① December 11, 2013 Lecture

Problem with the local $SU(2)_L \otimes U(1)_Y$ local gauge invariance

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_G$$

$$\mathcal{L}_0 = \bar{\psi}_L i \gamma^\mu D_\mu^L \psi_L + \bar{\psi}_R i \gamma^\mu D_\mu^R \psi_R$$

$$\psi_L = \begin{pmatrix} \nu_e \\ e_L^- \end{pmatrix}, \quad \psi_R = e_R^-$$

$$D_\mu^L \psi_L = \left(\partial_\mu + i g A_{i\mu} \frac{\tau_i}{2} + i g' B_\mu \frac{Y_L}{2} \right) \psi_L$$

$$D_\mu^R \psi_R = \left(\partial_\mu + i g' B_\mu \frac{Y_R}{2} \right) \psi_R$$

3 gauge fields $A_{i\mu}$ for $SU(2)_L$ and
1 gauge field B_μ for $U(1)_Y$

$$\mathcal{L}_G = -\frac{1}{4} W_{\mu\nu}^i W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$W_{\mu\nu}^i = \partial_\mu A_\nu^i - \partial_\nu A_\mu^i - g \epsilon_{ijk} A_\mu^j A_\nu^k$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$A_\mu' = A_\mu + \partial_\mu w + i g [A_\mu, w]$$

$$\left(A_\mu^{i\,\prime} = A_\mu^i + \partial_\mu w^i - g f_{ijk} A_{j\mu} w_k \right)$$

Problems : 1) e^- is massless
2) gauge bosons are massless

2

Introduce two complex scalar fields forming an SU(2) doublet:

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$ (corresponding to 4 gauge bosons in $SU(2) \times U(1)$)

This leads to another term in the Lagrangian

$$\mathcal{L}_S = (D_\mu \phi)^\dagger (D^\mu \phi) - V(\phi)$$

$$D_\mu \phi = (\partial_\mu + i g A_\mu + i g' B_\mu \tfrac{Y_H}{2}) \phi$$

$$V = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

$$\left(\mathcal{L}_0^{K-G} = \frac{1}{2} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2} m^2 \phi^2 \right)$$

for spin-0 free particle

One can also add a gauge-invariant Yukawa coupling:

$$\mathcal{L}_Y = -C_e\,[\,\bar{\psi}_R\,(\phi^\dagger \psi_L) + (\bar{\psi}_L\,\phi)\,\psi_R\,]$$

$\mathcal{L}_Y$ is now invariant under $SU_L(2)$

(due to the $SU(2)$ doublet of ϕ)

$U(w) = e^{-i w f Y}$ for $U(1)$ transformation

$\mathcal{L}_Y$ implies $Y_H = Y_L - Y_R$

or $Y_H = (-1) - (-2) = 1$

Since $Q = T_3 + Y/2$

we conclude that ϕ^+ has $Q = +1$

ϕ^0 has $Q = 0$

(4)

Solution:

1) Introduce extra scalar fields (doublet)
2) spontaneous symmetry breaking for the scalar fields under $SU(2)_L$ and $U(1)_Y$
3) Introduce new couplings between the scalar field doublet and the fermion field (Yukawa coupling)

Spontaneous symmetry breaking

1) Global $U(1)$ symmetry

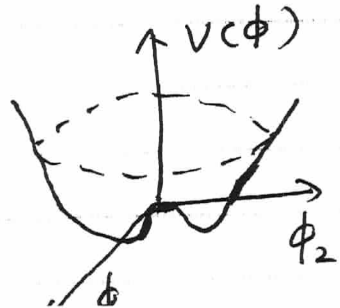
(for a complex scalar field singlet ϕ)

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

$$\mathcal{L} = (\partial_\mu \phi^+)(\partial^\mu \phi) - V(\phi)$$

$$V(\phi) = \frac{1}{4}\lambda(\phi^+\phi)^2 + \mu^2\phi^+\phi$$

$\mathcal{L}$ is invariant under global $U(1)$ transformation, $\phi \to \phi' = e^{i\alpha G}\phi$



$U(1)$ invariance $\Rightarrow$ symmetry under rotation along an axis perpendicular to ϕ_1, ϕ_2.

For $\mu^2 < 0$, $\lambda > 0$

minima occur at $\phi_1^2 + \phi_2^2 = \frac{-4\mu^2}{\lambda} \equiv v^2$

(5)

By selecting the physical vaccum at

$$\rho = v, \quad \theta = 0$$

where $\phi(x) = (\rho(x)/\sqrt{2}) \exp(i\theta(x)/v)$

is the scalar field,

the U(1) symmetry (rotation along the vertical axis) is spontaneously broken.

