Chapter 1

Introduction and Overview

Two important questions in physics are:

> "What is matter made of? (What are the elementary constituents of matter?)"
>
> "How do these elementary constituents interact? (What are the laws governing their interactions?)"

Answers to these questions evolved with time.

In the early 1930s, right after the discovery of the neutron, the building blocks of all matters were nicely identified as electrons, protons, and neutrons. Protons and neutrons appeared to be point-like and they built up nuclei. Combining a nucleus with electrons, an atom is formed. Adding atoms together, molecules would emerge.

This admirable state proved to be short-lived. Many new particles, which cannot be described in terms of *e*, *p*, *n*, were soon discovered. Moreover, the proton and neutron were found <u>not</u> to be point-like. They contain internal structures and appeared to be formed by smaller, more elementary objects.

The current answers to these two questions are:

Leptons and quarks are the elementary constituents of matter and they interact via strong, electromagnetic, weak, and gravitational forces.

Leptons: 3 'families'

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} \quad \begin{matrix} \leftarrow Q = 0 \\ \leftarrow Q = -1 \end{matrix}$$

Quarks: also three families

$$\begin{pmatrix} u \\ d \end{pmatrix} \quad \begin{pmatrix} c \\ s \end{pmatrix} \quad \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{matrix} \leftarrow Q = +\frac{2}{3} \\ \leftarrow Q = -\frac{1}{3} \end{matrix}$$

and their antiparticles

$$\begin{pmatrix} \bar{\nu}_e \\ e^+ \end{pmatrix} \quad \begin{pmatrix} \bar{\nu}_\mu \\ \mu^+ \end{pmatrix} \quad \begin{pmatrix} \bar{\nu}_\tau \\ \tau^+ \end{pmatrix} \quad \begin{pmatrix} \bar{u} \\ \bar{d} \end{pmatrix} \quad \begin{pmatrix} \bar{c} \\ \bar{s} \end{pmatrix} \quad \begin{pmatrix} \bar{t} \\ \bar{b} \end{pmatrix}$$

It might be useful to briefly review how these leptons and quarks were discovered. See Cahn and Goldhaber '*The Experimental Foundations of Particle Physics*' for details.

1) e^-

Electron was first discovered by J. Thomson in 1897 in the Cathode-ray experiment. He measured e/m and v of this particle and found that its property is independent of the material used as the cathode and independent of the gas, showing that it is a common constituent of different materials.

2) e^+

Discovered by C. Anderson in cosmic ray experiment using Wilson cloud chamber in 1932, this was the first observation of antiparticle confirming Dirac's theory.

3) $\bar{\nu}_e$

In 1919, Chadwick first observed a continuous energy spectrum of e^- from $^3\text{H} \rightarrow {}^3\text{He} + e^-$ decay. This greatly puzzled many physicists including Niels Bohr who questioned the validity of energy conservation. It also violates angular momentum conservation. In his famous 'Dear radioactive ladies and gentlemen' letter, Pauli proposed neutrino as a 'desperate remedy' to this problem. Fermi's theory of weak interaction successfully described β-decays. The direct detection of neutrino occurred much later. In 1953, Reines and Cowan, using neutrinos from the Hanford reactor, observed the reaction

$$\bar{\nu}_e + p \rightarrow e^+ + n$$

(They also considered detecting neutrinos from atomic bomb explosion!)

The difficulty in detecting neutrinos is due to the tiny probability for them to interact with the detectors. For example, the cross section $\sigma\left(\bar{\nu} + p \to e^+ + n\right)$ for 1 MeV $\bar{\nu}$ is $\sim 10^{-43}\mathrm{cm}^2$, implying a mean-free-path of 50 light-years long in water!

In 1955, R. Davis found that ν and $\bar{\nu}$ are different particles. Using $C_2\,Cl_4$ as a detector for neutrino, he could not observe the reaction

$$\bar{\nu}_e + {}^{37}_{17}C\ell \not\Rightarrow e^- + {}^{37}_{18}Ar$$

However, Davis succeeded to detect solar neutrinos in 1968 (although at a reduced rate, the 'solar neutrino problem')

$$\nu_e + {}^{37}C\ell \to e^- + {}^{37}Ar$$

This shows that neutrinos produced at reactors $\left(\bar{\nu}_e\right)$ are different from neutrinos produced in the sun (ν_e).

In 1987, ν's from supernova explosion were detected by two underground experiments: Kamiokande and IMB.

