PHY524 Cryogenics-5

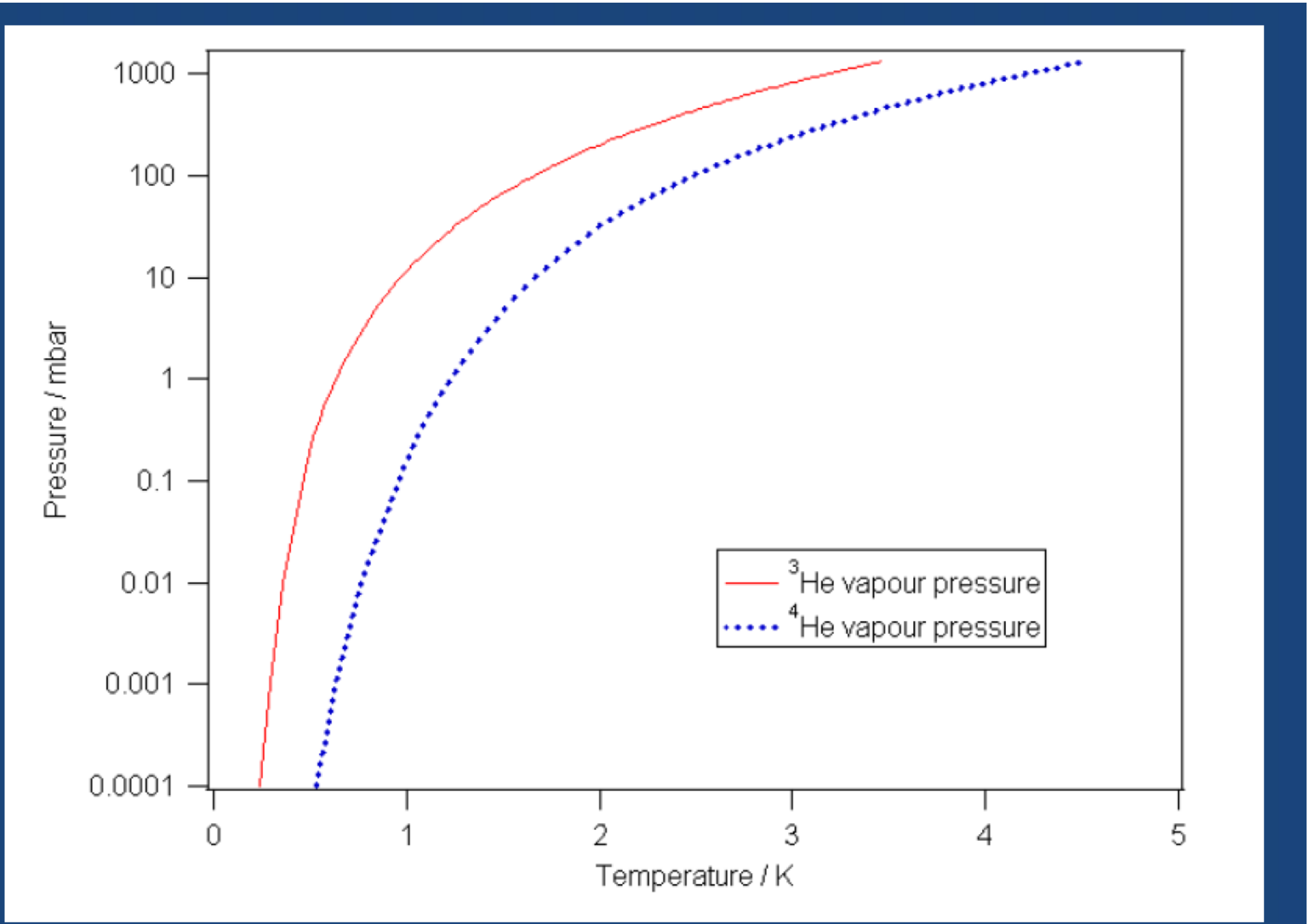
Liquefication, Helium & Dilution Refrigerator

11/20/2025

Material adopted from lectures by Henri Godfrin, Grenoble

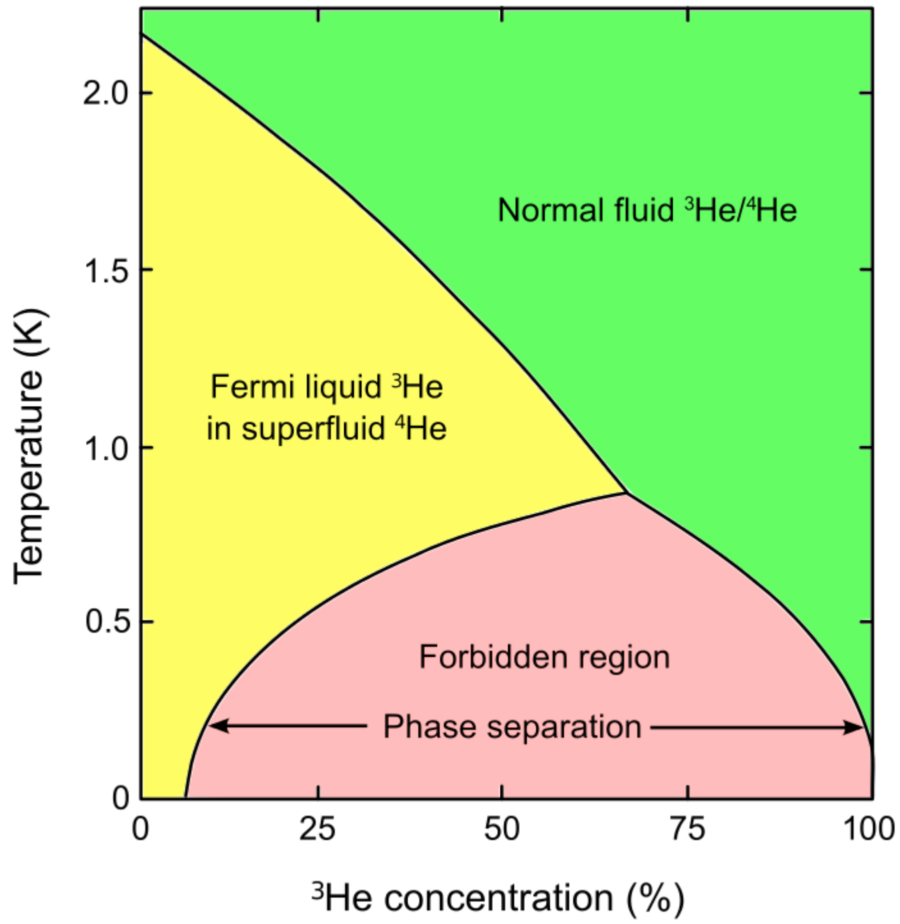
Helium vapor pressure

Note the log scale



Fast rolloff of the Helium-4 vapor pressure below the λ-point

Phase diagram of helium mixtures



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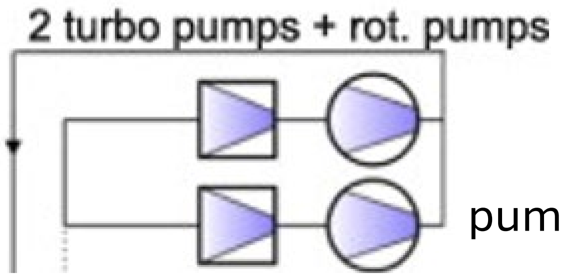
Diluting 3He into 4He depresses the superfluid transition temperature. Superfluid transition disappears completely for *x* > 67.5%

- As a mixture with *x*> 6.6% is cooled, it will **separate** into 2 phases.
- One phase (the "concentrated phase") will approach *x*=100%, the other (the "dilute phase") will approach *x*= 6.6% @ 0 K.

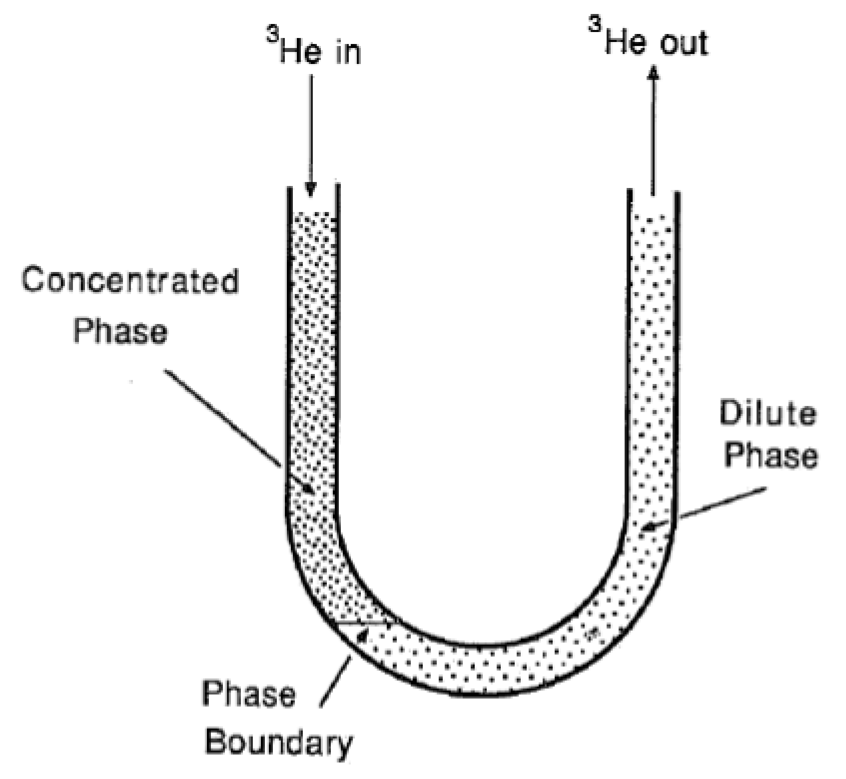
This finite solubility and phase separation is key to the dilution refrigerator process

DR cooling principles



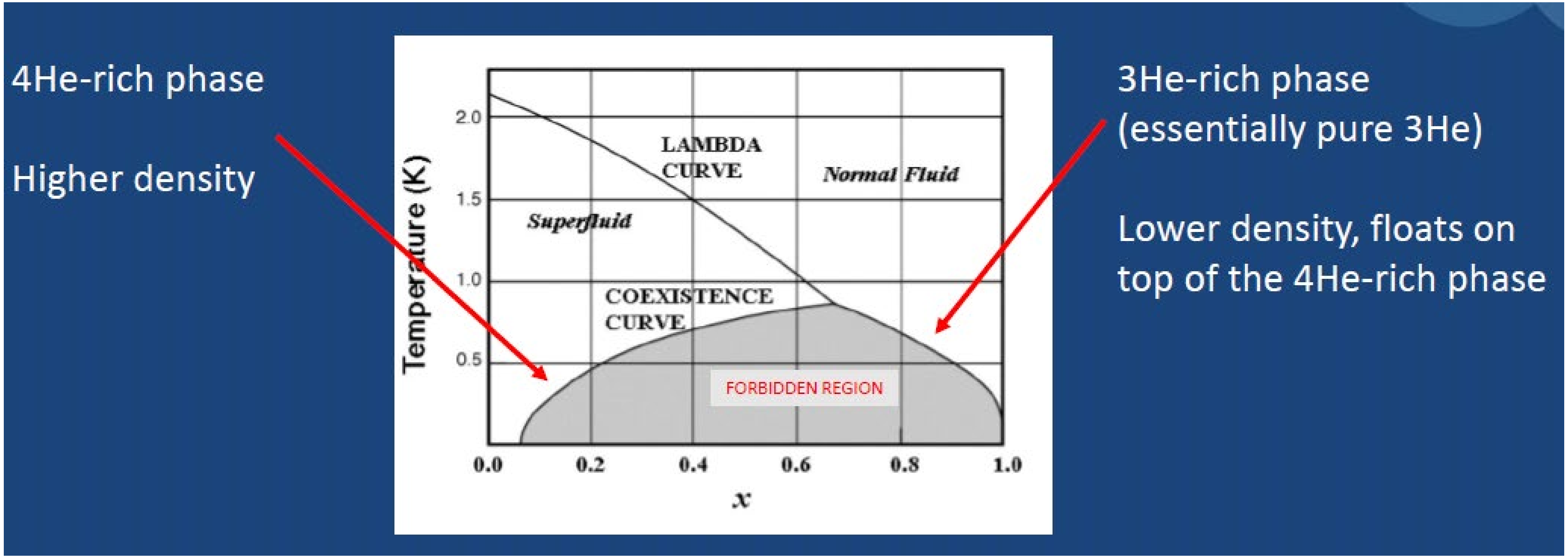
pumps

- **Simplest model for the refrigerator is a U-tube**, with the concentrated phase on one side and the dilute side on the other.
- As observed previously, the less dense **concentrated phase floats on top of the dilute phase** since the superfluid has much higher density.
- Removing 3He from the dilute phase surface upsets the equilibrium.



From Richardson & Smith (1998)

^{3}He-^{4}He mixtures



4He-rich phase

Higher density

3He-rich phase
(essentially pure 3He)

Lower density, floats on top of the 4He-rich phase

4He obeys **Bose statistics** and will condense into the quantum mechanical ground state at low enough temperature, i.e., superfluid.

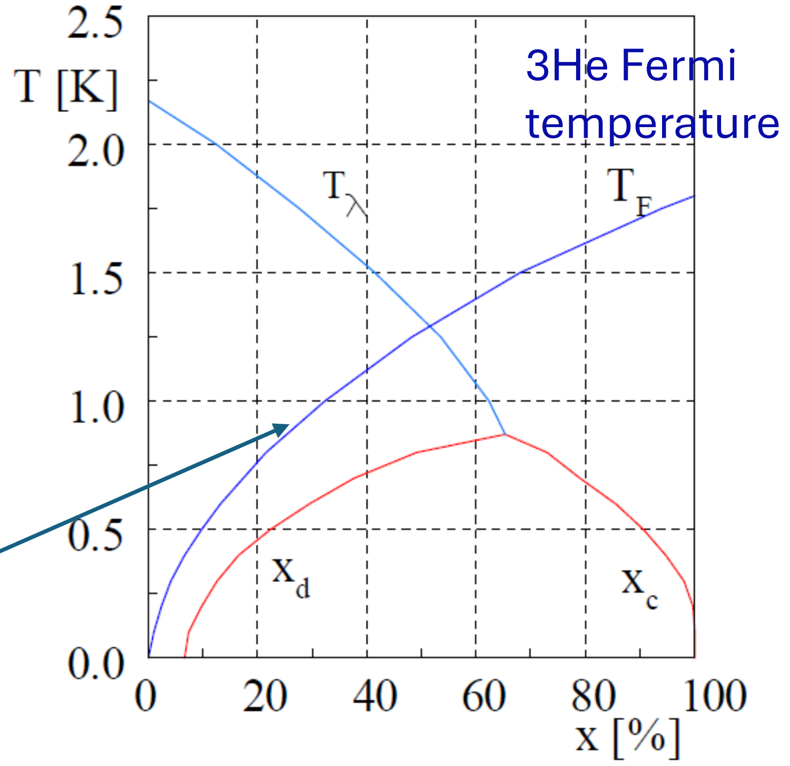
In a helium mixture, the condensed 4He acts as an “inert superfluid background” which contributes to the volume of the liquid and the effective mass of the 3He atoms, but has **zero heat capacity**.

The 3He obeys **Fermi statistics** and will behave in a similar way to conduction electrons in a metal.
However, whereas conduction electrons have a Fermi temperature of 10,000 K, **the Fermi temperature of the 3He is around 1 K**.

