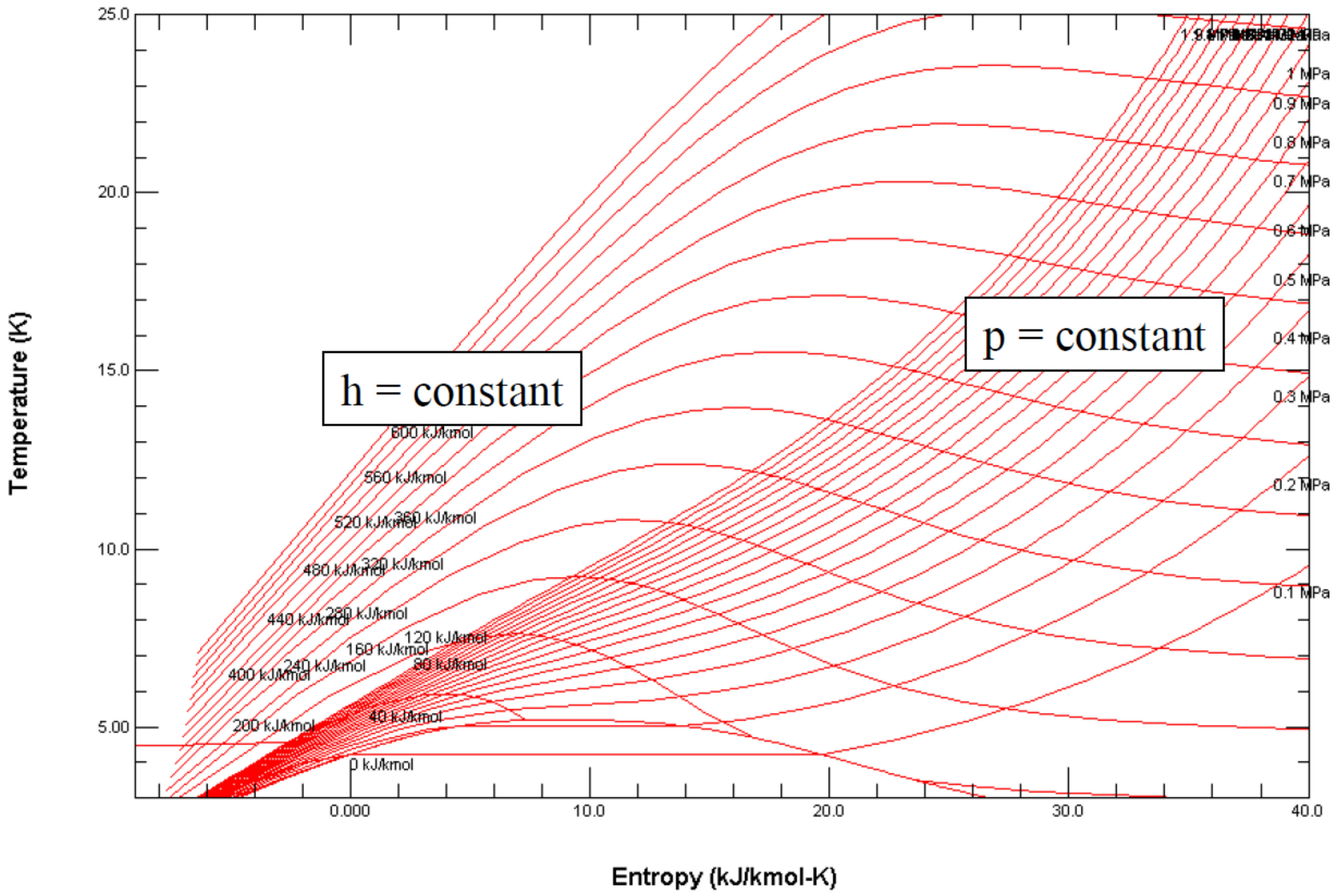
No triple point!