4) $\mu^{\pm}$

Found by Neddermeyer and Anderson (and Street and Stevenson) in 1937 in cosmic ray experiments. Originally, they were thought to be the pi-meson, proposed by Yukawa as the mediator of strong force. Later, it was found they did not interact strongly with nuclei and cannot be the mediator of strong forces. Rather, $\mu^{\pm}$ behaved just like heavy electron/positron.

5) ν_μ

Using 'neutrino beam' generated at the AGS accelerator, Lederman, Schwartz, Steinberger in 1962 found these neutrinos to be different from ν_e. They observed

$$\bar{\nu}_\mu + p \to \mu^+ + n$$

but not

$$\bar{\nu}_\mu + p \not\rightarrow e^+ + n$$

Therefore, this showed that the neutrinos produced in the accelerator are different from the electron neutrinos. This new type of neutrino is called the muon neutrino.

6) $\tau^\pm$

Discovered by Perl et al. at SLAC in 1975

$$e^+e^- \rightarrow \tau^+\tau^-$$

7) ν_τ

Three possible events were reported in ~2000 by the 'DONUT' experiment at Fermilab.

Small ν_τ beam flux, together with the difficulty to identify

$$\nu_\tau + n \rightarrow \tau^- + p$$

made the experiment difficult.

Mass of the Leptons

For charged leptons, their masses are well determined to be $m\ (e^\pm) = 0.511$ MeV, $m\ (\mu^\pm) = 105.66$ MeV, $m\ (\tau^\pm) = 1777$ MeV. It remains a mystery why their masses are what they are.

In the Standard Model, neutrinos are assumed to be massless. In recent years, several experiments suggested that neutrinos have non-zero masses. This remains a very active area of intense research.

The masses of neutrinos can be determined from precise measurements of decay kinematics. In particular, the following decay processes have been used:

For electron neutrinos:

$${}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e \ (+18.6 \text{ KeV})$$

The end-point (maximal) energy of e^- is a sensitive measure of the $\overline{\nu}_e$ mass.

For muon neutrinos:

$$\pi^+ \to \mu^+ + \nu_\mu$$

In this two-body decay, the μ^+ momentum is directly related to the ν_μ mass:

$$m_{\nu_\mu}^2 = m_\pi^2 + m_\mu^2 - 2m_\pi \left(m_\mu^2 + P_\mu^2\right)^{1/2}$$

For tau neutrinos:

$$\tau^+ \to \pi^+\pi^-\pi^+\overline{\nu}_\tau,\ \ \tau^+ \to \pi^+\pi^0\pi^0\overline{\nu}_\tau,\ \ \tau^+ \to 5\pi\overline{\nu}_\tau$$

From the above 'direct measurements' of ν mass, we obtain

$$m(\overline{\nu}_e) < 3 \text{ eV}, \quad m(\nu_\mu) < 0.19 \text{ MeV}, \quad m(\nu_\tau) < 18.2 \text{ MeV}$$

Neutrino Oscillations

Another very interesting method to detect a non-zero neutrino mass is the study of neutrino oscillation. We now briefly discuss the phenomenon of neutrino oscillation.

The neutrino states ($|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$) that are produced in weak-interaction in association with particular charged leptons are called 'flavor eigenstates'. These flavor eigenstates are linear combination of the 'mass eigenstates.'

Now, consider an electron neutrino produced in the SUN in the reaction

$$p + p \to d + e^+ + \nu_e$$

The mass eigenstates ν_1, ν_2 can be expressed in terms of the flavor eigenstates ν_e, ν_μ:

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \text{ or } \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

The plane-wave ν_e is traveling along the x direction, at time t, the wave function of ν_e is now

$$\left|\nu_e\right\rangle_t = \cos\theta\left|\nu_1\right\rangle e^{-i\frac{(E_1t-p_1x)}{\hbar}} + \sin\theta\left|\nu_2\right\rangle e^{-i\frac{(E_2t-p_2x)}{\hbar}}$$