Phase Separation & Finite Solubility

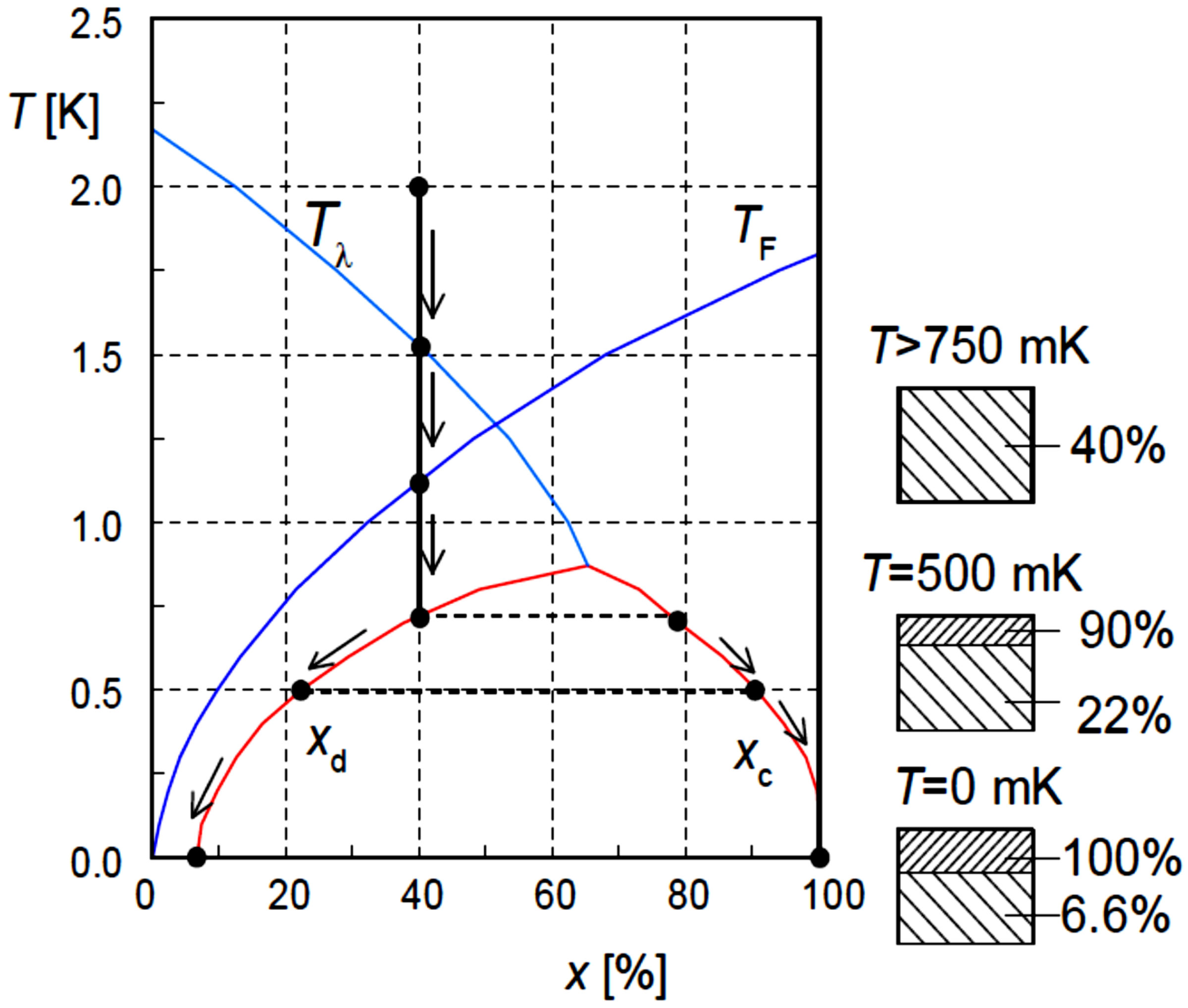
- The van der Waals forces between 4He and 3He are identical, since their electronic structure is identical. However, the 3He atoms have larger zero-point motion (due to their smaller mass), hence the 4He atoms occupy a smaller volume than the 3He atoms, and the 3He atoms will feel a stronger attraction to a 4He atom.
- However, recall that the 3He atoms must obey the **Pauli Exclusion Principle**: If we add additional 3He atoms in the liquid, they have to go into successively higher energy states
- Eventually, all the states up to the Fermi energy will be filled with two 3He atoms. *The binding energy of the 3He has to decrease if the number of atoms is increased.*
- **The chemical potential of a 3He atom** in the dilute mixture is thus:

$$\frac{\mu_{3,d}(0)}{N_0} = -\epsilon_{3,d}(x) + k_B T_F(x)$$



3He Fermi temperature

Solubility decreases upon cooling



Removing 3He from the top of the dilute phase will reduce the concentration below the 6.6% equilibrium concentration.

- The osmotic pressure difference will drive the flow of 3He across the phase boundary.

Since we will be dealing with helium mixtures with varying concentration and temperature, we will see osmotic pressure differences.

History

- Dilution refrigerators are a real workhorse apparatus for millikelvin cryogenics; they are the only cooling machine capable of obtaining millikelvin temperatures **continuously**.
- Very useful at higher Sub-1 Kelvin temperatures as well due to high cooling power compared to other refrigeration processes in the same range.
- Cooling comes from the heat of mixing of Helium-3 and Helium-4, the two common helium isotopes.

The dilution refrigerator principle was suggested by **Heinz London in 1952**

H. London, G.R. Clarke, and E. Mendoza proposed a prototype of continuous refrigerator in 1962

It was realized in 1964 in the Kamerlingh Onnes Laboratorium at Leiden University
Das, P.; Ouboter, R. B.; Taconis, K. W. (1965). "A Realization of a London-Clarke-Mendoza Type Refrigerator". Low Temperature Physics LT9. p. 1253.
(Tmin~220mK)

B.S. Neganov and co-workers in **Dubna** and H.E. Hall and co-workers in **Manchester** went below
100 mK (1966). Dubna rapidly reached **25 mK**.

The principles and methods of dilution refrigeration have been substantially developed by J. Wheatley et al. at **La Jolla**.

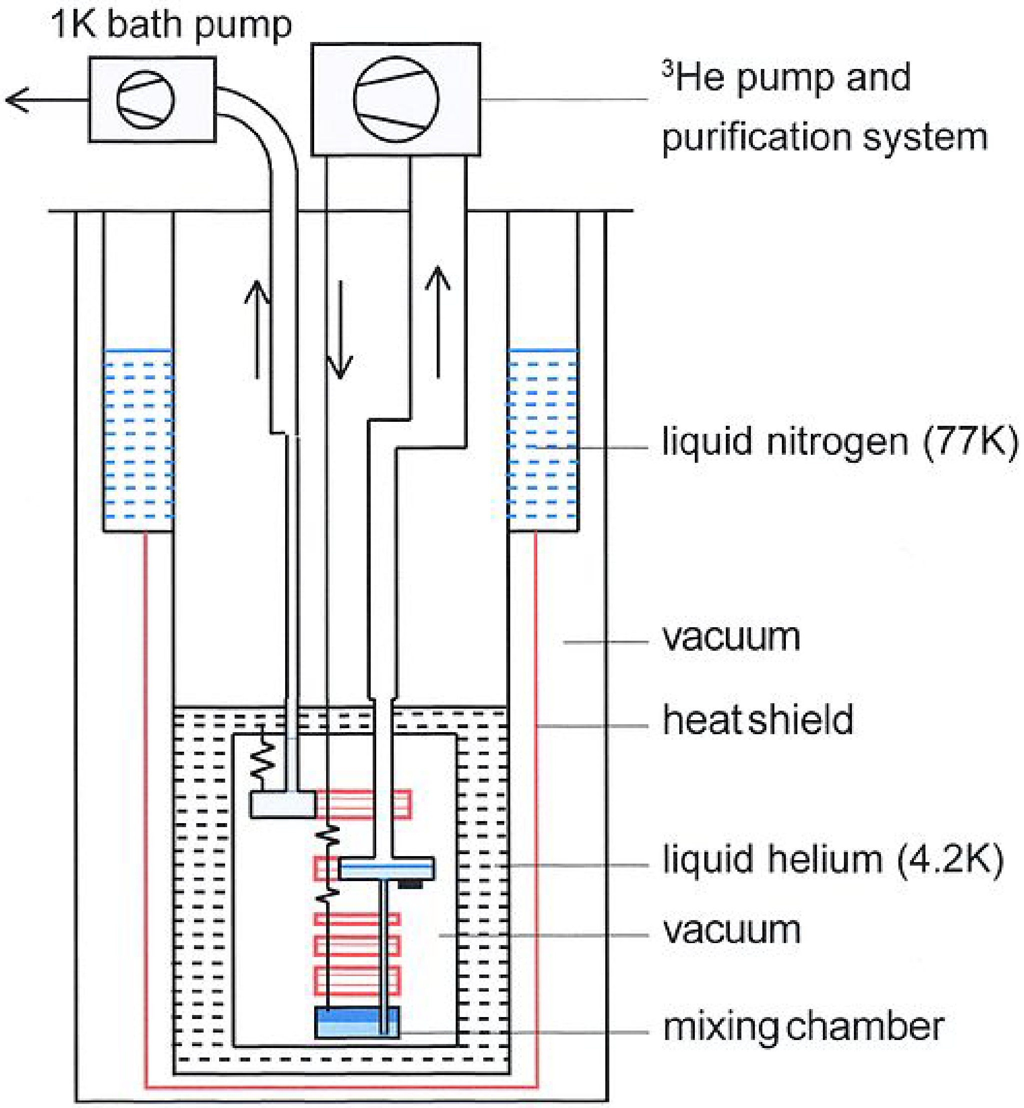
Modern « wet » refrigerators are based on the **Grenoble design** by Frossati and coworkers.
The development of sintered silver heat exchangers Tmin led to **Tmin~2 mK**.

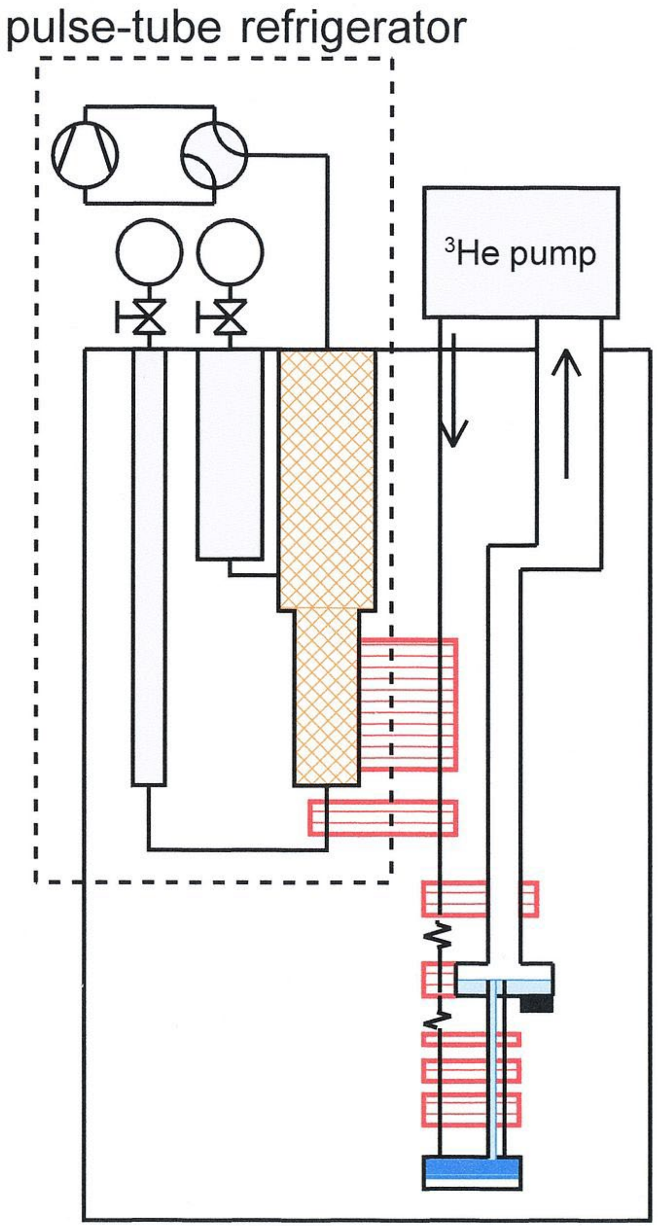
« Dry » refrigerators were developed
- by K. Uhlig et al. **on *GM coolers*** (1993),
- and **on *Pulse-tube coolers*** by K. Uhlig et al. (2002) and H. Godfrin (1999 to 2003, date of first commercial unit, delivered by l’ Air Liquide)

Lancaster has the present record of low temperatures, **1.75 mK**

Wet vs Dry Dilution Refrigerator (DR)