Expand ϕ around the vacuum $(\rho = v, \theta = 0)$, we have

$$\phi(x) = \frac{1}{\sqrt{2}}(v + h(x)) \exp(-i\theta(x)/v)$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \mu^2 h^2 + \frac{1}{2}\partial_\mu \theta \partial^\mu \theta + \mu^4/\lambda + \cdots$$

h excitation has mass of $\sqrt{2}(-\mu^2)^{1/2}$

θ excitation is massless
(no quadratic term in θ)

This reflects the zero curvature along the θ direction around the vacaum.

$\mu^2 > 0$ (no SSB) $\Rightarrow$ $[(-\mu^2)^{1/2}, (\mu^2)^{\frac{1}{2}}]$ masses

$\mu^2 < 0$ (SSB) $\Rightarrow$ $(\sqrt{2}(-\mu^2)^{1/2}, 0)$ masses

6

Goldston theorem :

Each spontaneously broken symmetry introduces a massless particle .

2) Global SU(2) SSB

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix} \quad \begin{matrix} (K^+, K^-) \\ (K^0, \bar{K}^0) \end{matrix}$$

(The charges for (ϕ^+, ϕ^0) are analogous to (ν, e^-) doublet)

$$\mathcal{L} = (\partial_\mu \phi)^\dagger (\partial^\mu \phi) - \mu^2 \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2$$

4 degrees of freedom , each with the same mass μ (when $\mu^2 > 0$) .

$\mathcal{L}$ is invariant under

$$SU(2) \; : \; \phi' = \exp(-i \vec{\alpha} \cdot \vec{\tau}/2)\, \phi$$

and

$$U(1) \; : \; \phi' = \exp(-i\alpha)\, \phi$$

(4 conserved charges (symmetries) , 3 from SU(2) and one from U(1))

when $\mu^2 < 0$, minimum occurs at

$$\phi^+ \phi = -2\mu^2/\lambda \equiv v^2/2$$

$$\phi^+ \phi = v^2/2$$

$$\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 = v^2$$

The chosen vaccum might not break all possible symmetries $\Rightarrow$ number of massless Goldstone bosons might be less than the maximal numbers

Choose the vaccum such that

$$\phi = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad \phi_1 = \phi_2 = \phi_4 = 0$$

Excitation around the vacuum

$$\hat{\phi} = \exp\{-i(\hat{\theta}(x)\cdot\vec{\tau}/2)\,v\,\}\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+\hat{H}(x)) \end{pmatrix}$$

$$\mathcal{L} = \frac{1}{2}\partial_\mu \hat{H}\,\partial^\mu \hat{H} - \mu^2 \hat{H}^2 + \cdots$$

$$(\mu,\mu,\mu,\mu) \longrightarrow (0,\ 0,\ 0,\ \sqrt{2}\,\mu)$$

SSB leads to three massless Goldstone bosons

Why not 4 Goldstone bosons (one from $U(1)$ three from $SU(2)$)?

For $\delta\hat{\phi} = -i\epsilon(1+\tau_3)\hat{\phi}$ transformation

we have $\delta\hat{\phi} = 0$

Hence the $(\underset{U(1)}{1} + \underset{SU(2)}{\tau_3})$ symmetry is still preserved by the vacuum.

* where are these massless spin-0 Goldstone bosons?

(8)

3) Local $U(1)$

$$\mathcal{L} = [(\partial^\mu + iqA^\mu)\phi]^\dagger [(\partial_\mu + iqA_\mu)\phi]$$
$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\lambda(\phi^\dagger\phi)^2 + \mu^2(\phi^\dagger\phi)$$

Adding the gauge field (massless) A^μ, implies ~~that we now have four field degrees of~~ freedom (2 for complex scalar field, and 2 for massless gauge field)

$$\phi'(x) = e^{-i\alpha(x)}\phi(x)$$

$$A'^\mu(x) = A^\mu(x) + \frac{1}{q}\partial^\mu\alpha(x)$$

Repeating the steps for the global $U(1)$, we can expand the $\hat{\phi}(x)$ around the vaccum, which has spontaneous broken symmetry:

$$\hat{\phi}(x) = \frac{1}{\sqrt{2}}(v + \hat{h}(x))\exp(-i\hat{\theta}(x)/v)$$

The gauge freedom in $A^\mu(x)$ allows us to choose

$$\hat{\alpha}(x) = -\hat{\theta}(x)/v$$

and ~~hence~~

$$\hat{\phi}'(x) = e^{-i\hat{\alpha}(x)}\hat{\phi}(x) = \frac{1}{\sqrt{2}}(v + \hat{h}(x))$$