E_1 and E_2 are the energies of the two mass eigenstates, while P_1 and P_2 are the momenta of the mass eigenstates. Note that the neutrino energies are in general much greater than the neutrino masses, and the neutrino travels near the speed of light. The probability amplitude for ν_e to remain a ν_e after travelling for a time t is

$$\left\langle\nu_e\middle|\nu_e\right\rangle_t = \cos^2\theta e^{-i\frac{(E_1t-p_1x)}{\hbar}} + \sin^2\theta e^{-i\frac{(E_2t-p_2x)}{\hbar}}$$

and the probability $P(\nu_e \rightarrow \nu_e)$ is

$$P(\nu_e \rightarrow \nu_e) = \left|\left\langle\nu_e\middle|\nu_e\right\rangle_t\right|^2 = 1-\sin^2 2\theta\sin^2\left[\frac{(E_2-E_1)t-(P_2-P_1)x}{2\hbar}\right]$$

Assuming the two mass eigenstates have the same momentum, then

$$E_1^2 = P^2 + m_1^2 \qquad E_2^2 = P^2 + m_2^2$$

$$E_2 - E_1 = \frac{m_2^2 - m_1^2}{(E_2+E_1)} \simeq \frac{\Delta m^2}{2E}$$

Therefore,

$$P(\nu_e \rightarrow \nu_e) = 1-\sin^2 2\theta\sin^2\left(\frac{\Delta m^2}{4\hbar E}t\right)$$

Now, $\hbar c$ = 197 MeV • fm, $ct = L$ is the distance of travel, and we obtain

(1) $P(\nu_e \rightarrow \nu_e) = 1-\sin^2 2\theta\sin^2\left(\frac{1.27\Delta m^2 L}{E}\right)$

where L is in meter, Δm^2 in $(\text{eV})^2$, and E in MeV

(2) Similarly, $P\left(\nu_e \to \nu_\mu\right) = \sin^2 2\theta \sin^2\left(\dfrac{1.27\Delta m^2 L}{E}\right)$

Note: $P\left(\nu_e \to \nu_e\right) + P\left(\nu_e \to \nu_\mu\right) = 1$

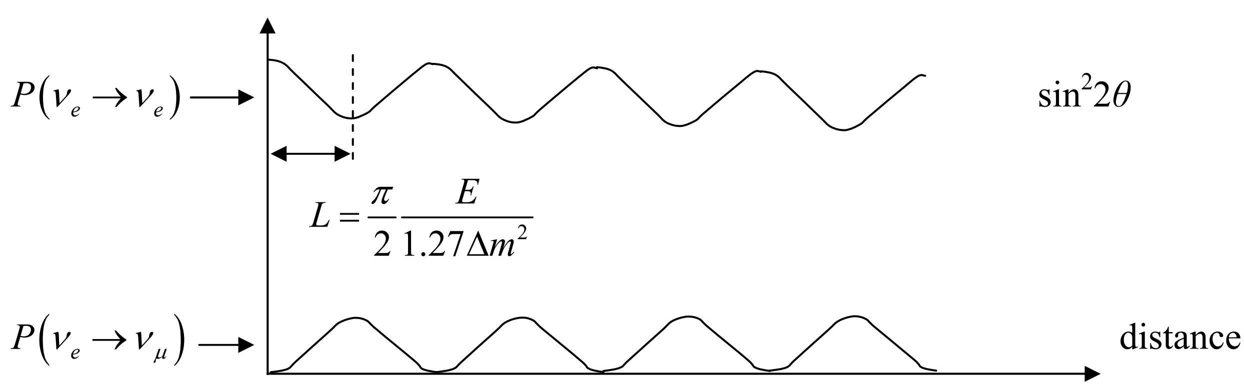
Equations (1) and (2) show that the sensitivity to Δm^2 depends on the energy of the neutrino (E) and the distance (L) between the source of the neutrino and the detector

$$\Delta m^2 \sim \frac{E}{L}$$

where Δm^2 is in (eV)^2, E in MeV and L in meter

For typical neutrino oscillation experiments using reactor neutrinos, accelerator neutrinos, atmospheric neutrinos, and solar neutrinos, we have

Experiment	ν type	E_ν (MeV)	L (m)	Δm^2 (ev^2)
Reactor	$\bar{\nu}_e$	~10	$10 - 10^5$	$1\text{-}10^{-4}$
Accelerator	$\nu_\mu,\ \bar{\nu}_\mu,\ \nu_e,\ \bar{\nu}_e$	$10 - 10^4$	$10 - 10^5$	$1\text{-}10^{-4}$
Atmospheric	$\nu_\mu,\ \bar{\nu}_\mu,\ \nu_e,\ \bar{\nu}_e$	10^3	$10^4 - 10^7$	$10^{-1} - 10^{-4}$
Solar	ν_e	< 10	10^{11}	$10^{-10} - 10^{-11}$