A ‘wet’ DR:
A 1K pot from Lhe is used to cool 3He

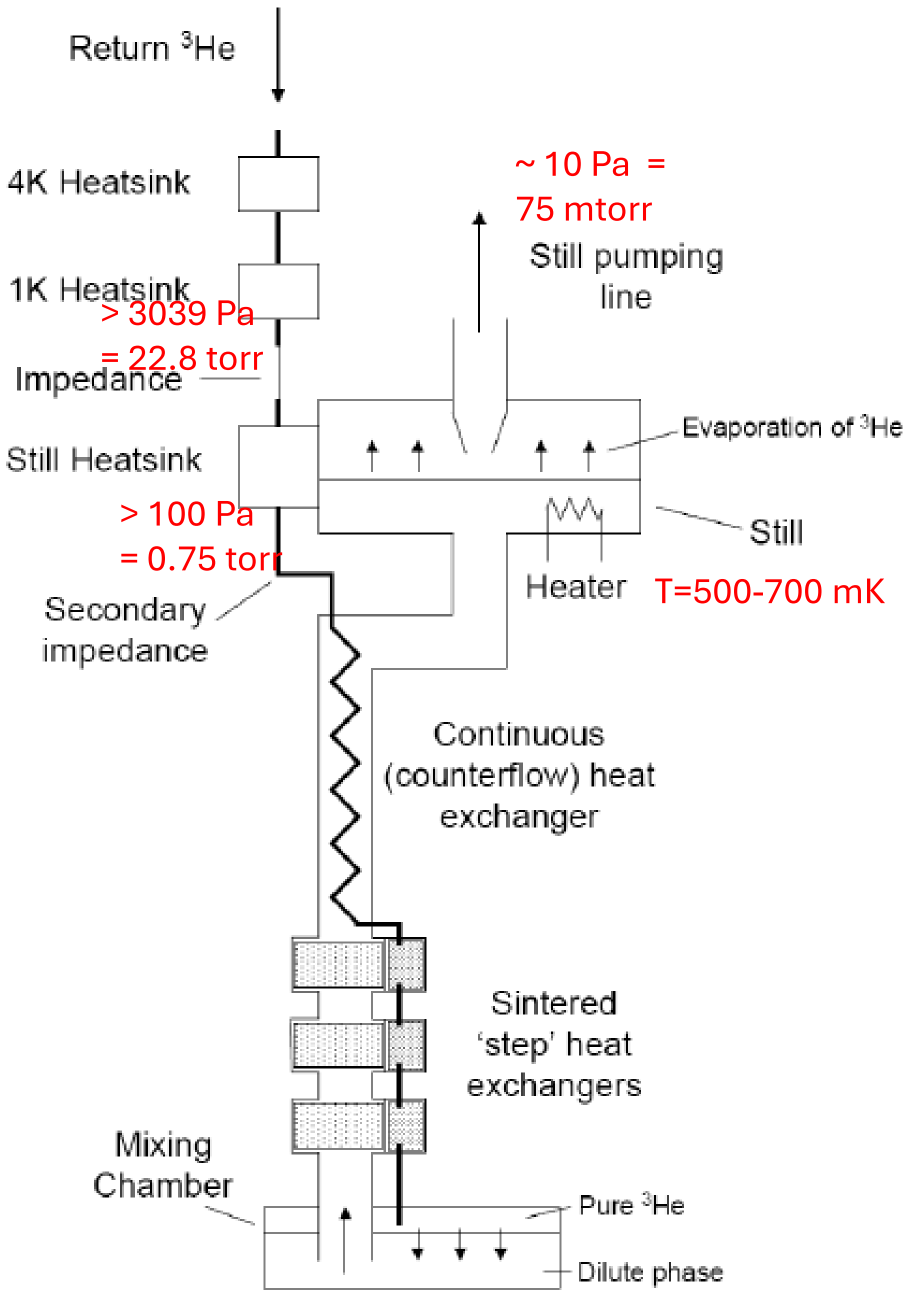




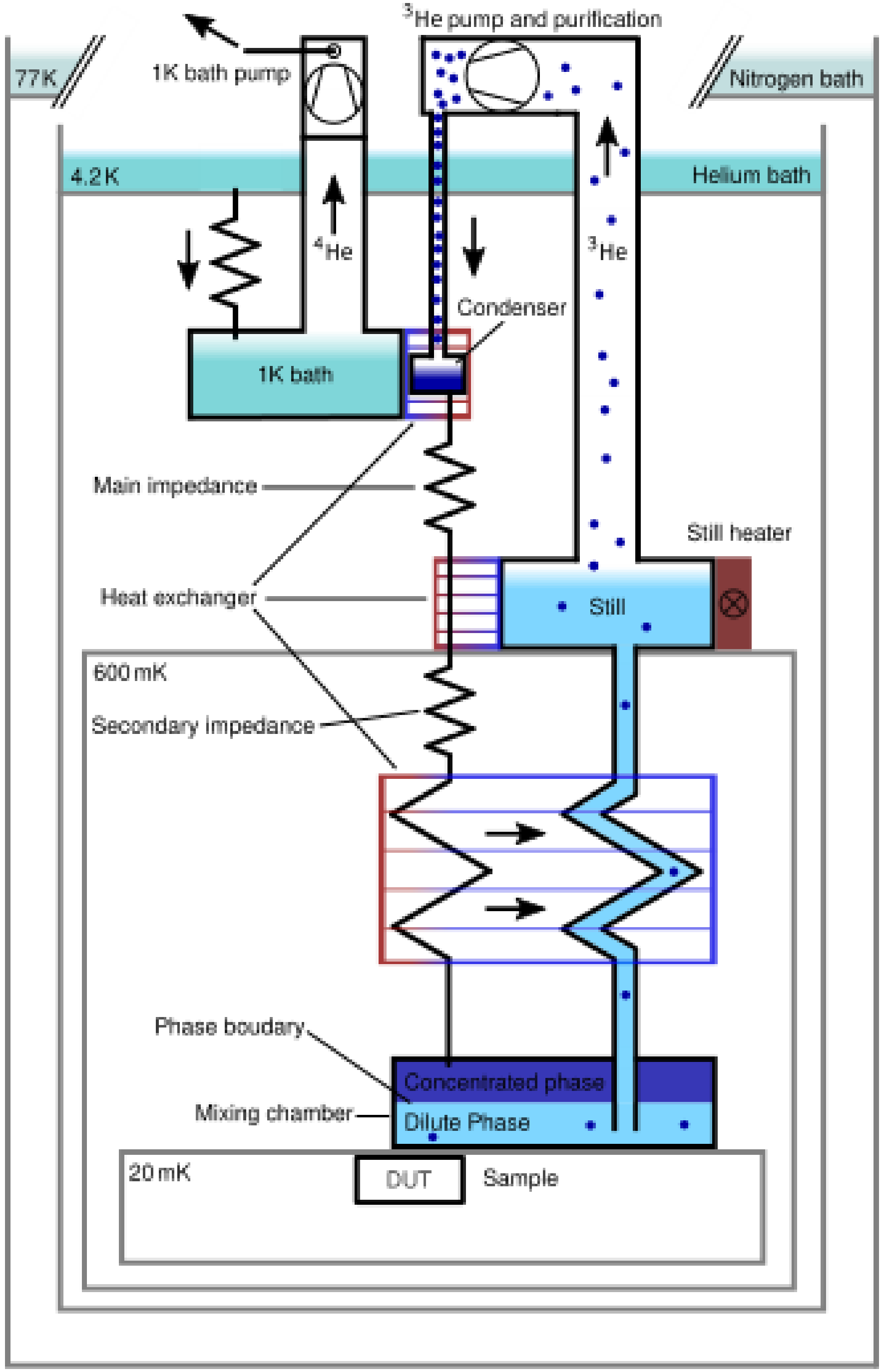
A ‘dry’ DR:
In the left corner is the **two-stage PTR** cooling the radiation shields and the incoming 3He (also via the regenerator of the PTR)

PTR: see previous lecture

DR loop



- 3He/4He mixture from the room temperature pump system is cooled (by 4K heatsink & 1K pot) and liquified at/above the still.
- Cooled further on the way to the **mixing chamber (MC).**
- 3He/4He phase separates inside the **MC.**
- 3He is pumped away from the **still.**
  - a heater is in the still to ensure enough 3He are in vapor.

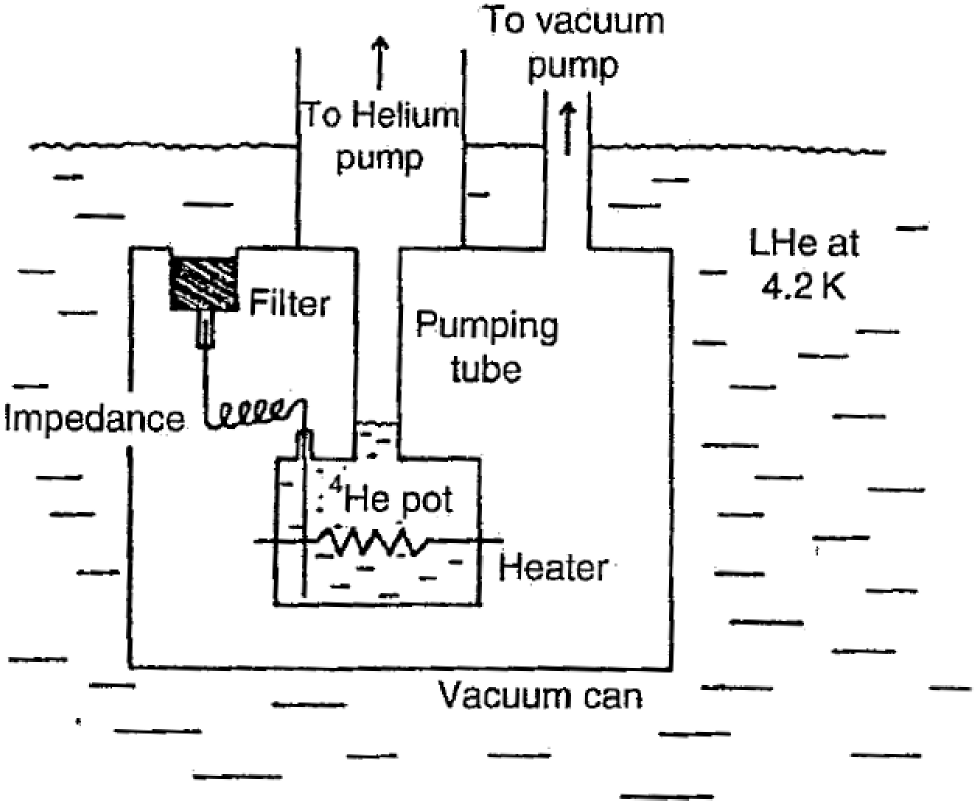


1K pot: precool incoming 3He gas

- A helium-4 bath connected to a pump. Approximately 50% of the liquid volume will be evaporated during pump down.

- 1K pot: a small pot of Helium-4 that is fed from a bath at atmospheric pressure.
  - Much lower pumping capacity required.
  - Requires a cryogenic vacuum seal for the inner vacuum can.
  - Must be aware of contamination blocking the flow impedance.

- 1K pot is used to cool incoming He3 for further condensation in the still and mixing chamber.

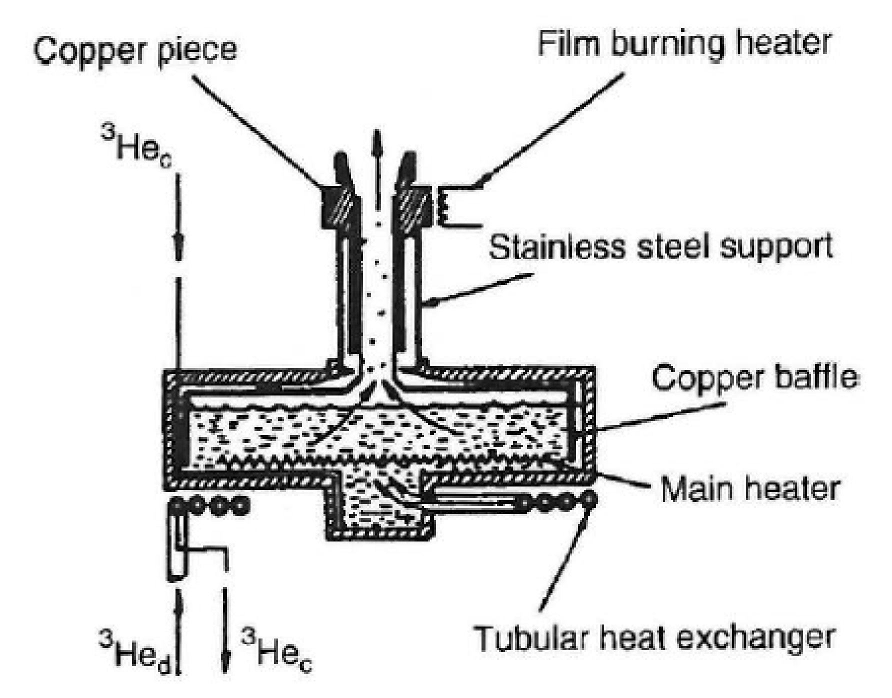


Still: remove 3He from dilute phase

- The main function of the still is to all 3He to be pumped away from dilute phase of the system.

- The optimum still temperature is ~600 – 700 mK. Pumping on 3He will tend to cool towards 400 mK (similar to a 3He fridge). A heater is used to raise the temperature to drive the helium evaporation.

- To keep the amount of 4He being pumped to a minimum, the still will typically incorporate a system to suppress the superfluid film creep from the dilute phase.

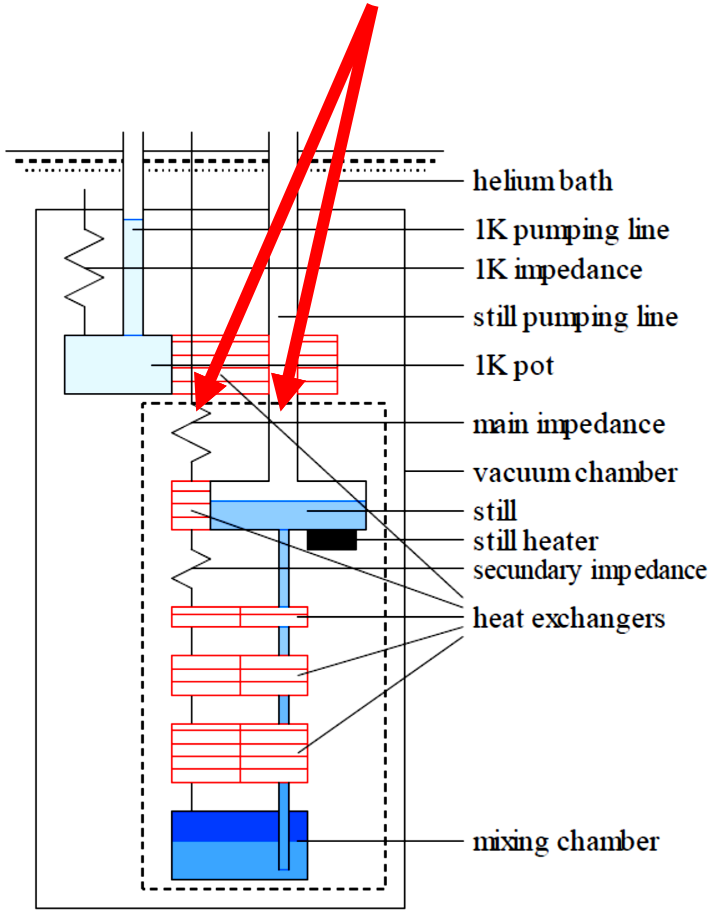


Pobell (2007)

Why is the1K pot needed?

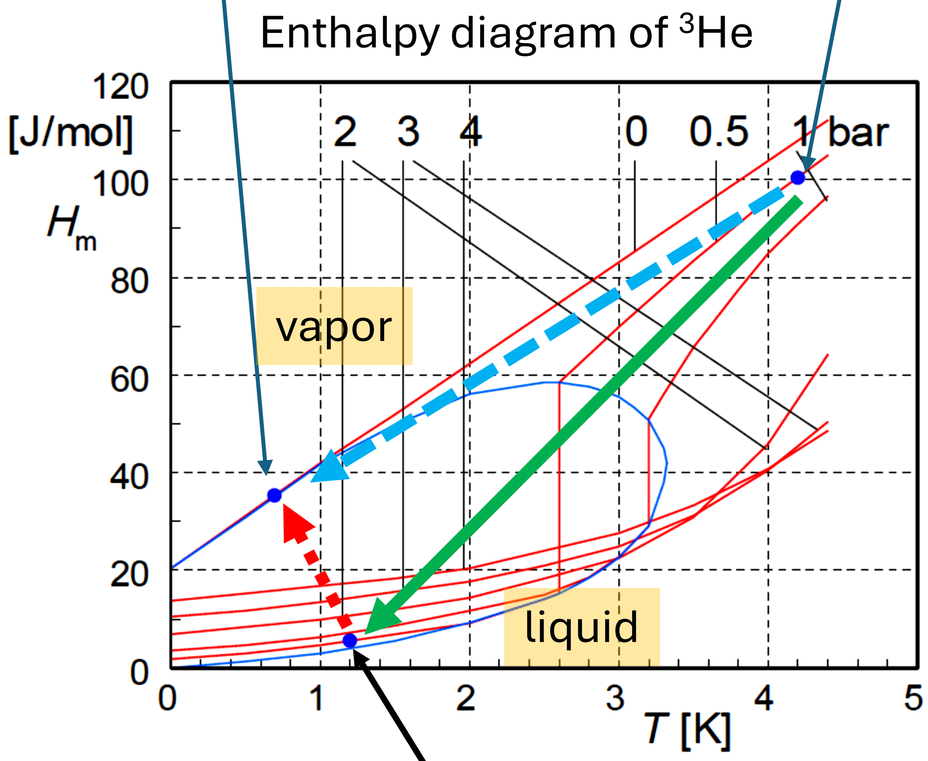
A: Precooling the incoming 3He gas; Balance cooling power available in still for a stable continuous-flow operation.

Global enthalpy balance of DR:
Every **incoming 3He atom** can be cooled by an **outgoing 3He atom in still**:



^{3}He leaves the **still at 0.7 K and P~0**. It's enthalpy is 35 J/mol.

Incoming ^{3}He at **P=0.5 bar and 4.2 K** has a enthalpy of 99 J/mol


Enthalpy diagram of ^{3}He

With a 1K pot at 1.2 K, ^{3}He can be condensed to liquid and has enthalpy of 6 J/mol!

The deficit in enhalphy between the outgoing & incoming 3He is huge: 64 J/mol !

$$\dot{Q} = \overset{*}{n}_3(35-99) = -64\frac{\mathrm{J}}{\mathrm{mol}}\overset{*}{n}_3.$$

Without 1K pot: *negative* balance, External cooling needed.

2. Using a 1K pot at 1.2 K:

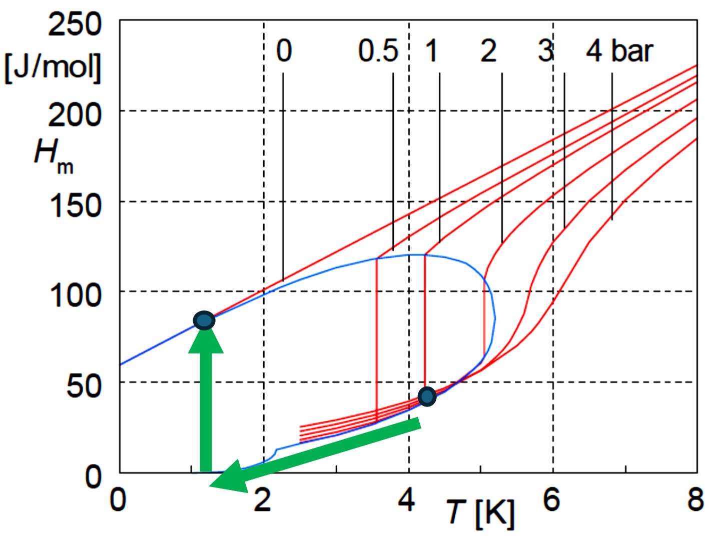
$$\dot{Q}_S = \overset{*}{n}_3(35-6) = 29\frac{\mathrm{J}}{\mathrm{mol}}\overset{*}{n}_3.$$

Incoming 3He is already liquefied and has a small enthalpy.
A *positive* balance → An external **heating** (usually at the still…) needed.