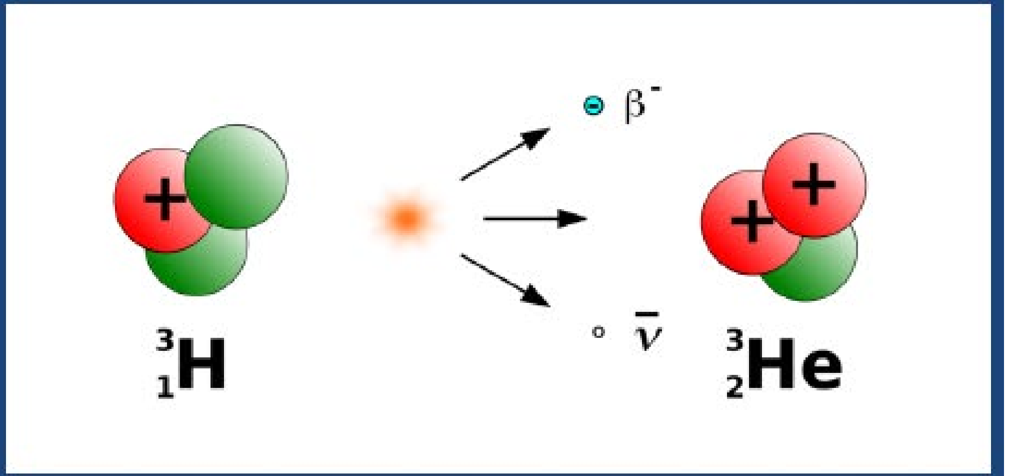
Cryostats operate at point 2 to avoid boiling

T-S Diagram for Helium



Production of Helium-3

- Helium-3 is rare. Current stocks come from the decay of tritium (12 year half life) used in thermonuclear weapons.
- Occurs in very low abundance in sea water
- May occur in abundance on the lunar surface, but this of course in inaccessible (for now)
- Other than cryogenics, uses include neutron detection, often in security applications, and medical imaging.



Beta decay of tritium (from physicsstack.org)

Availability and economics of Helium-3

- Production and distribution of Helium-3 in the US is managed by the Department of Energy Isotope Program.

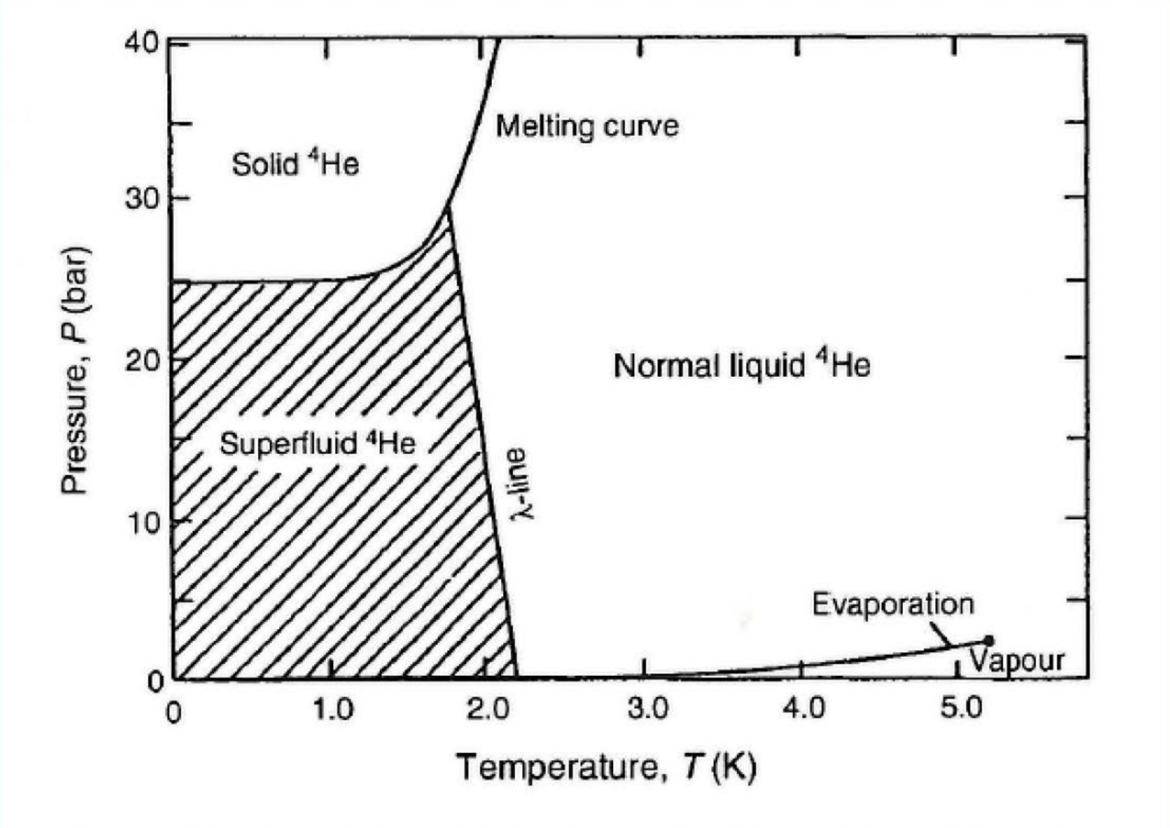
- Also available from sources in Russia and Europe.