Two types of ν-oscillation experiments

a) Disappearance experiments

For example $$P(\nu_e \to \nu_e) < 1$$

This requires good knowledge on the initial ν_e flux, and is suitable for $\sin^2 2\theta \geq 0.1$

b) Appearance experiments

Like $P(\nu_e \to \nu_\mu)$. Sensitive to much smaller $\sin^2 2\theta$

Sources of Neutrinos:

Solar:

$$p + p \to d + e^+ + \nu_e + 0.42 \text{ MeV}$$
$$e^- + {}^7Be \to {}^7Li + \nu_e + 0.862 \text{ MeV}$$
$${}^8B \to {}^7Be^* + e^+ + \nu_e + 15 \text{ MeV}$$

Reactor:

$\bar{\nu}_e$ from β-decays of fission products of U and Pu

Atmospheric:

$$p + N_2 \to \pi^+ (\pi^-) + x$$

(~ 90% of cosmic ray is p, ~ 10% ^{4}He and ions)

$$\pi^+ \to \mu^+ + \nu_\mu \text{ (lifetime} \sim 10^{-8} \text{ sec)}$$
$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu \text{ (lifetime} \sim 10^{-6} \text{ sec)}$$

Similarly,

$$\pi^- \to \mu^- + \bar{\nu}_\mu$$
$$\mu^- \to e^- + \bar{\nu}_e + \nu_\mu$$

Therefore, one expects

$$\frac{N(\nu_\mu + \bar{\nu}_\mu)}{N(\nu_e + \bar{\nu}_e)} \simeq 2$$

Accelerator:

$$\pi^+ \to \mu^+ + \nu_\mu$$
$$\pi^- \to \mu^- + \overline{\nu}_\mu$$
$$\mu^+ \to e^+ + \nu_e + \overline{\nu}_\mu$$
$$\mu^- \to e^- + \overline{\nu}_e + \nu_\mu$$
$$K^+ \to \mu^+ + \nu_\mu$$
$$K^+ \to \pi^0 e^+ \nu_e$$

The observation of oscillations in solar neutrinos, atmospheric neutrinos, and reactor neutrinos can be analyzed in the scheme of 3-generation ν-oscillation.

$$\nu_\alpha = \sum_i V_{\alpha i}^* \; \nu_i$$

flavor-eigenstates (ν_e, ν_μ, ν_τ) mass eigenstates (ν_1, ν_2, ν_3)

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{23} & \sin\theta_{23} \\ 0 & -\sin\theta_{23} & \cos\theta_{23} \end{bmatrix} \begin{bmatrix} \cos\theta_{13} & 0 & \sin\theta_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin\theta_{13} e^{i\delta} & 0 & \cos\theta_{13} \end{bmatrix} \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} & 0 \\ -\sin\theta_{12} & \cos\theta_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{23}S_{13}e^{i\delta} & C_{12}C_{23} - S_{12}S_{23}S_{13}e^{i\delta} & S_{23}C_{13} \\ S_{12}C_{23} - C_{12}C_{23}S_{13}e^{i\delta} & -C_{12}S_{23} - S_{12}C_{23}S_{13}e^{i\delta} & C_{23}C_{13} \end{bmatrix}$$

where

$$C_{12} = \cos\theta_{12} \qquad S_{12} = \sin\theta_{12} \qquad \text{etc.}$$

V is a unitary matrix. $VV^+ = 1$ and, as we will discuss later, $\delta \neq 0$ implies CP symmetry is violated.

One can work out the expressions for neutrino oscillation. For example

$$P\left(\nu_e \to \nu_e\right) = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{13} - \sin^2 \Delta_{12} \left(C_{13}^4 \sin^2 2\theta_{12} + S_{12}^2\right) \sin^2 2\theta_{13}$$
$$+ S_{12}^2 \sin^2 2\theta_{13} \left[{}^1\!/_2 \sin 2\Delta_{12} \sin 2\Delta_{13} + 2\sin^2 \Delta_{13} \sin^2 \Delta_{12} \right]$$

where

$$\Delta_{12} = \frac{\left(m_2^2 - m_1^2\right) L}{4E_\nu}$$
$$\Delta_{13} = \frac{\left(m_3^2 - m_1^2\right) L}{4E_\nu}$$

If $\theta_{12} \neq 0$ and $\theta_{13} = 0$

then $P\left(\nu_e \to \nu_e\right) = 1 - \sin^2 2\theta_{12} \sin^2 \Delta_{12}$

and we reproduce the expression for two-generation ν-oscillation.