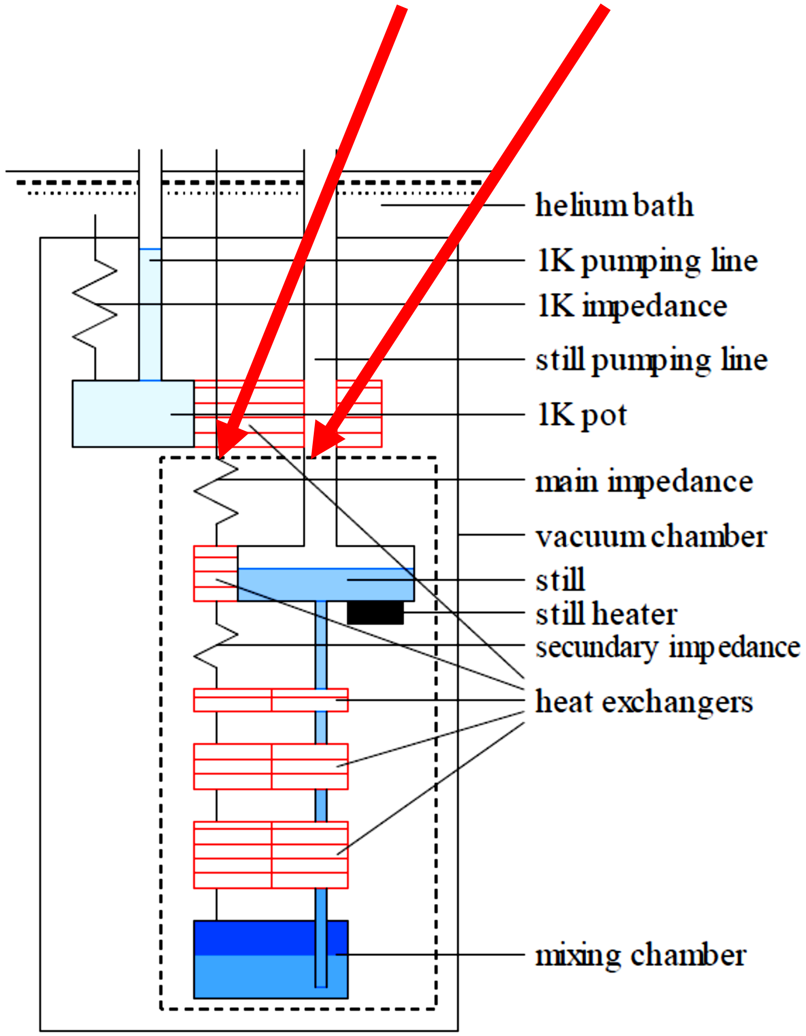
Global enthalpy balance of DR

(only pure ^{3}He flows in and out the sub-system)

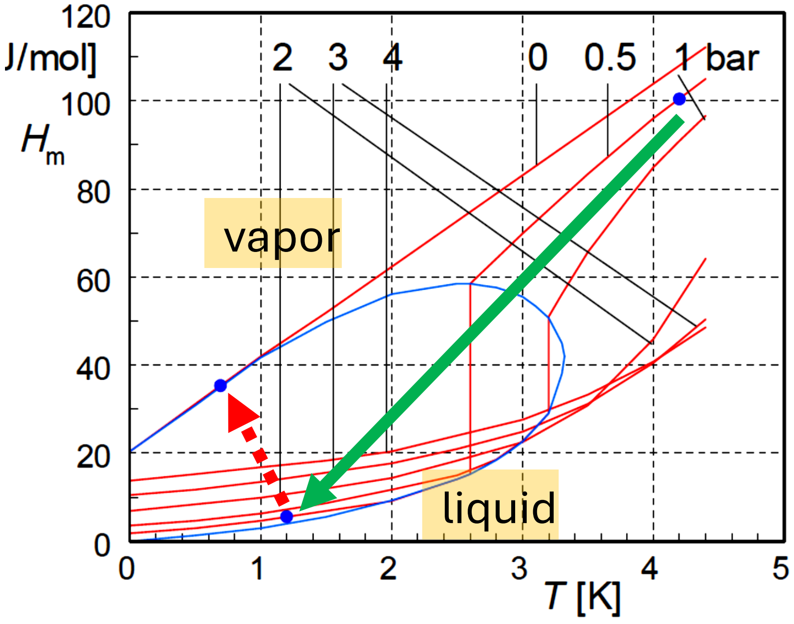
Enthalpy diagram of 4He



1K pot cooling power:
Cool liquid from 4.2K to 1.2K, then latent heat of liquid at 1.2 K



Enthalpy diagram of 3He

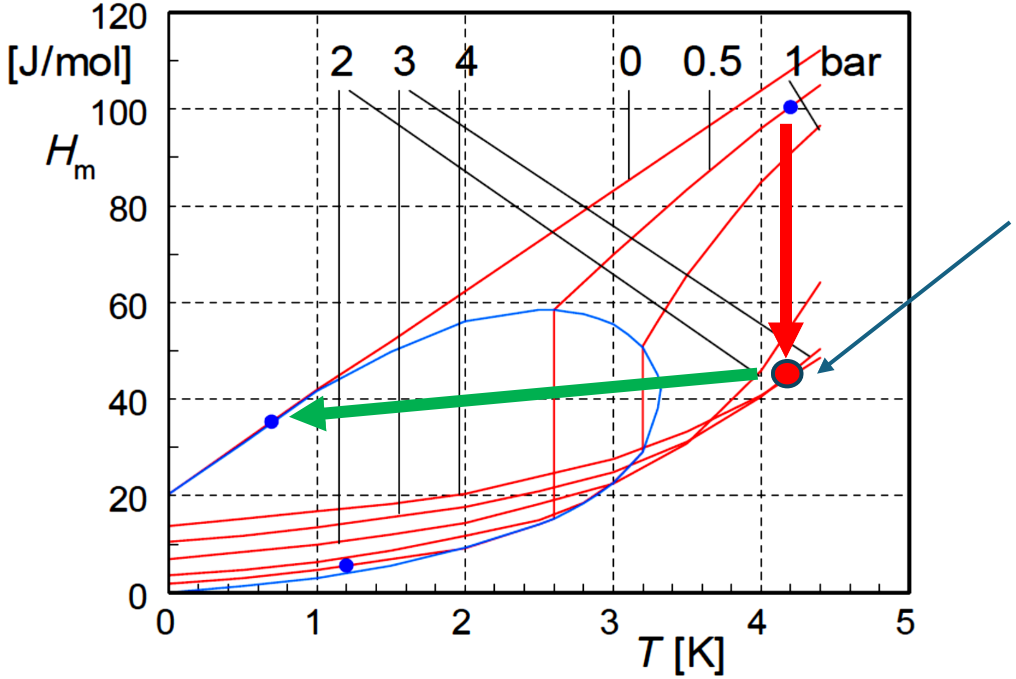


Cooling power needed to change the ^{3}He from *gas at 0.5 bar and 4.2 K* to *liquid at 0.5 bar and 1.2 K*, is

$$\dot{Q}_{1\mathrm{K}} = \overset{*}{n}_3\,(99 - 6) = 93\,\frac{\mathrm{J}}{\mathrm{mol}}\,\overset{*}{n}_3$$

Alternatively, precool the incoming ^{3}He at ~ 3 bars

Enthalphy diagram of 3He



At 4.2 K the minimum enthalpy (vs. Pressure) is $H_m \approx 42$ J/mol near **3 bars**,

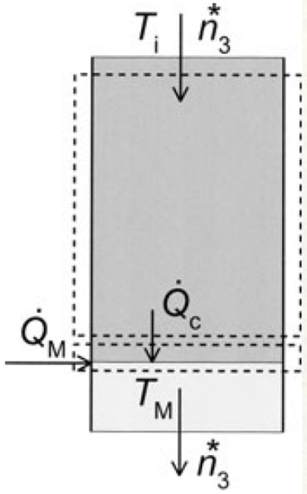
just above the enthalpy of the vapor at 0.7 K (35 J/mol).

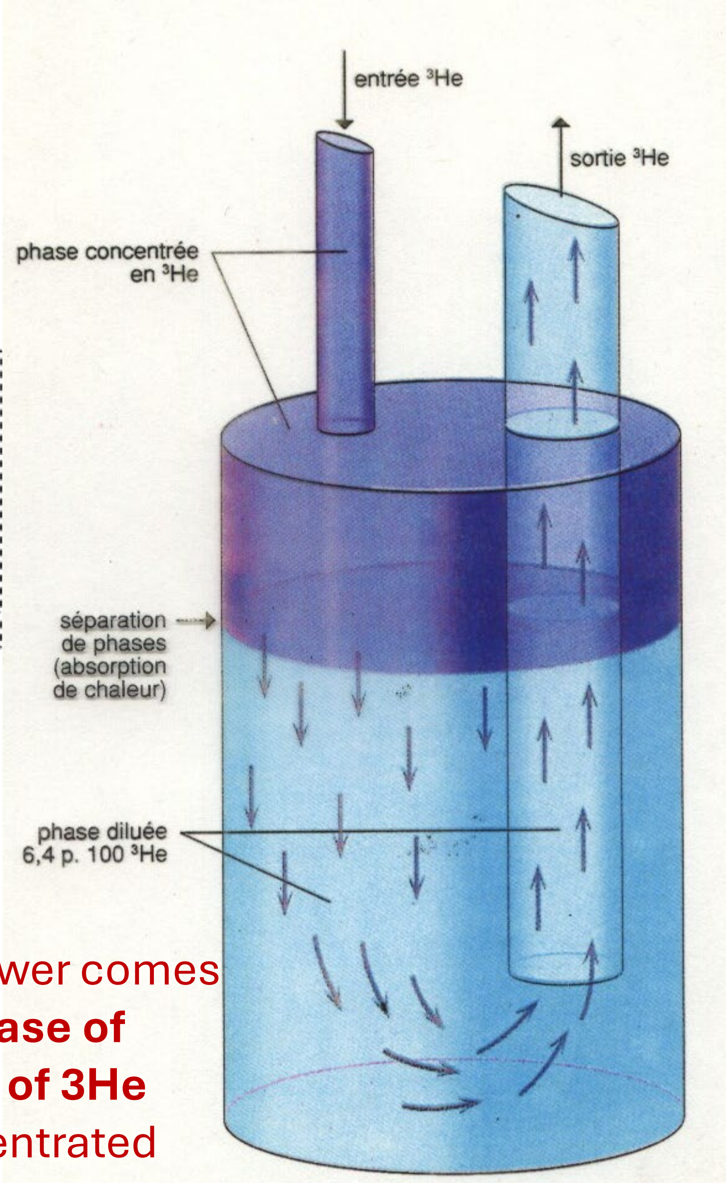
→ steady state is possible using a *heat exchanger* to precool the incoming 3He at 3 bars (gas) by outgoing 3He vapor from still at 0.7 K!!!

A compressor needed (essentially during initial condensation)

J. Kraus, Cryogenics **17**, 173 (1977)
A.T.A.M. de Waele, A.B. Reekers, H.M. Gijsman, Cryogenics **17**, 175 (1977)

The Mixing Chamber: cooling is due to *expansion*, i.e., 3He expanding from concentrated into dilute phase





The cooling power comes from **the increase of molar volume of 3He** from the concentrated phase to the dilute phase!

second law, reversible cooling power

$$\dot{Q}_\mathsf{M} = \overset{*}{n}_3 T_\mathsf{M} \left[S_3 \left(T_\mathsf{M}, x_\mathsf{s} \right) - S_\mathsf{c} \left(T_\mathsf{M} \right) \right]$$

Entropy in dilute 3He

Entropy in concentrated, pure 3He (x≅1)

since

Fermi liquid: $S_\mathsf{F} = \dfrac{\pi^2}{2} R \dfrac{T}{T_\mathsf{F}(x)}$

so

$$\dot{Q}_\mathsf{M} = \overset{*}{n}_3 \frac{\pi^2}{2} R T_\mathsf{M}^2 \left(\frac{1}{T_\mathsf{F}(x_\mathsf{s})} - \frac{1}{T_\mathsf{F}(1)} \right)$$

since

$$T_\mathsf{F}(x) \sim \left[\frac{N_\mathsf{A}}{V_\mathsf{m}(x)} \right]^{2/3}$$

$$\dot{Q}_M = \overset{*}{n}_3 \left(H_3 - H_{3c} \right)$$

we get

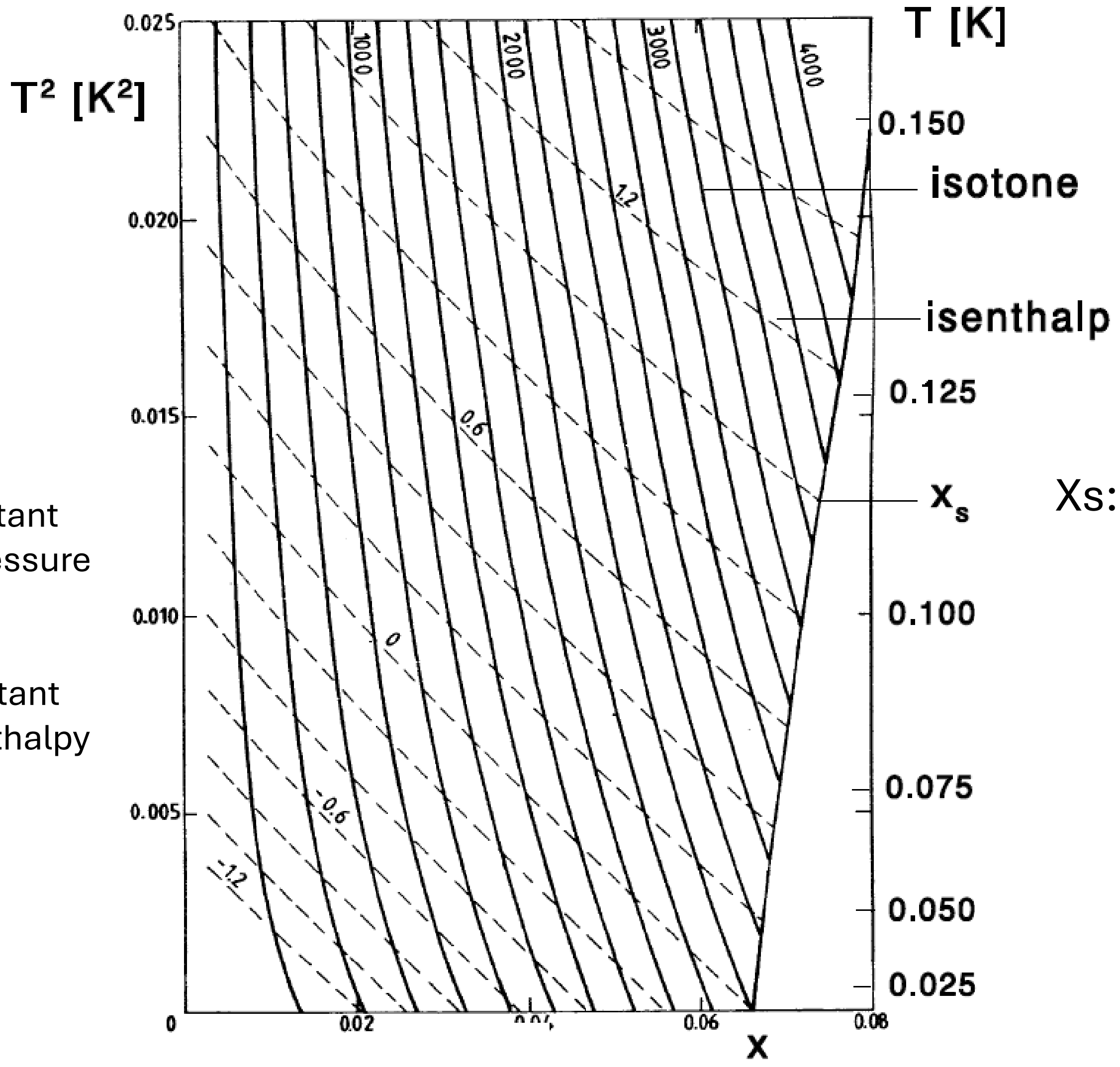
$$\dot{Q}_\mathsf{M} \sim \overset{*}{n}_3 T_\mathsf{M}^2 \left(V_\mathsf{ms}^{2/3} - V_\mathsf{mc}^{2/3} \right)$$

Molar volumes:

with $V_\mathsf{ms} = 426\ \mathsf{cm}^3/\mathsf{mol}$ and $V_\mathsf{mc} = 37\ \mathsf{cm}^3/\mathsf{mol}$

$T_F(x_s)$ = 393 mK and $T_F(1)$ = 1.8 K

Osmotic enthalpy of ^{3}He mixtures



Solid: constant osmotic pressure (Pa)

Dash: constant osmotic enthalpy H3 (J/mol)

special cases

	approximate H_3
saturated concentration $x = x_\mathrm{s}(T)$	$96T^2$
constant concentration $x = x_\mathrm{s}(0)$	$84T^2$
constant osmotic pressure $\Pi = \Pi_\mathrm{s}(0)$	$54T^2$
pure ^{3}He	$12T^2$

Xs: the saturation concentration of the dilute phase.