The Lagrangian density becomes

$$\mathcal{L} = -\frac{1}{4}(\partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu)(\partial^\mu \hat{A}^\nu - \partial^\nu \hat{A}^\mu) + \underbrace{\frac{1}{2} q^2 v^2 \hat{A}_\mu \hat{A}^\mu}_{\text{massive gauge boson}} + \frac{1}{2}\partial_\mu \hat{h} \partial^\mu \hat{h} - \underbrace{\mu^2 \hat{h}^2}_{\text{massive scalar particle}} + \cdots$$

The gauge boson becomes massive (at the expense of the 'disappearance' of the Goldstone boson $\hat{\theta}(x)$)

mass of the scalar field $= \sqrt{2}\mu$

mass of the boson $= qv$

initial degrees of freedom : (2 for scalar field, 2 for massless gauge boson)

after symmetry breaking : (1 for scalar field, ~~3 for massive gauge boson~~)

(3 degrees of freedom for the massive gauge boson)

Now, we return to $SU(2)_L \times U(1)_Y$, and add the scalar field (and spontaneously broken symmetry) to it

We now have, dropping the $'$ symbols

$$\mathcal{L}_S = (D_\mu \phi)^\dagger (D^\mu \phi) + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2$$

$$\mathcal{L}_G = -\frac{1}{4} W^i_{\mu\nu} W_i^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$

$$\mathcal{L}_\ell = \bar{\Psi}_L i\gamma^\mu D^L_\mu \Psi_L + \bar{\Psi}_R i\gamma^\mu D^R_\mu \Psi_R$$

$$\mathcal{L}_{\ell Y} = -C_e \left[\bar{\Psi}_R (\phi^\dagger \Psi_L) + (\bar{\Psi}_L \phi) \Psi_R \right]$$

① $\mathcal{L}_S$ (scalar fields term) $\quad \left(\phi = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+H) \end{pmatrix} \right)$

$$D_\mu \phi = \left(\partial_\mu + i g A_\mu + i g' B_\mu \frac{Y_H}{2} \right) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}$$

$$A_\mu = \frac{1}{2} \tau_i A_{i\mu} = \frac{1}{2} (\tau_1 A_{1\mu} + \tau_2 A_{2\mu} + \tau_3 A_{3\mu})$$

$$= \frac{1}{\sqrt{2}} (\tau_+ W_\mu + \tau_- W_\mu^\dagger) + \frac{1}{2} \tau_3 A_{3\mu}$$

$\tau_\pm = \frac{1}{2}(\tau_1 \pm i \tau_2)$, $\quad W_\mu = \frac{1}{\sqrt{2}} (A_{1\mu} - i A_{2\mu})$

(raising and lowering operator in weak isospin) $\quad W_\mu^\dagger = (W_\mu)^\dagger$

$$(D_\mu \phi)^\dagger (D_\mu \phi)$$

$$= \frac{1}{4} g^2 v^2 W_\mu^\dagger W^\mu + \frac{1}{8} v^2 (g A_{3\mu} - g' B_\mu)^2 + \cdots$$

$$\Updownarrow$$

$$M_W^2 W_\mu^\dagger W^\mu \Rightarrow \boxed{M_W = \frac{1}{2} g v}$$

expect $\frac{1}{2} m_A^2 A_3^2$, $\frac{1}{2} m_B^2 B^2$. The non-zero cross terms implies A_3 and B are mixed

$\cancel{A_\mu = \sin\theta_W A_{3\mu} +}$

$$A = \sin\theta_W A_3 + \cos\theta_W B$$

$$Z = \cos\theta_W A_3 - \sin\theta_W B$$

$$\frac{1}{8} v^2 (g A_{3\mu} - g' B_\mu)^2$$

$$= \frac{1}{8} v^2 \{ A_\mu^2 (g\sin\theta_W - g'\cos\theta_W)^2 + Z_\mu^2 (g\cos\theta_W + g'\sin\theta_W)^2$$

$$+ \cancel{\text{cross}}\ 2 A_\mu Z^\mu (g\sin\theta_W - g'\cos\theta_W)(g\cos\theta_W + g'\sin\theta_W) \}$$