Similarly, if $\theta_{13} \neq 0$ and $\theta_{12} = 0$

then $P\left(\nu_e \to \nu_e\right) = 1 - \sin^2 2\theta_{13} \sin^2 \Delta_{13}$

as one expects.

From the solar neutrino experiments, we obtain

$$\Delta m_{12}^2 = m_2^2 - m_1^2 = 7 \times 10^{-5} eV^2$$

and

$$\tan^2 \theta_{12} = 0.42 \qquad \left(\theta_{12} \simeq 33^\circ\right)$$

From the atmospheric neutrino experiment, we have

$$\left|\Delta m_{23}^2\right| = \left|m_3^2 - m_2^2\right| = 2 \times 10^{-3} eV^2$$

$$\sin^2 2\theta_{23} = 1 \qquad \left(\theta_{23} = 45^\circ\right)$$

From reactor neutrino experiment at short baseline, which did not observe neutrino oscillation signals, we can set a limit for θ_{13}.

$$\sin^2\theta_{13} \le 0.03$$

Therefore, we have

$$\begin{aligned}
\sin^2\theta_{12} &\simeq \frac{1}{3} & \cos^2\theta_{12} &\simeq \frac{2}{3} \\
\sin^2\theta_{23} &\simeq \frac{1}{2} & \cos^2\theta_{23} &\simeq \frac{1}{2} \\
\sin^2\theta_{13} &\simeq 0 & \cos^2\theta_{13} &\simeq 1
\end{aligned}$$

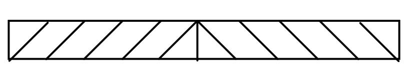
The mixing matrix V becomes

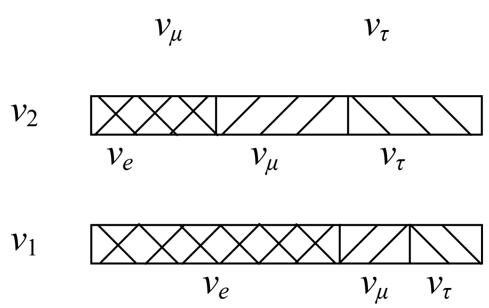
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & \sim 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \\ \sqrt{\frac{1}{6}} & -\sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

and the mass eigenstates are

$$\begin{aligned}
\nu_1 &\simeq \sqrt{\frac{2}{3}}\ \nu_e - \sqrt{\frac{1}{6}}\ \nu_\mu + \sqrt{\frac{1}{6}}\ \nu_\tau \\
\nu_2 &\simeq \sqrt{\frac{1}{3}}\ \nu_e + \sqrt{\frac{1}{3}}\ \nu_\mu - \sqrt{\frac{1}{3}}\ \nu_\tau \\
\nu_3 &\simeq \qquad\quad \sqrt{\frac{1}{2}}\ \nu_\mu + \sqrt{\frac{1}{2}}\ \nu_\tau
\end{aligned}$$

Graphically, we have

ν_3  (ν_3 does not have ν_e component)

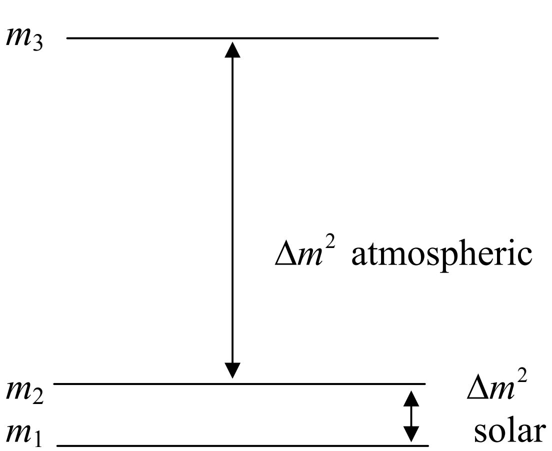


(ν_2 is an equal mixture of ν_e, ν_μ, ν_τ)