Cooling power at the mixing chamber (MC)

c: pure
s: saturated

The relation for the molar enthalpy of pure ^{3}He as a function of temperature is

$$H_{3\mathrm{c}} = 12T^2. \tag{28}$$

It should be noted that the numbers in the expressions (25)-(28) are only approximate.

In terms of the osmotic enthalpy the cooling power of the mixing chamber can be written as

$$\dot{Q}_{\mathrm{M}} = \overset{*}{\dot{n}}_3 \left[H_{3\mathrm{s}}\left(T_{\mathrm{M}}, x_{\mathrm{s}}\right) - H_{3\mathrm{c}}\left(T_{\mathrm{i}}\right)\right]. \tag{29}$$

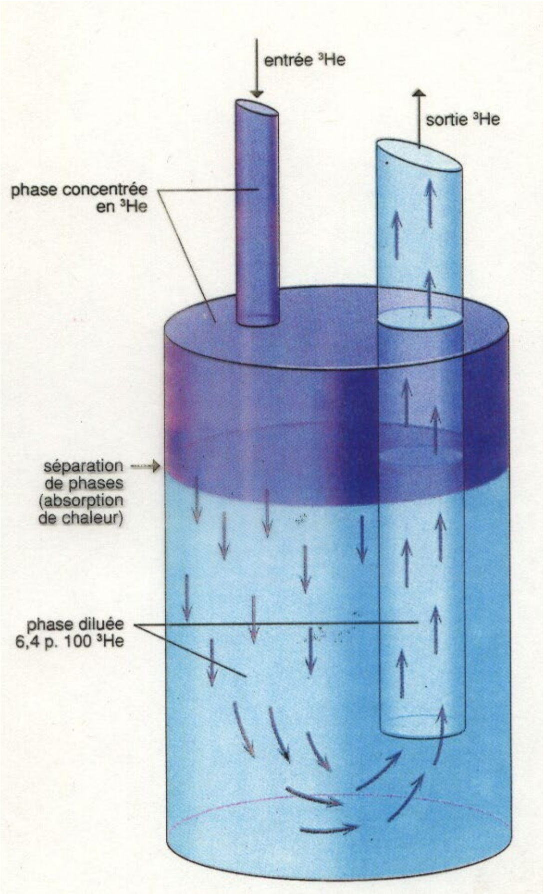
Now it is no longer necessary to limit the expression to reversible mixing. The temperature T_{i} is the temperature of the ^{3}He entering the mixing chamber in the concentrated phase. With Eqs.(26) and (28) Eq.(29) gives

$$\boxed{\dot{Q}_{\mathrm{M}} = \overset{*}{\dot{n}}_3 \left(96T_{\mathrm{M}}^2 - 12T_{\mathrm{i}}^2\right).} \tag{30}$$

If $\dot{Q}_{\mathrm{m}} = 0$. Using Eqs.(30) gives the famous relation

$$\boxed{T_{\mathrm{M}} = \frac{1}{\sqrt{8}}T_{\mathrm{i}} \approx \frac{T_{\mathrm{i}}}{2.8}} \tag{31}$$

which states that there is a fixed ratio between the mixing chamber temperature and the temperature of the incoming fluid. This property shows the importance of the precooling of the incoming ^{3}He. In this case $T_{\mathrm{M}} \neq T_{\mathrm{i}}$ so the mixing process is not reversible and Eq.(8) does not hold.



Suggested reading:
https://courses.physics.illinois.edu/phys523/fa2025/CryogenicsFiles/MinifridgeJaakkoKoivuniemi1998.pdf

Homework - due 12/2

In the dipstick DR, we will condense about 2 liters of STP 3He gas into liquid. The pump will allow a circulation rate of 35 micro-mole/sec.

Estimate the following:

1. The amount of 3He in moles.
2. The volume of liquid 3He after being condensed in the mixing chamber.
3. The cooling power of the mixing chamber at 100 mK. The cooling power depends on the incoming temperature of 3He entering the mixing chamber. Estimate the cooling power of the MC at 100 mK for the following incoming 3He temperature:

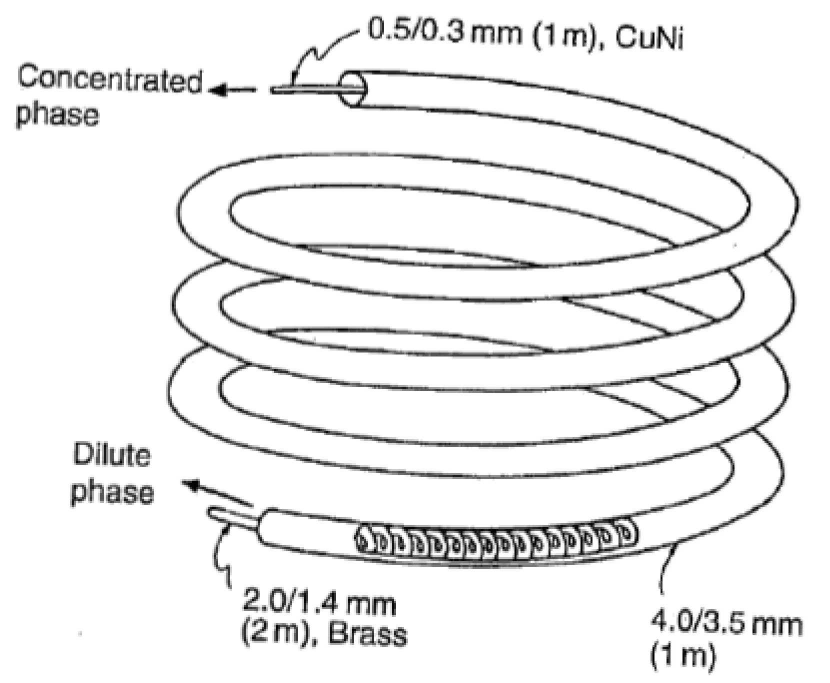
Ti	500 mK	400 mK	300 mK	200 mK
Cooling power (micro-W)				

4. How cold does the incoming 3He has to be for the MC to reach 100 mK?

Supplemental slides - reference

Heat exchanger design above 50 mK

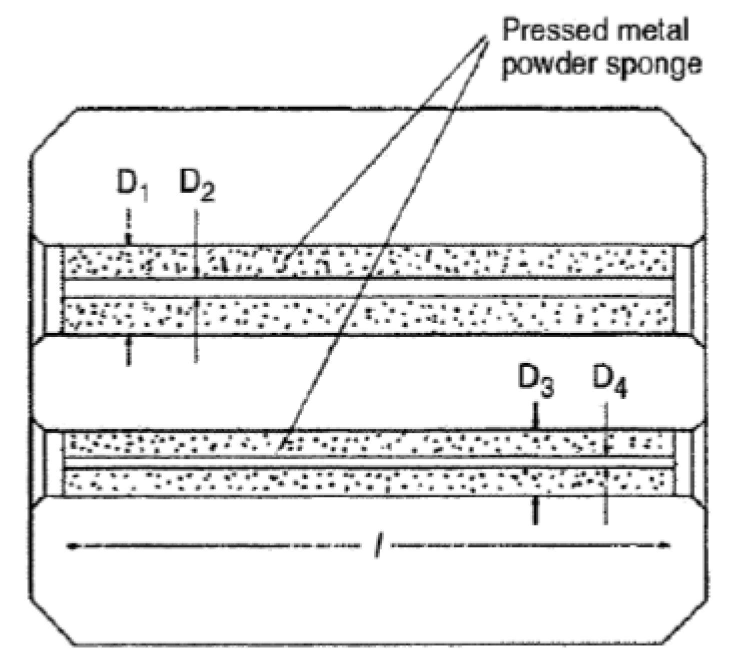
- At higher temperatures, continuous heat exchangers work well. These are typically of the concentric (or coil-in-tube) configuration.
- Heat is transferred from the dilute phase in the annular space and the concentrated phase in the coiled capillary.
- Heat transfer along the fluid and the capillary are negligible.



Pobell (2007)

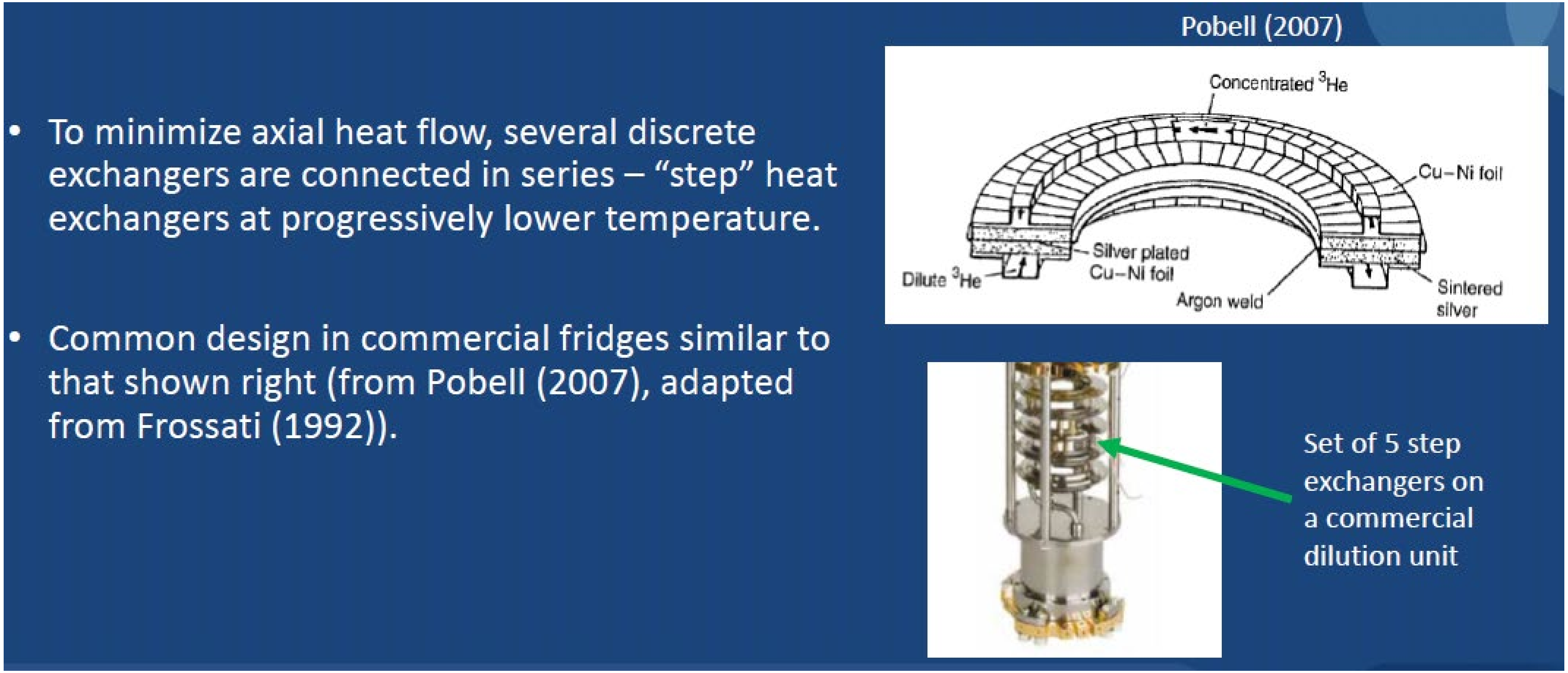
Heat exchanger design below 50 mK

- Designs using continuous heat exchangers only can reach ~30 mK.
- For lower temperatures or very high flow rates, larger surface area heat exchangers are needed to overcome the increasing Kaptiza resistance.
- Simple form is a sintered metal block to give **large surface area**, with a machined channel to **reduce flow impedance** (and hence viscous heating, see later).



General discrete heat exchanger layout (from Pobell, 2007)

Heat exchanger design below 50 mK

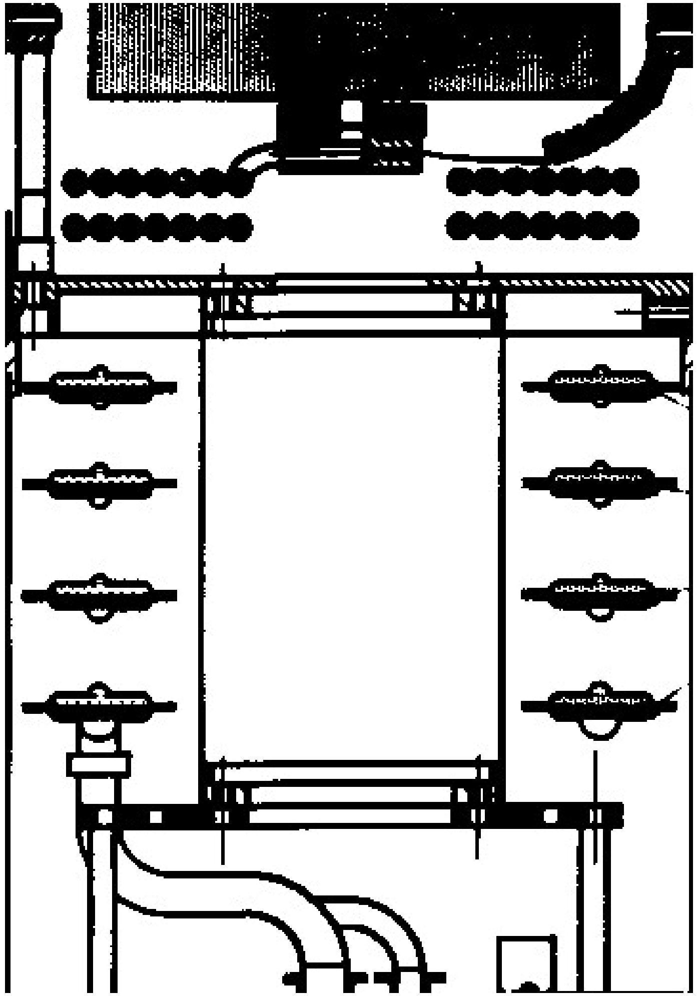


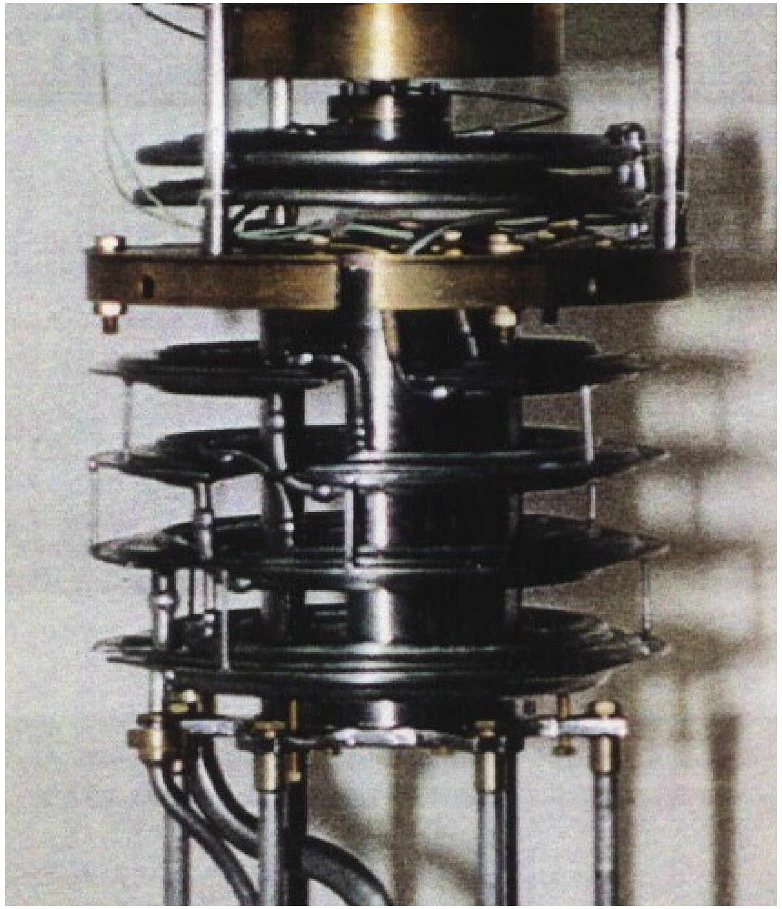
Pobell (2007)

- To minimize axial heat flow, several discrete exchangers are connected in series – “step” heat exchangers at progressively lower temperature.
- Common design in commercial fridges similar to that shown right (from Pobell (2007), adapted from Frossati (1992)).