- Cost of Helium-4 approximately $1 per stp liter of gas. Cost of Helium-3 is $1000 per stp liter through the DOE Isotope Program, and up to $3000 per stp liter on the open market.

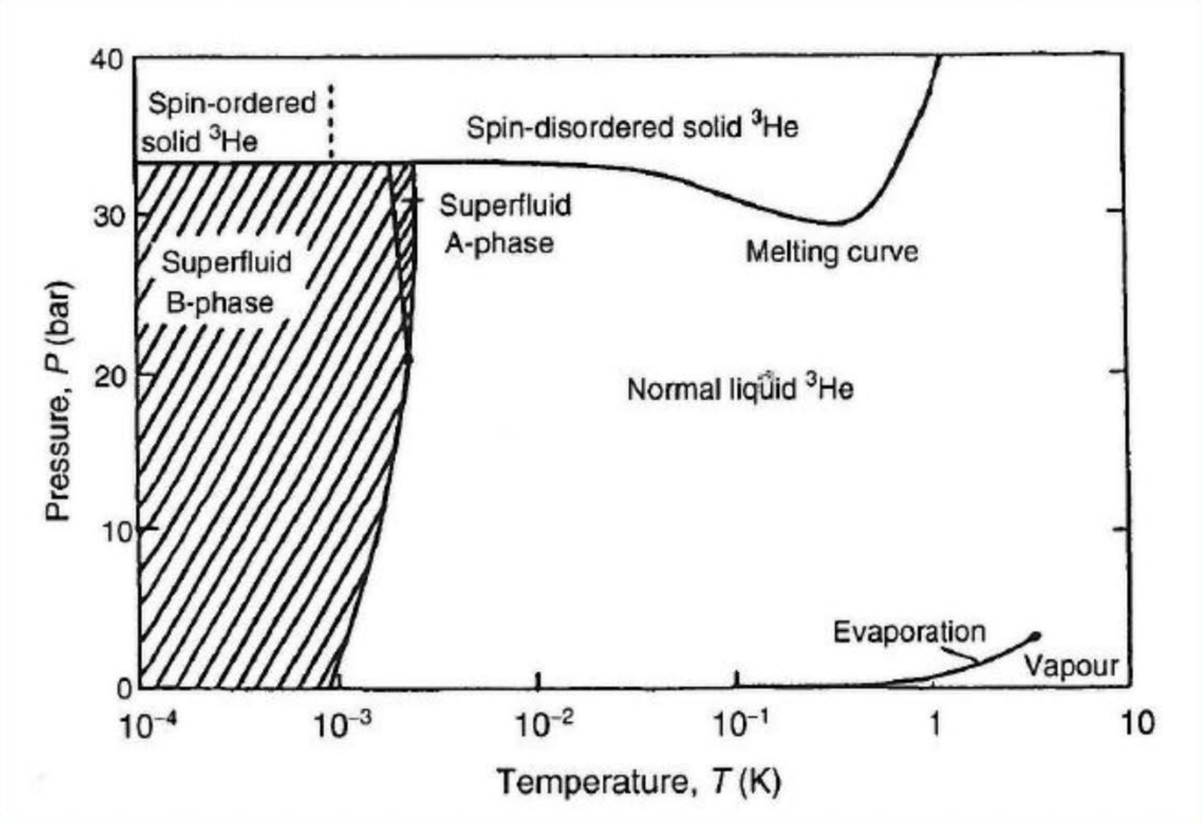
Liquid Helium and Isotopes

	4He	3He
Boiling Point (at 1 atmosphere, K)	4.21	3.19
Critical temperature (K)	5.20	3.32
Maximum superfluid transition temperature (K)	2.1768	0.0025
Density (grams cm^{-3})	0.1451	0.082
Latent Heat (at normal boiling point, kJ/kg)	20.9	8.49
Classical molar volume (cm^3 mol^{-1})	12	12
Actual molar volume (cm^3 mol^{-1})	27.58	36.84
Gas-to-liquid expansion ratio	866	662