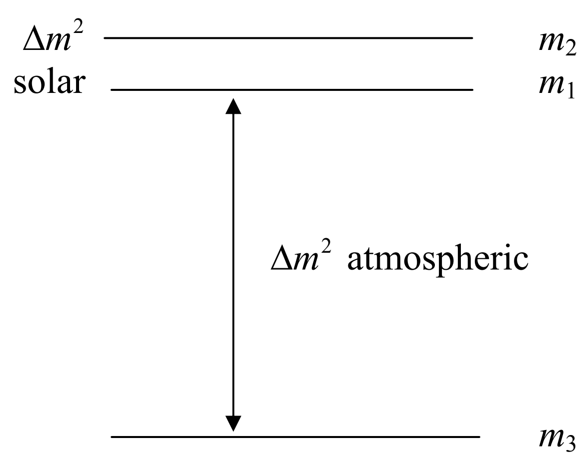
(ν_1 is mostly ν_e)

There are two possible solutions for the masses m_1, m_2, m_3 (since the sign of Δm^2_{23} is not yet known).

(1)



(2)



Further references on ν-oscillation and ν-mass

1) Barger et al.
arXiv: hep-ph/0308123
(http://xxx.lanl.gov/abs/hep-ph)

2) Garcia and Maltoni
arXiv: 0705.1800

3) Altarelli and Feruglio
hep-ph/0306265

4) *Review of Particle Physics* (Particle Data Book)
You can request a copy at (http://pdf.lbl.gov/pdgmail)

Lifetime of Leptons

$e^{\pm}$: They are believed to be stable, since $e^{\pm}$ are the lightest charged particles. Charge conservation and energy conservation imply that $e^{\pm}$ are stable. Nevertheless, some experimentalists still searched for signals of e^- decay. If e^- suddenly disappears in an atom, it would leave a hole behind, which is quickly filled by an outer-shell electron. Hence, an x-ray will be emitted. Search for such x-rays sets a limit for the electron lifetime to be $\tau > 4.2 \times 10^{24}$ years.

$\mu^{\pm}$: The mean lifetime for $\mu^{\pm}$ is 2.2×10^{-6} sec. The dominant decay mode is

$$\mu^+ \to e^+ \nu_e \bar{\nu}_\mu$$

Extensive searches for the following decay modes have been made

$$\mu^+ \to e^+ \gamma \quad \text{(Branching Ratio} < 1.2 \times 10^{-11})$$
$$\mu^+ \to e^+ e^- e^+ \quad \text{(Branching Ratio} < 1.0 \times 10^{-12})$$

The non-observation of these decay modes strongly suggests that there is a conservation law for Lepton Family Numbers. We can assign the lepton family numbers for each lepton as follows:

	L_e	L_μ	L_τ
e^-, ν_e	+1	0	0
e^+, $\bar{\nu}_e$	-1	0	0
μ^-, ν_μ	0	+1	0
μ^+, $\bar{\nu}_\mu$	0	-1	0
τ-, ν_τ	0	0	1
τ^+, $\bar{\nu}_\tau$	0	0	-1

This conservation law forbids $\mu^+ \to e^+\gamma$ decay, since $L_\mu = -1$, $L_e = 0$ for the initial state, while $L_\mu = 0$, $L_e = -1$ for the final state. This law also implies that $n \to p + e^- + \nu_e$ is forbidden, since $L_e = +2$ on the right-hand side and $L_e = 0$ on the left-hand side. Instead, $n \to p + e^- + \bar{\nu}_e$ is allowed.

$\tau^{\pm}$: The mean lifetime for $\tau^{\pm}$ is 2.9 x 10^{-13} sec. The much shorter lifetime is a result of its heavy mass (1777 MeV). Many decay channels are now possible, including $\tau^{+} \rightarrow \mu^{+}\nu_{\mu}\bar{\nu}_{\tau}$ (B.R. = 17.3%).