Set of 5 step exchangers on a commercial dilution unit

Heat exchangers & cold plate





Sintered silver heat exchangers

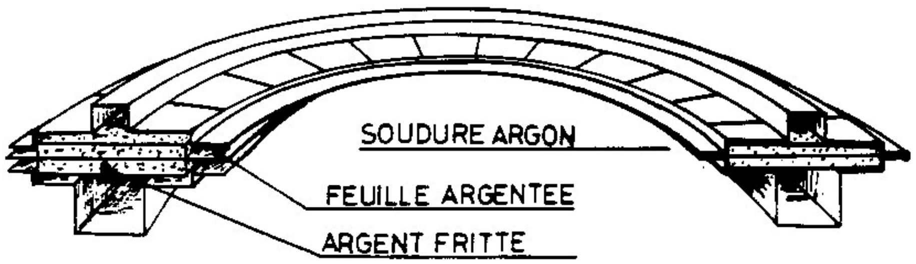


Fig. 20a
Coupe d'un échangeur continu utilisant de la poudre d'argent frittée

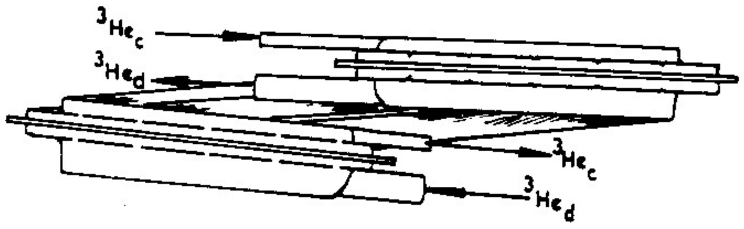
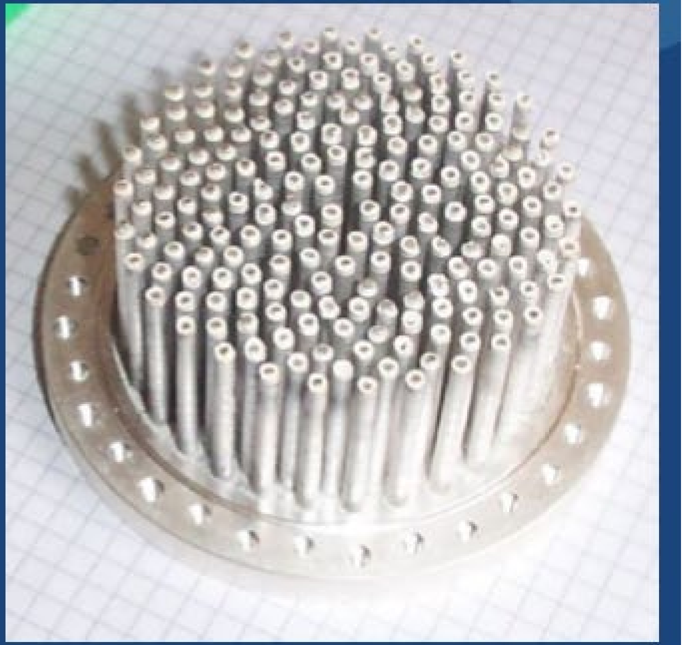


Fig. 20b
La forme spirale facilite la connection avec les autres échangeurs et empêche la formation de points bas côté concentré, où la phase diluée pourrait s'accumuler

Mixing chamber

- The mixing chamber is a relatively simple. Major requirement is for a large contact area between the helium phases and the metal.
- This is typically achieved through sintered structures, such as the silver sponge example shown right.
- In this design, the diameter of the mixing chamber is approximately 100 mm and each sponge has a surface area of 10 m^2.



Mixing chamber construction example from Uhlig (2015)

“Dry” vs “Wet” Refrigerators

Cooled by Pulse Tube to ~50 K

Cooled by Pulse Tube to ~4 K

Cooled and partially liquified by JT

Cooled and liquified by Still

Mixing Chamber

“Dry” Process

Cooled by Liquid Helium to ~4 K

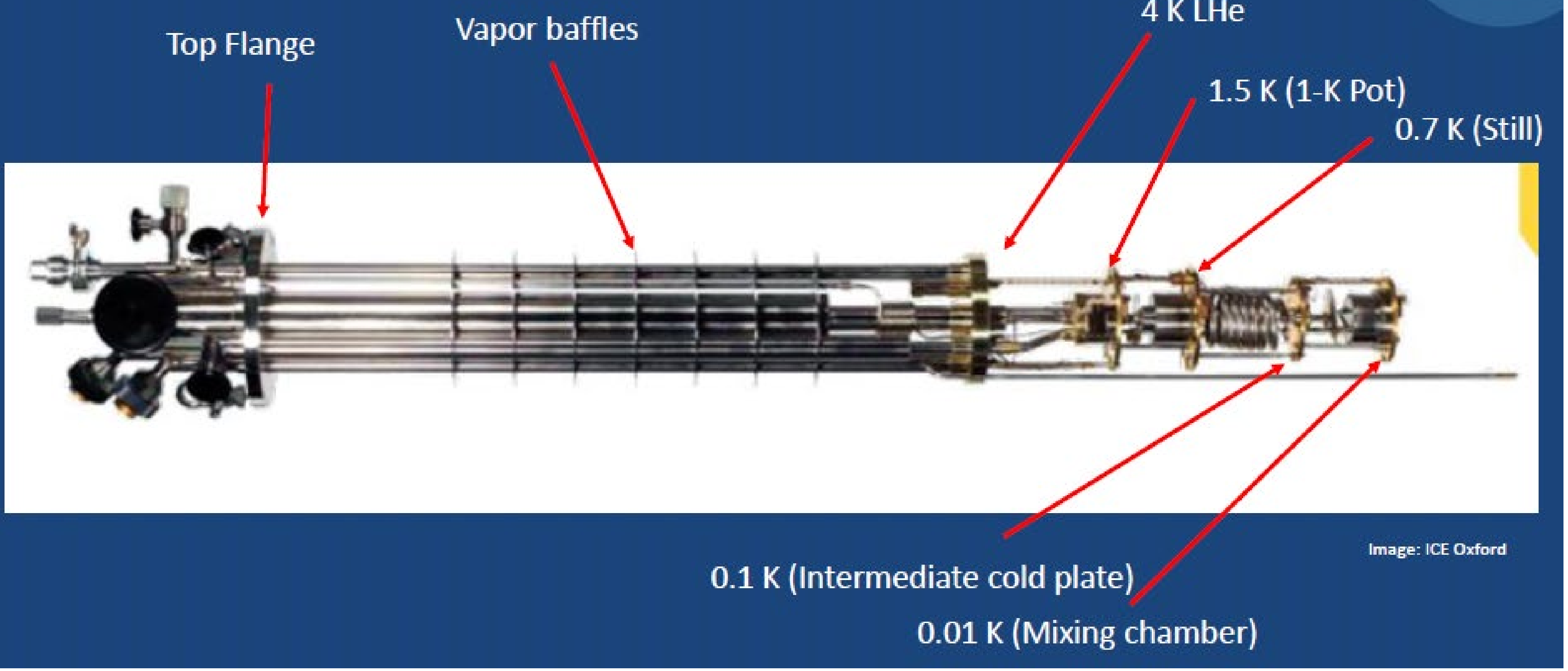
Cooled and liquified by 1-K Pot

Cooled by Still

Mixing Chamber

“Wet” Process

"Wet" Layout



Top Flange

Vapor baffles

4 K LHe

1.5 K (1-K Pot)

0.7 K (Still)

0.1 K (Intermediate cold plate)

0.01 K (Mixing chamber)

Image: ICE Oxford

“Cryogen Free” Refrigerator Layout

2 stage pulse tube cooler

Additional heat exchanger

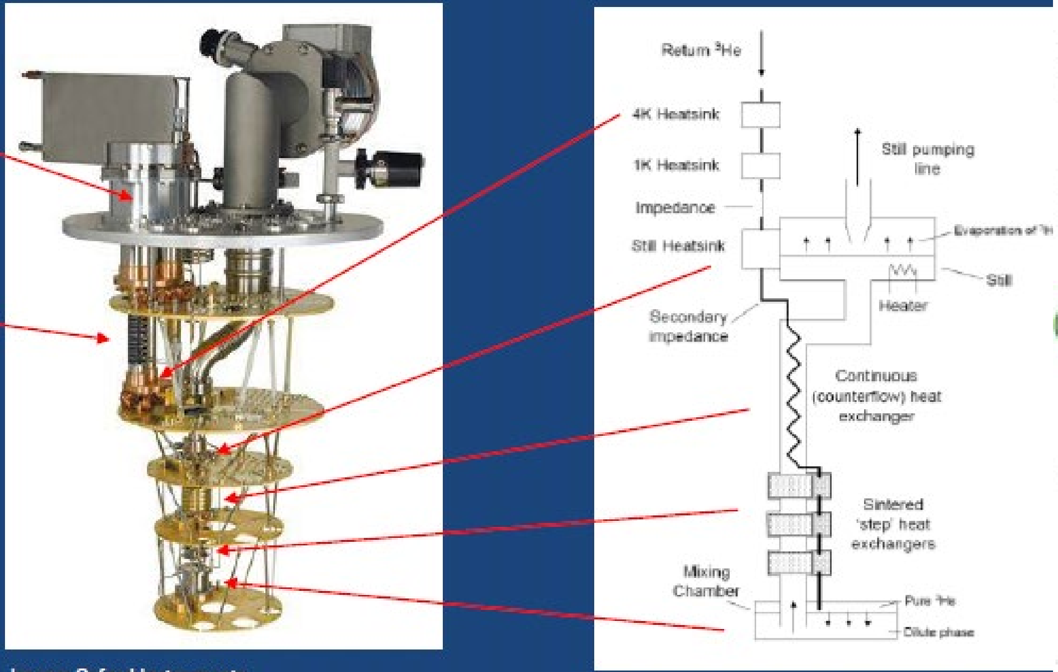
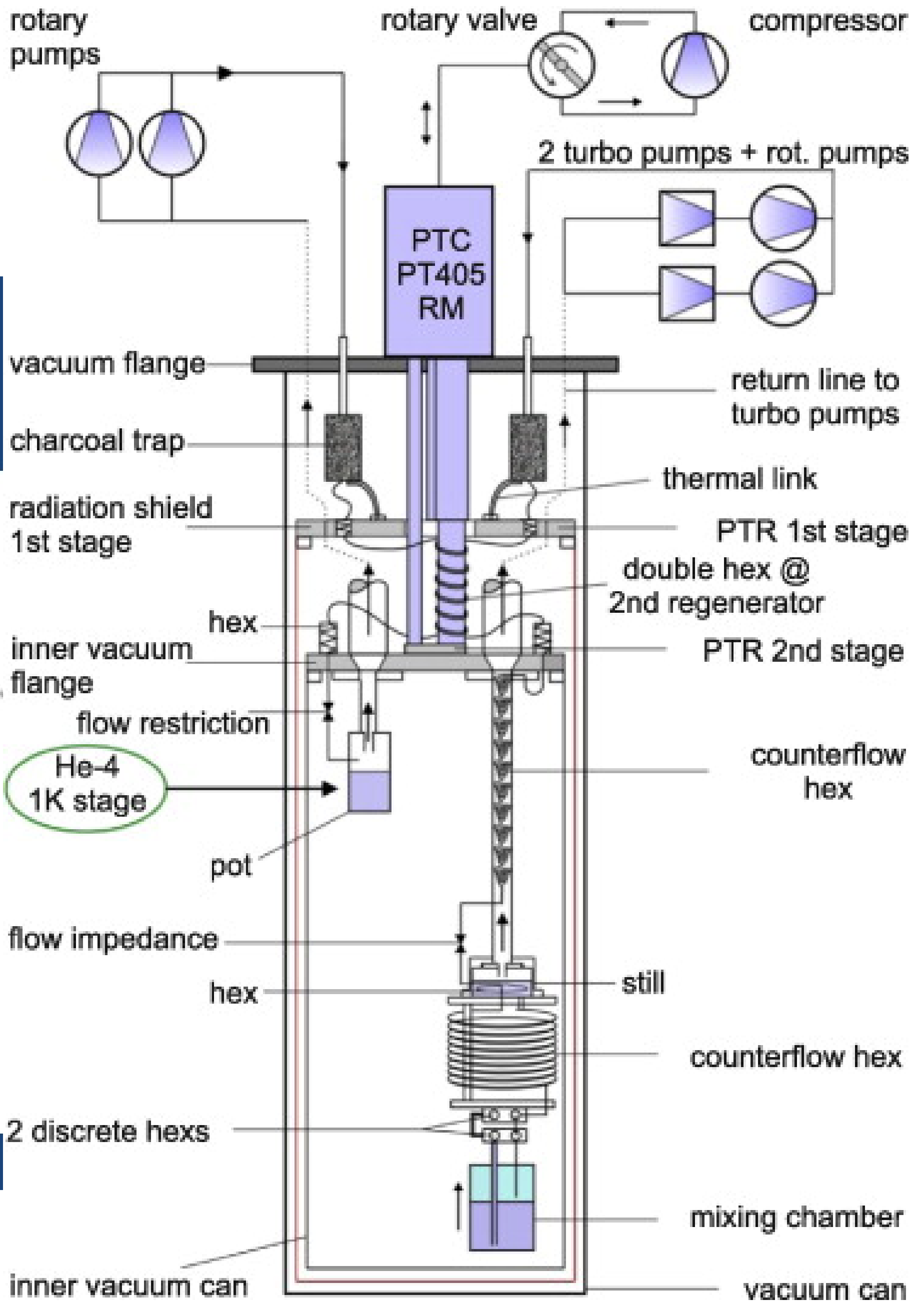


Image: Oxford Instruments

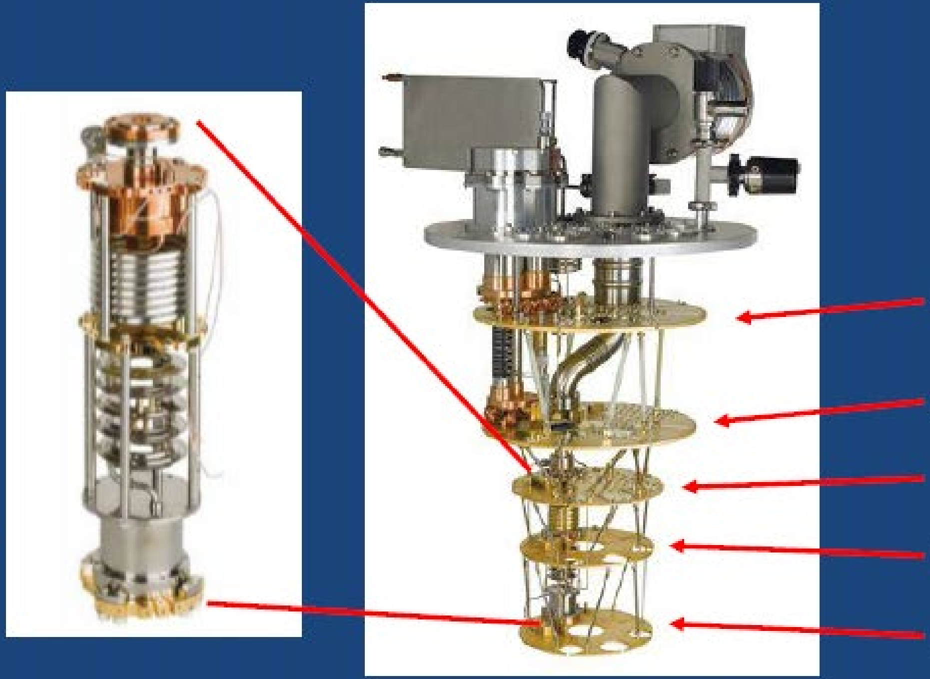


Cryogen Free Layout – Temperature Stages

Still

ICP

Mixing Chamber



50 K (First stage pulse tube)

4 K (Second stage pulse tube)

1 K (Still)

0.1 K (Intermediate cold plate)

0.01 K (Mixing chamber)

Images: Oxford Instruments

Improvements to the precooler design

- An interesting property of the pulse tube cooler was measured in the early 2000s in that excess cooling capacity on the regenerator between the two stages exists.
- Some heat can be dissipated on the regenerator with little impact on the 4 K cooling performance.
- Adding a coil to the regenerator provides a useful additional heat sink for the inflowing 3He, reducing the cooling requirements at the 4-K stage.