$A \longrightarrow$ photon

$$\Rightarrow \quad g\sin\theta_W - g'\cos\theta_W = 0$$

cross ~~se~~ term vanishes

$$g\sin\theta_W = g'\cos\theta_W$$

$$\boxed{\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}} \qquad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$M_Z = \frac{1}{2} v (g\cos\theta + g'\sin\theta)$$

$$= \frac{1}{2} g v \frac{1}{\cos\theta_W}$$

~~$M_Z =$~~ $\boxed{M_W = \cos\theta_W M_Z}$

The potential term in $\mathcal{L}_S$

$$V(\phi) = \tfrac{1}{4}\mu^2 v^2 + \mu^2 H^2 + \lambda\left(v H^3 + \tfrac{1}{4} H^4\right)$$

$$\boxed{M_H = \sqrt{-2\mu^2}} \; \boxed{\cancel{= \sqrt{2\lambda v^2} = \sqrt{2\lambda}\, v}}$$

② $\mathcal{L}_\ell$ and $\mathcal{L}_{\ell Y}$ (Lepton terms)

$$\mathcal{L}_{\ell Y} = -C_e\left[\bar{\Psi}_R(\phi^+\Psi_L) + (\bar{\Psi}_L\phi)\Psi_R\right]$$

$$\phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}, \quad \Psi_R = e_R, \quad \Psi_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$$

$$\mathcal{L}_{\ell Y} = -\frac{C_e}{\sqrt{2}}\left(v\,\bar{e}e + H\,\bar{e}e\right)$$

me term e-H coupling

$$\boxed{m_e = \frac{C_e v}{\sqrt{2}}}$$

(also coupling to Higgs is proportional to C_e or the fermion mass)

$$\mathcal{L}_\ell = \bar{\Psi}_L i\gamma^\mu D_\mu^L \Psi_L + \bar{\Psi}_R i\gamma^\mu D_\mu^R \Psi_R$$

$= \bar{\nu}_L i\gamma^\mu \partial_\mu \nu_L + \bar{e} i\gamma^\mu \partial_\mu e$ ← kinetic terms

$+ \left(-\frac{g}{\sqrt{2}}\right)\left(J^{\mu +} W_\mu + J^\mu W_\mu^+\right)$ ← Charged current

$+ \left(-g \sin\theta_W\, j_\mu^{em} A^\mu\right.$ ← EM current

$\left. - \frac{g}{\cos\theta_W} j_\mu^{Z} Z^\mu\right)$ ← neutral current

$J_\mu = j_\mu^1 - i j_\mu^2$ ← V-A

$= \bar{\Psi}_L \gamma_\mu \tau_- \Psi_L = \bar{e}_L \gamma_\mu \nu_L$

$= \frac{1}{2}\bar{e}\gamma_\mu(1-\gamma_5)\nu$

$J_\mu^+ = \bar{\nu}\gamma_\mu e$

where

$$j_\mu^Z = \bar{\psi}_L \gamma_\mu Z_L \psi_L + \bar{\psi}_R \gamma_\mu Z_R \psi_R$$

$$Z_L = T_{3L} - Q \sin^2\theta_W$$

$$Z_R = - Q \sin^2\theta_W$$

$$\boxed{g \sin\theta_W = e} \qquad (g' \cos\theta_W = e)$$

Five parameters (before the symmetry breaking)

g, g' SU(2) coupling, U(1) coupling

λ, μ^2 for the scalar field potential

C_e for the Yukawa scalar - Fermion coupling

<u>After symmetry breaking</u>

$$\boxed{M_W,\ e,\ \cancel{M_Z}\ m_e,\ M_H,\ e,\ \theta_W}$$

$$\tan\theta_W = g'/g$$

$$e = g \sin\theta_W$$

$$m_e = \frac{1}{\sqrt{2}} C_e \cancel{\sqrt{-\mu^2/\lambda}}\ v = \frac{1}{\sqrt{2}} C_e \sqrt{-\mu^2/\lambda}$$

$$M_W = \frac{1}{2} g v = \frac{1}{2} g \sqrt{-\mu^2/\lambda}$$

$$\boxed{M_H = \sqrt{-2\mu^2}} \leftarrow \text{remain to be found}$$

(have been found ?!)

$G_F = 1.166 \times 10^{-5}\ GeV^{-2}$

$G_F/\sqrt{2} = g^2/(8M_W^2)$

$\frac{4\pi}{e^2} = 1/\alpha = 137.035989$

$g \sin\theta_W = e$

From G_F, θ_W, α, one can predict $M_W \sim 81\,GeV$

From $M_W = \frac{1}{2} g v$, one can obtain

$v \simeq 246\ GeV/c^2$

From the mass of Higgs, $M_H = \sqrt{-2\mu^2}$, and v

one determine μ^2 and λ

$(v = \sqrt{-\mu^2/\lambda})$

Connection between the Higgs scalar field and dark energy? (vacuum energy?)

Connection between Higgs scalar field and inflation during the Big Bang?