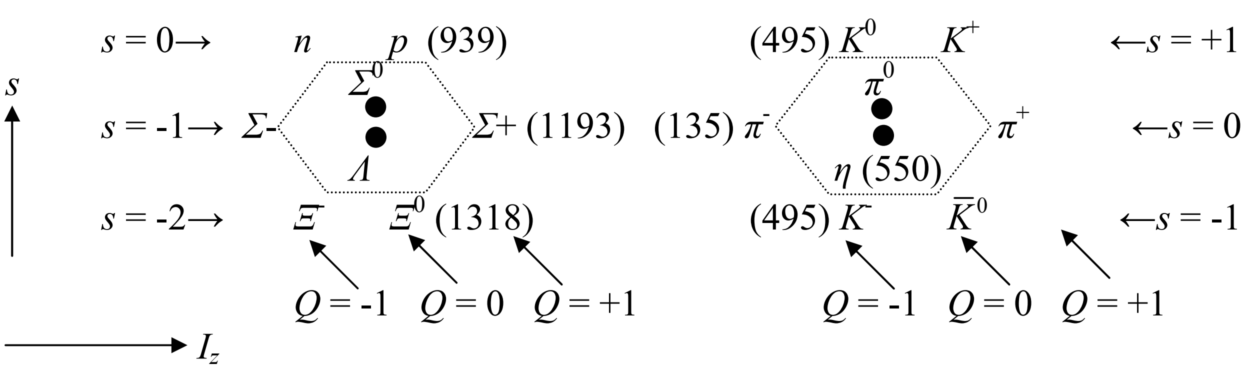
We now consider the quarks. Unlike the leptons, the quarks have never been observed experimentally as free particles. There is much indirect evidence, including spectroscopy of hadrons as well as high-energy collision phenomena, for the existence of quarks. We now believe there are six types of quarks grouped into three families, just like the 6 leptons grouped into three lepton families:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \quad \begin{matrix} \leftarrow Q = 2/3 \\ \leftarrow Q = -1/3 \end{matrix}$$

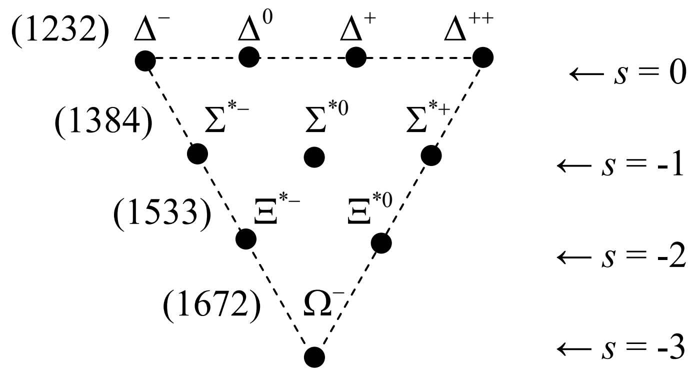
The up (u), down (d), strange (s) quarks were discovered 'theoretically' long after particles containing them were observed experimentally.

In the early 1960s, Gell-Mann and Néeman noticed that many of the known particles can be arranged into hexagonal array or triangular array:

J = ½ baryon octet J = 0 Meson octet



$J = \frac{3}{2}$ baryon decuplet

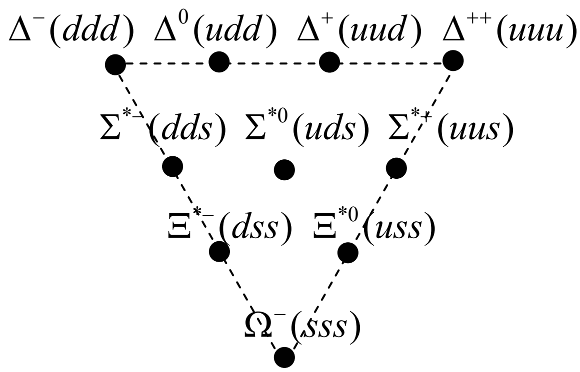


When Gell-Mann and Néeman proposed this 'Eight-fold way', Ω^- was not yet discovered. Gell-Mann predicted the mass and lifetime of Ω^-, which was later discovered in 1964, with properties precisely as predicted.

Why do baryons and mesons fit into these curious patterns? In 1964, Gell-Mann and Zweig independently proposed that hadrons (baryons and mesons) are composed of quarks. They considered three quarks (up, down, strange) with the following properties:

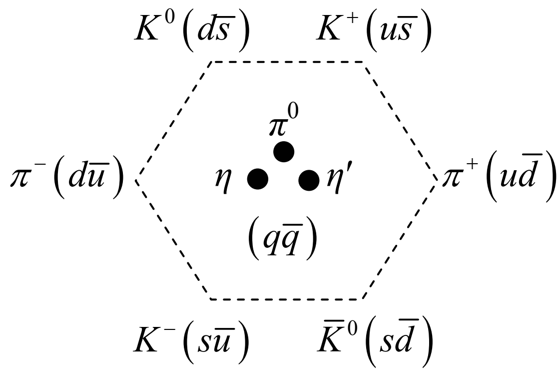
	Q (charge)	S (strangeness)	I	I_z
u (up)	$+\frac{2}{3}$	0	$\frac{1}{2}$	$+\frac{1}{2}$
d (down)	$-\frac{1}{3}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
s (strange)	$-\frac{1}{3}$	-1	0	0
$\bar{u}$	$-\frac{2}{3}$	0	$\frac{1}{2}$	$-\frac{1}{2}$
$\bar{d}$	$+\frac{1}{3}$	0	$\frac{1}{2}$	$\frac{1}{2}$
$\bar{s}$	$+\frac{1}{3}$	+1	0	0