Additional heat exchanger (Uhlig, personal communication)

Well, you decided to have a DR…
(or you had no choice!)

- There are many kinds of dilution refrigerators:

- Classical, Helium (and nitrogen?) bath: Ultralow temperatures, measurements with superconducting coils, special samples…). Several manufacturers: Air Liquide, Cryoconcept, Leiden Cryogenics, Oxford Instruments, etc.

- With partial cryogenic bath and shields (DRILL, Sionludi) (neutrons, X-rays, etc…) “Table cryostat” Air Liquide, Oxford Instruments,…

- Pulse-tube Dilution refrigerators: Air Liquide, Leiden Cryogenics, BlueFors Cryogenics, Oxford Instruments, Cryoconcept, Vericold,…

- "top loading" : fast measurements with small samples, Oxford Instruments,…

- Inserts (Neumeier, Garching), ILL

- Horizontal (Niinikovskii, CERN)

- ^{4}He circulation (de Waele, Eindhoven; Schumacher, Grenoble)

Some choices...

Precooling stage:

- With 1K pot?
- Joule-Thompson stage?

Pumping: a rotary pump and...?

- diffusion pump?
  - Roots?
  - Turbo-molecular pump?
  - Adsorption pump?

Design considerations

Obtaining low temperatures and high cooling power (dn/dt >100 µmoles/sec):

1) optimisation by a careful calculation:
    - heat exchangers (sintered silver powder step exchangers).
    - conduction and viscous heating in the low temperature parts.
    - pressure drop in the pumping lines.
    - condensation of the mixture

2) Optimisation of the mechanical parts:
    - vibrations
    - viscous heating

3) Empirical optimisation (know-how!):
    - circulation of 4He
    - interaction of the dilute 3He on the excitations of 4He

4) Optimisation of the reliability:
    - simple design
    - modularity
    - foresee leak testing by elements

5) Training the operators!

Where do I put my DR now…???

- Check-list:
  - Space available: area, height…
  - Door width (can the DR Dewar enter the room??)
  - Do I need to make holes in the floor or ceiling?
  - Dewar, pumping lines…
  - Room for the helium and nitrogen storage dewars?
  - Room for the pumps?
  - Electrical power? Water? Compressed air?
  - Helium recovery lines?
  - Anti-vibration system?
  - Shielded room?

- Adapt the design to the needs.
- Adapt the design to the constraints

DTA : Dilution Cryostats MINIDIL OD70 & OD45

Standard MINIDIL and detail of a lower part

DTA : Dilution Cryostats (Table Dilution)

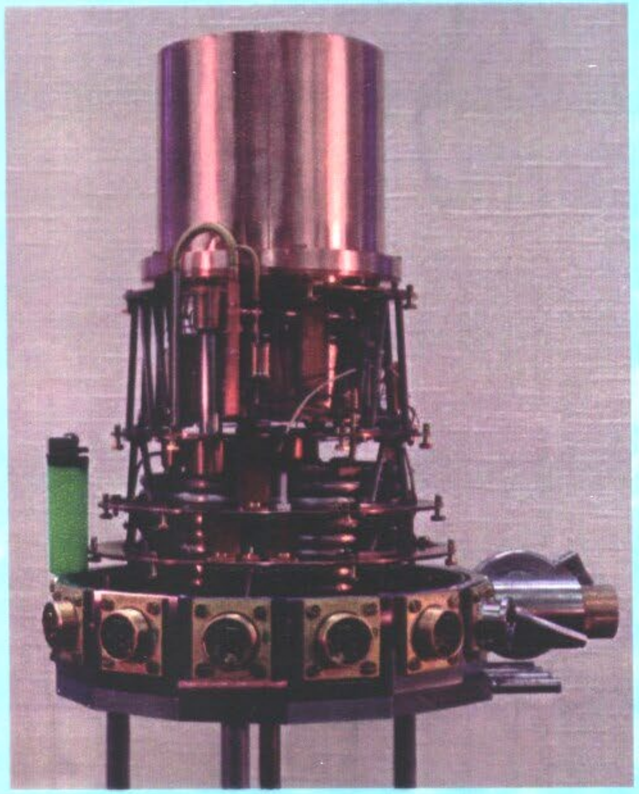
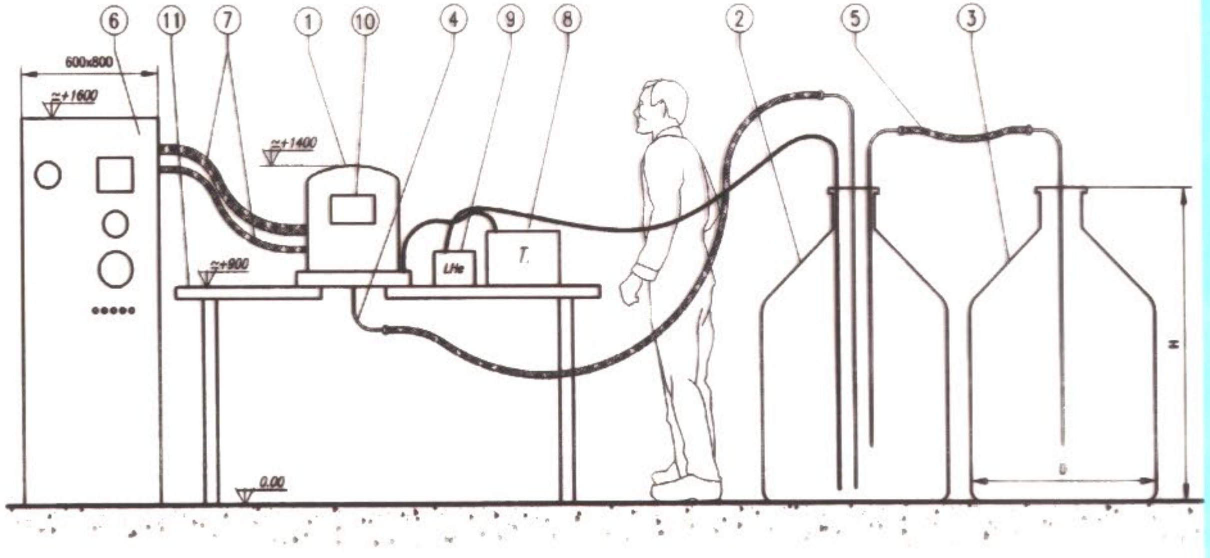


TABLE DILUTION LAYOUT



HELIUM RESERVOIR			
TYPE	H	D	VOLUME LHe
RH100	1300	520	100 L
RH250	1550	820	250 L

POSITION N°2:
NO HOLE IN THE GROUND

1- TABLE DILUTION OD ≃ 300mm
2- LHe RESERVOIR FOR ALIMENTATION OF THE 4K STAGE OF THE TABLE DILUTION
3- REFILLING RESERVOIR OF 1
4- ALIMENTATION LINE OF THE 4K STAGE (FIXED AND MOVABLE PARTS)
5- FLEXIBLE TRANSFER LINE FOR REFILLING
6- GAS HANDLING SYSTEM
7- PUMPING AND INJECTION LINES
8- TEMPERATURE REGULATOR CONTROLER (TRMC 2)
9- LEVEL METER CABINET (LHe) AND ITS SENSOR
10- EXPERIMENTAL ACCESS (OD 130mm HEIGH 130mm)
11- TABLE

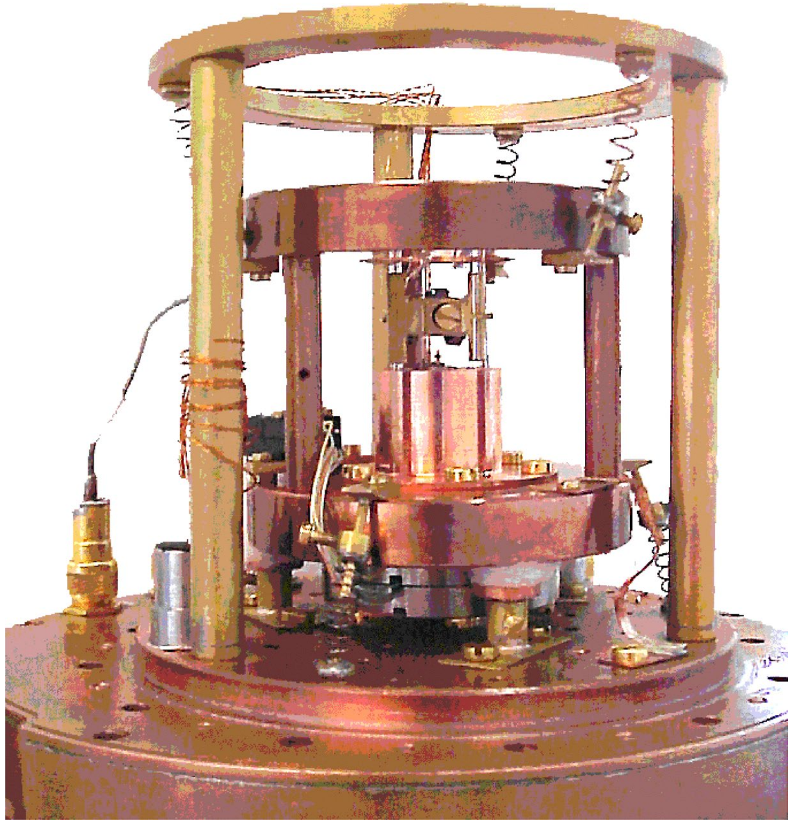
TABLE DILUTION & Lay OUT of this NEW system

AIR LIQUIDE

Table-top Dilution Refrigerator 50 mK

CNRS – Air Liquide

A. Benoit



STM, AFM, optical access...

CryoConcept

HoR
200

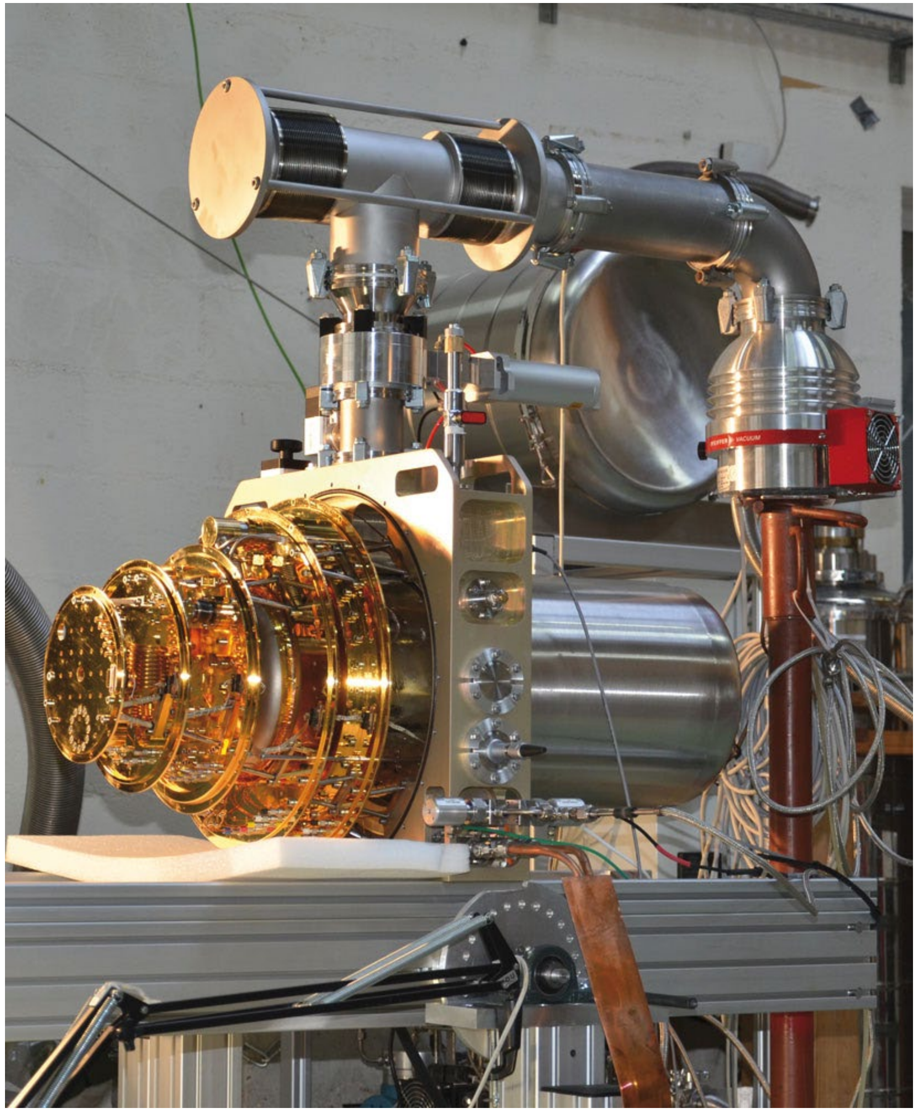
Base temperature < 30mK

Stability @ 30mK 0,1 mK

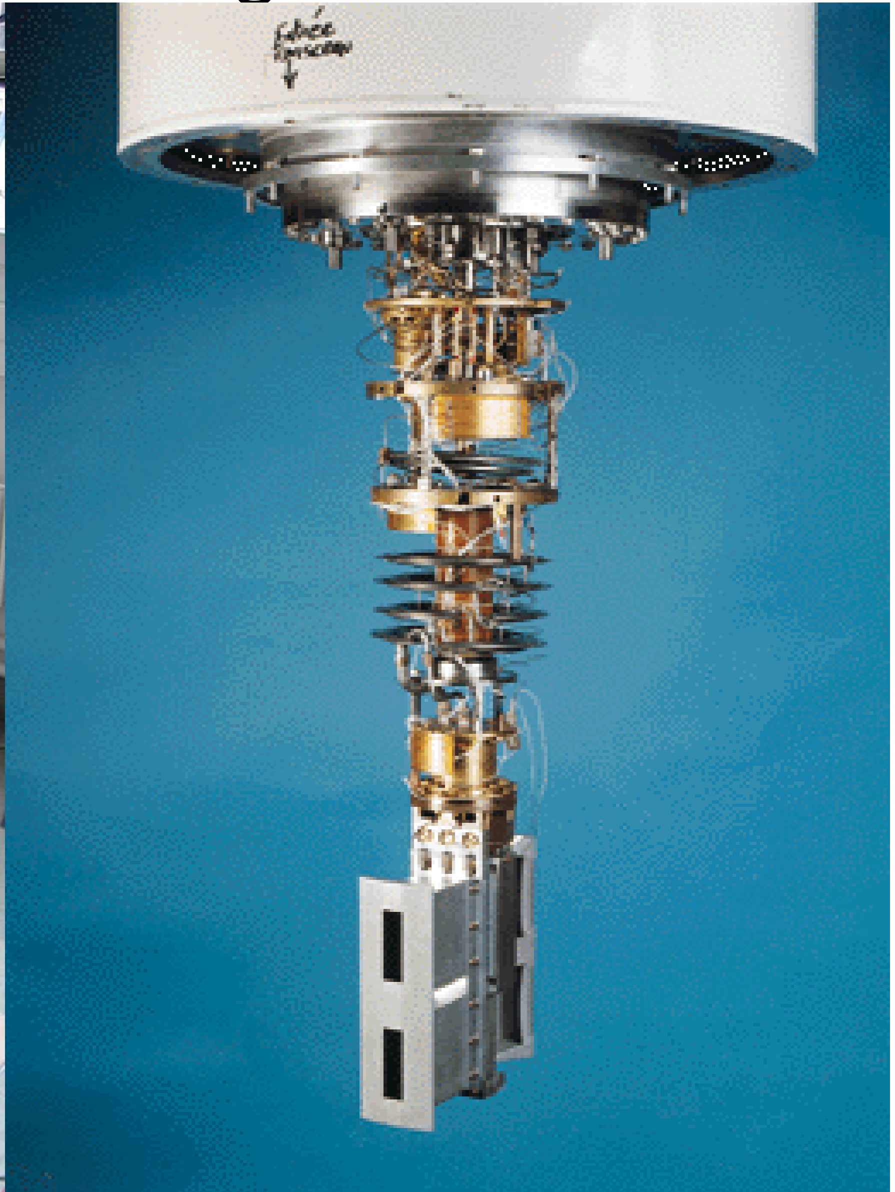
250µw @ 100mK

Less than 16 hours for 300K-30mK

Liquid helium hold time 48 hours
(up to 10 days with automatic refill)