Helium Phase Diagram



Helium-4 Phase Diagram



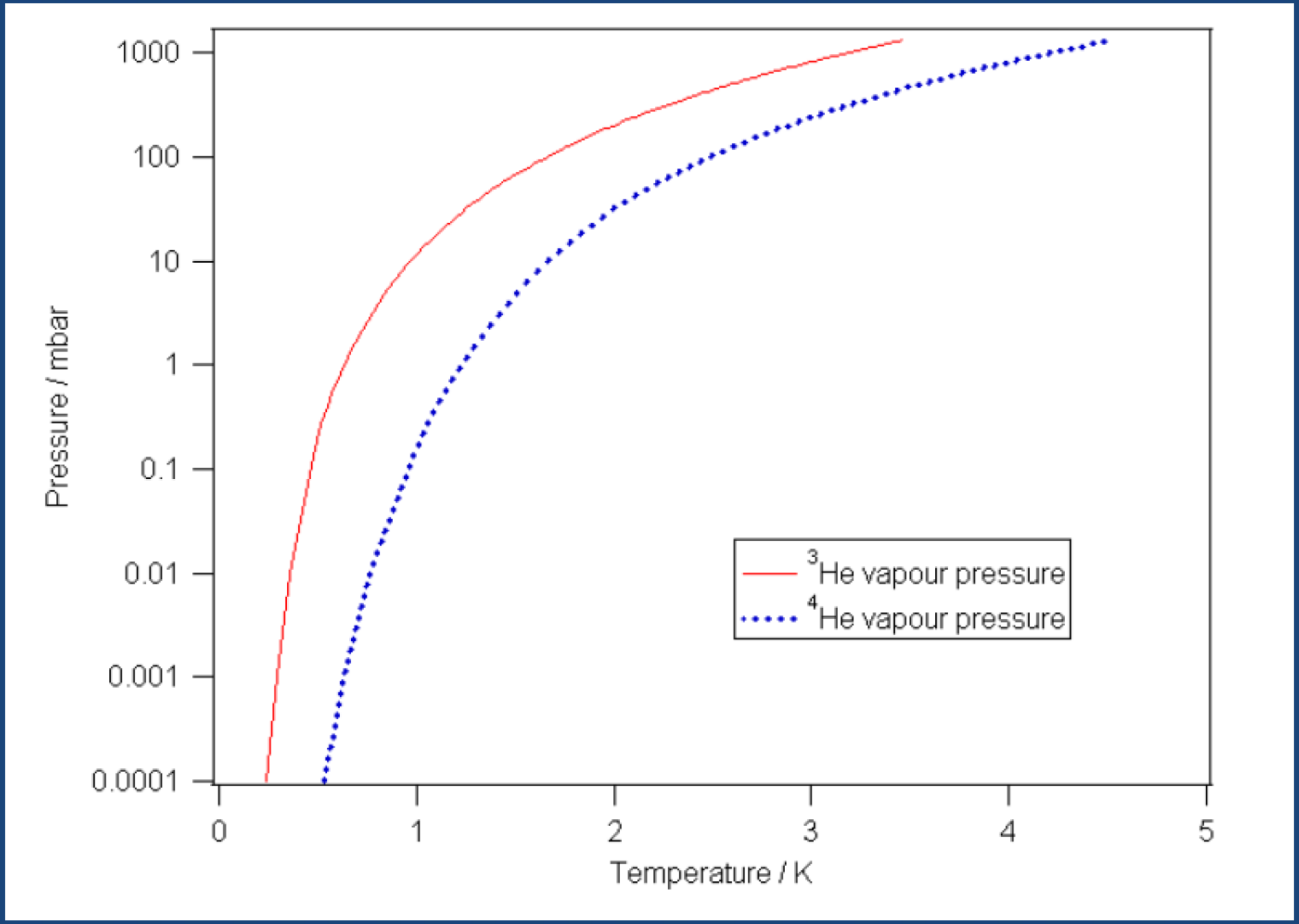
Helium-3 Phase Diagram

Adapted from Pobell (2007)

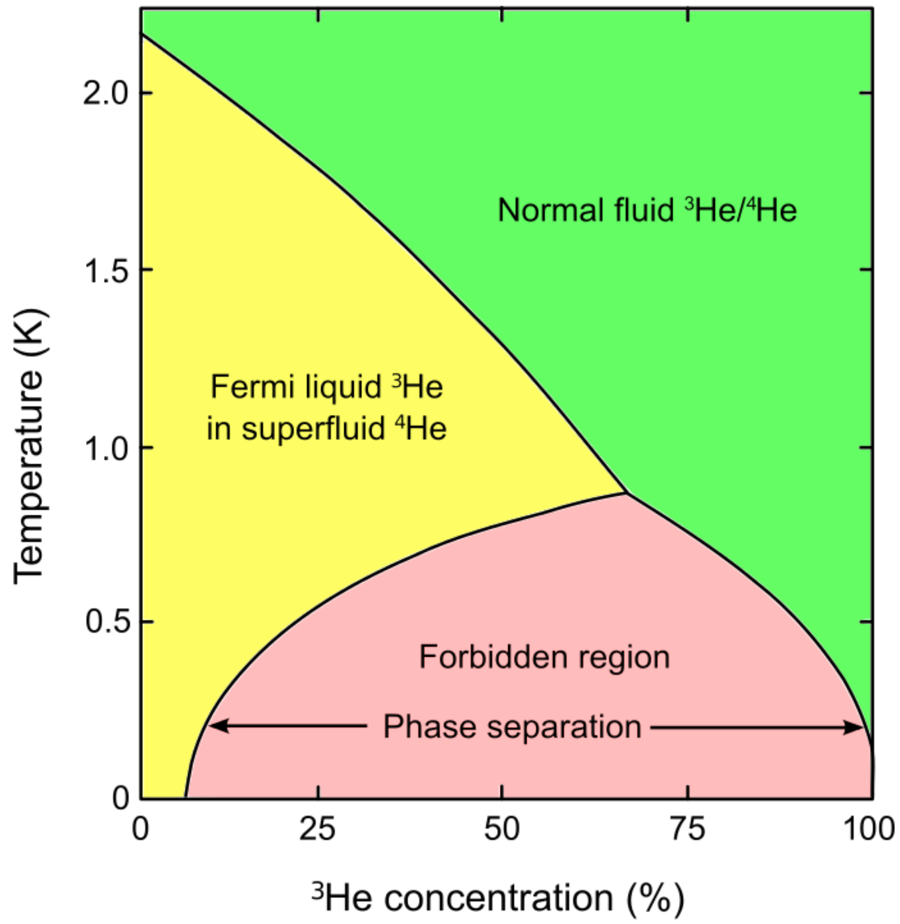
Helium vapor pressure

Note the log scale

Fast rolloff of the Helium-4 vapor pressure below the λ-point



Phase diagram of helium mixtures



Diluting 3He into 4He depresses the superfluid transition temperature. Superfluid transition disappears completely for *x* > 67.5%

- As a mixture with *x*> 6.6% is cooled, it will **separate** into 2 phases.
- One phase (the "concentrated phase") will approach *x*=100%, the other (the "dilute phase") will approach *x*= 6.6% @ 0 K.

This finite solubility and phase separation is key to the dilution refrigerator process