DRILL – 5 mK DR for neutron scattering

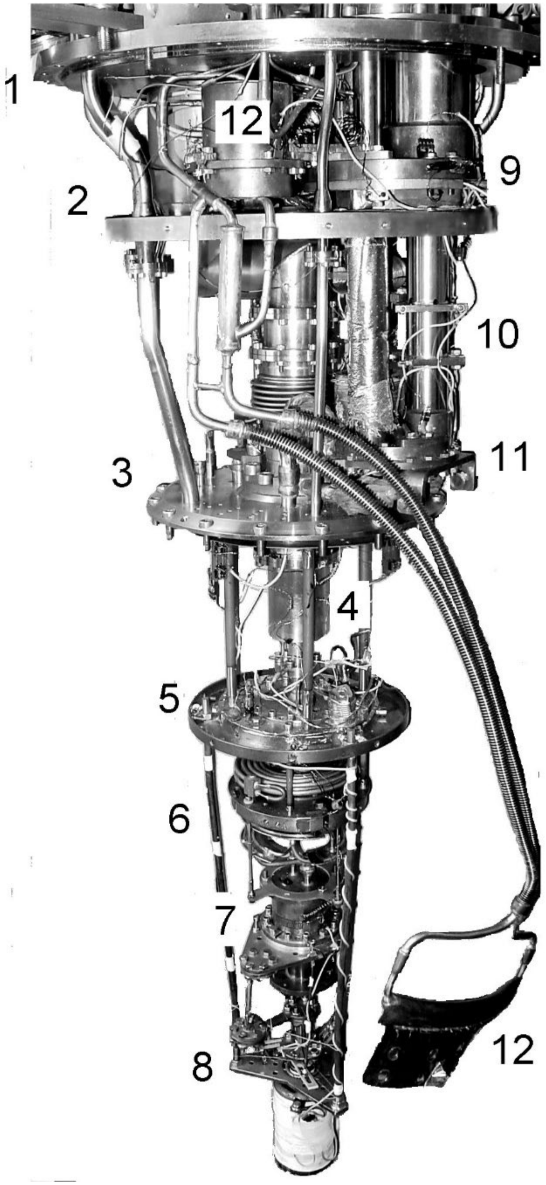


DN1: Dilution refrigerator for ultra-low temperatures

PT-DR1 (1999) and PT-DR2

(Commercial unit delivered 2003)

H. Godfrin, S. Triqueneaux (CNRS & Air Liquide)



Th. Prouvé, S. Triqueneaux, H. Godfrin

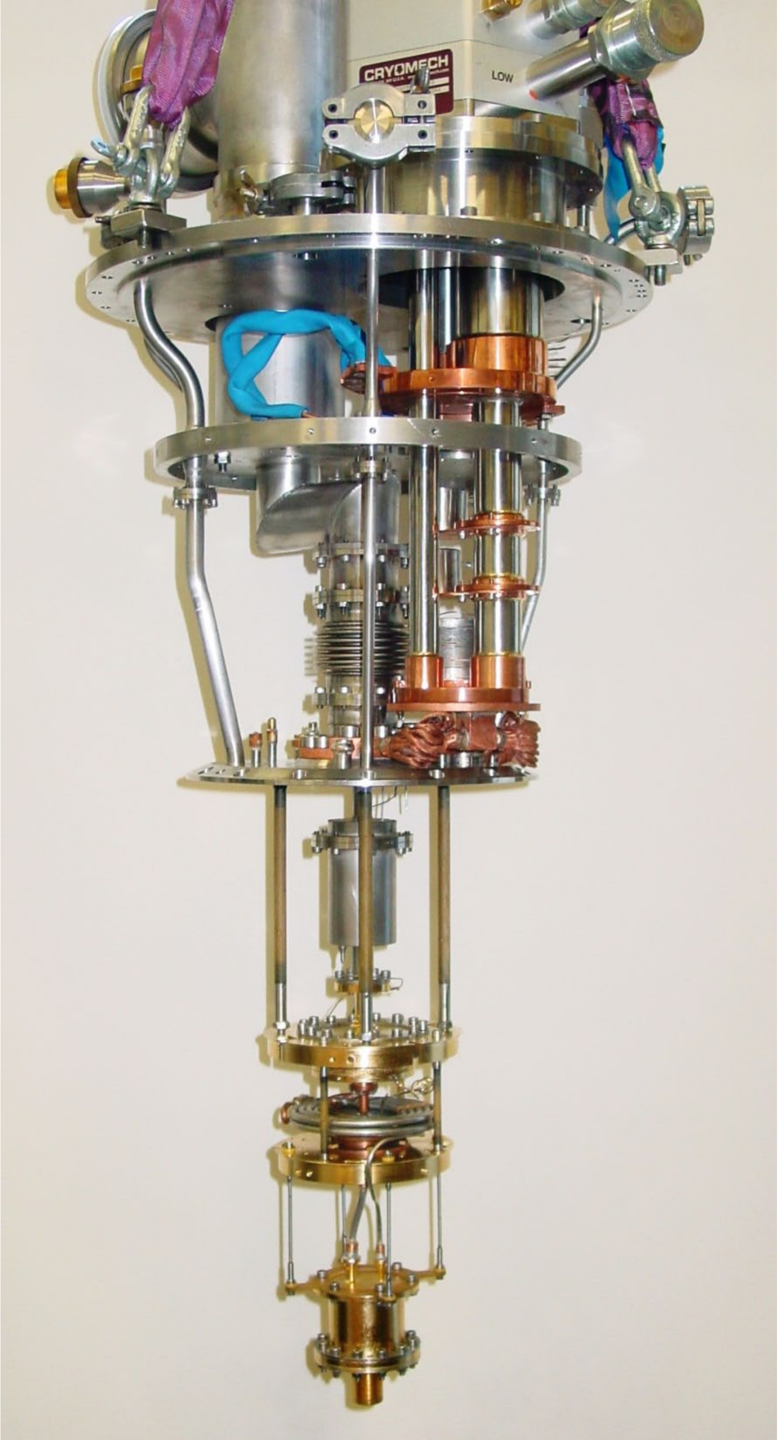
Pulse-Tube dilution refrigerator PT-DR2

(industrial collaboration with l'Air Liquide)

Commercial unit

PT-DR3

CNRS - Air Liquide



CryoConcept

HD 400

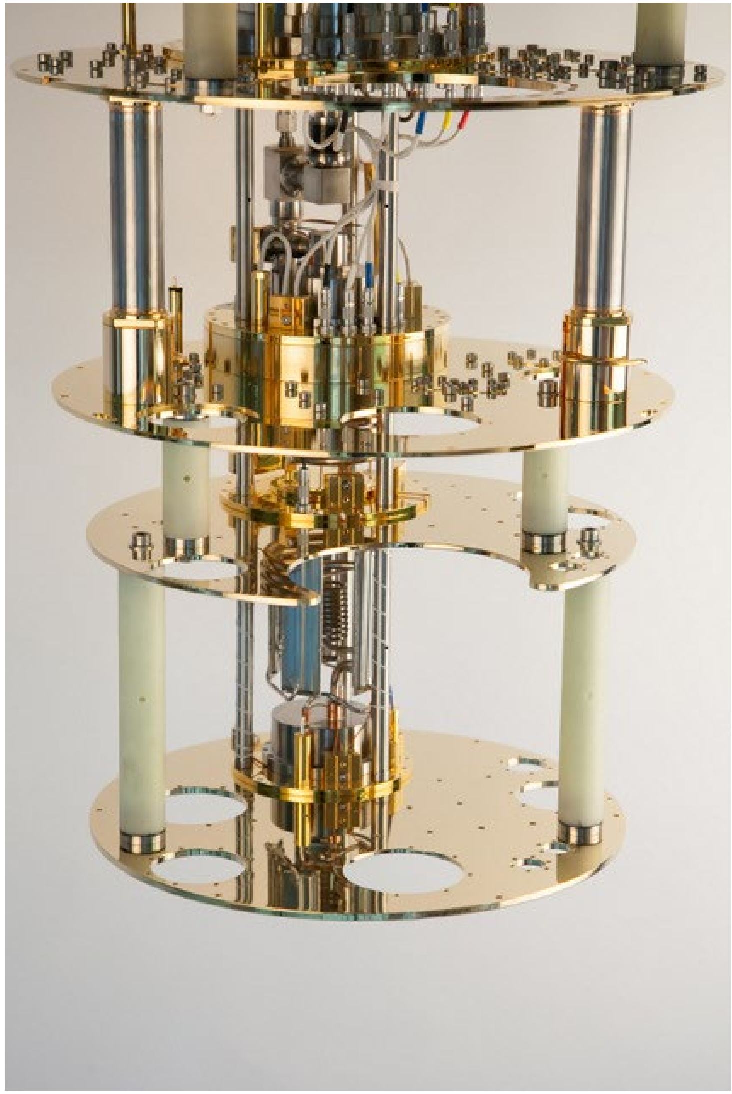
Base temperature < 10mK

400µw @ 100mK

Stability @ 20mK 0,1 mK

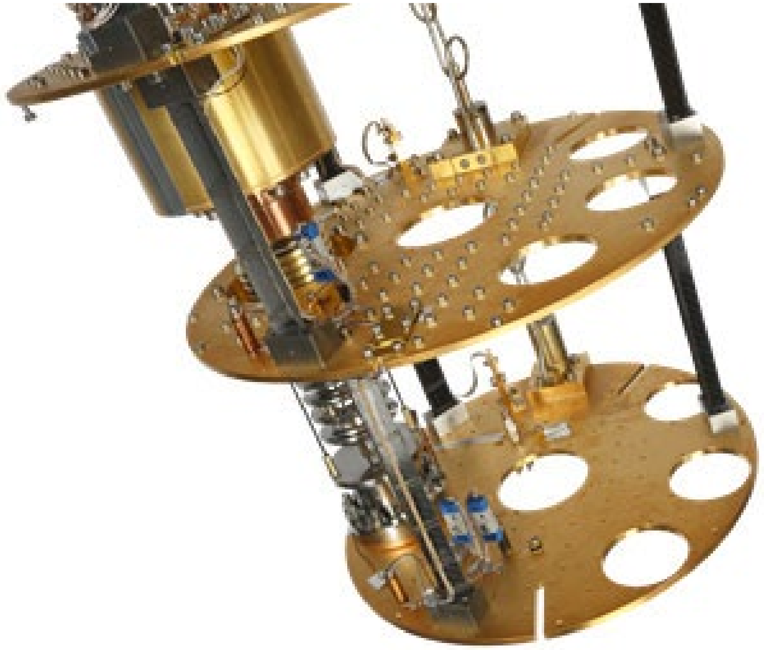
10 µw @ 20 mK

High temperature 30K

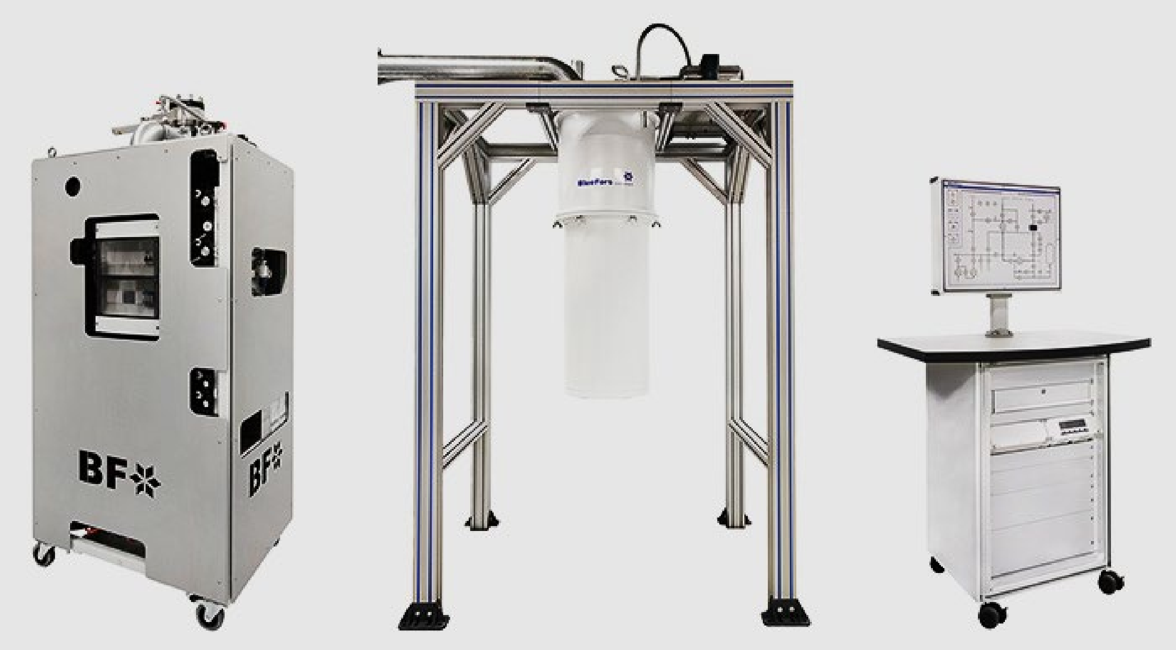


OXFORD INSTRUMENTS

The Business of Science®

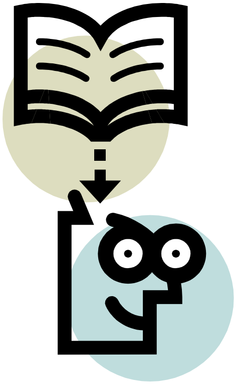


	With DU7-500 dilution unit	**With DU7-300 dilution unit**
Base T:	8 mK typical, < 10 mK guaranteed	8 mK typical, < 10 mK guaranteed
At 100 mK:	500 μW typical, 450 μW guaranteed	300 μW typical, 250 μW guaranteed
At 20 mK:	15 μW typical, 12 μW guaranteed	10 μW typical, 8 μW guaranteed
^{3}He requirement:	18 litres STP	11 litres STP


BF
BF
BlueFors

Playing with the mixture…

- Calculate all the volumes of all circuits inside the RD, and also in pumps, traps, lines, etc…
- Calculate the amount of 3He and 4He gas needed
- Remember that the still has less than 1% concentration of 3He…
- Mixing Chamber should be large for stability…
- Start with minimum amount of mixture!
- Best operation with small amount of mixture in the still, and small amount of concentrated phase in mixing chamber



Condensation of the mixture

- Pre-cooling of cryostat at 4 K
- Pump the exchange gas (pump or adsorption)
- Check tanks pressure
- Check LN2 (eventually LHe) traps
- Circulate mixture through the traps.
- Long condensation process (hours)
- Condensation temperature must be low to avoid additional delays.
- Start circulation. With tricky cryostats, start with high flow rates (4He low points)

Operating the RD...

- Flow rate is determined by the still power, not by the pumps!
- This is NOT true for PT-DRs (no 1K pot, large heat of condensation)
- Choose adequate flow rate (high power, or T_{min} ?
- With high circulation rates, ^{4}He poisoning of the RD.
- High temperature (T> 1 K) operation: remove mixture, use only the mixture of the pump.

Warm-up

- Removing the mixture takes a long time
- Heating the MC and Still: not very efficient!
- Exchange gas? Caution! Unstable, keep $P < 10^{-4}$ mBar
- Close tanks, put rest of the mixture in the pump, close the pump.
- Traps: throw away (pumping) what was not pumped when the nitrogen trap was cold.
- Do not allow water condensation on RD…

Troubleshooting

- 1 K pot hot, pressure is low. Filling capillary blocked. Remove LHe so that the bath level is below the intake, keep 4He pressure in Pot above bath pressure. Heater on capillary…
- High inlet pressure. Air? Hydrogen? Water?
- Low still pressure. Still empty? Check T vs. P!
- No cooling power. Interface level in MC? Heat leak to MC? Apply heat and check cooling power at higher temperatures. ^{3}He/^{4}He ratio OK?
- Heating spikes, oscillations. Superfluid leak to Vacuum can?

Good luck!

- Building a dilution refrigerator is NOT easy
- Repairing a dilution refrigerator is even worse.

We told you….!

Pulse-tube $^3He/^4He$ Dilution refrigerators

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