$J = 3/2$ baryons decuplet



qqq	Q	S	baryon	I_z
uuu	2	0	Δ^{++}	$3/2$
uud	1	0	Δ^{+}	$1/2$
udd	0	0	Δ^{0}	$-1/2$
ddd	-1	0	Δ^{-}	$-3/2$
uus	1	-1	Σ^{*+}	+1
uds	0	-1	Σ^{*0}	0
dds	-1	-1	Σ^{*-}	-1
uss	0	-2	Ξ^{*0}	$+1/2$
dss	-1	-2	Ξ^{*-}	$-1/2$
sss	-1	-3	Ω^{-}	0

$J = 0$ meson octet (nonet)



$q\bar{q}$	Q	S	meson	I_z
$u\bar{d}$	1	0	π^+	+1
$d\bar{u}$	-1	0	π^-	-1
$u\bar{s}$	1	1	K^+	+ ½
$d\bar{s}$	0	1	K^0	- ½
$s\bar{d}$	0	-1	$\bar{K}^0$	+ ½
$s\bar{u}$	-1	-1	K^-	- ½
$u\bar{u}$	0	0		0
$d\bar{d}$	0	0	π^0, η, η'	0
$s\bar{s}$	0	0		0

Note that no mesons with $Q = \pm 2$ or $S = \pm 2$ are allowed. Indeed, no such 'exotic' mesons have ever been found.

Despite the success of the quark model for constructing mesons and baryons, the quarks were for a long time not considered as real physical entities. Despite extensive searches, no free quarks were ever found!

What saved the quark model was the surprising discovery of J/ψ particle in November 1974.

J/ψ was discovered at Brookhaven in the reaction

$$P + Be \to J/\psi + x \to e^+ + e^- + x$$

and at SLAC: $$e^+ + e^- \to J/\psi \to e^+ + e^-$$

The surprisingly long lifetime observed for J/ψ (~ 10^{-20} sec, rather than the typical 10^{-23} sec for massive mesons composed of <u>u</u>, <u>d</u>, <u>s</u> quarks) suggested that a new type of quarks is involved. Indeed, J/ψ is understood as the bound state of a 'charm' quark and its antiquark

$$J/\psi : c\overline{c} \text{ bound state}$$

The quark model predicts all sorts of new mesons and baryons consisting of *c* as well as the usual *u*, *d*, *s* quarks. The charmed baryons $\left(\Lambda_c^+ = udc,\ \Sigma_c^{++} = uuc\right)$ were found in 1975, and charmed mesons $\left(D^0 = c\overline{u} \text{ and } D^+ = c\overline{d}\right)$ were discovered in 1976.

In 1977, the bottom quark (*b*) was discovered. The heavy meson $\Upsilon = b\overline{b}$ was found at Fermilab via

$$P + \text{nucleus} \to \Upsilon + x \to \mu^+ + \mu^- + x$$

b-mesons and *b*-baryons were also found subsequently.

The discovery of the top (*t*) quark had to wait for a much higher energy accelerator. In 1995, the top quark was discovered at the Fermilab $p - \overline{p}$ collider. The mass of the top quark was found to be ~ 175 GeV! Due to its large mass, the top quark is very unstable. In fact, the top quark decays before it has a chance to form $t\overline{t}$ or other top-mesons or top-baryons.

Quark Mass

Since quarks are not observed as free particles, their masses cannot be measured directly and must be determined indirectly from their influences on hadron masses. The Particle Data Group (PDG) provides the following estimates of the quark masses:

Quark Flavor	Mass (Bare Mass)
u	15 to 5 MeV
d	3 to 9 MeV
s	60 to 170 MeV
c	1.15 to 1.35 GeV
b	4.0 to 4.4 GeV
t	174.3 ± 5.1 GeV

The large spread reflects the model dependence. The 'effective' mass of quarks within the confines of a hadron is expected to be greater. Estimates of the 'constituent' quark masses are

$$m_u \simeq m_d \simeq 0.31 \text{ GeV}$$
$$m_s \simeq 0.50 \text{ GeV}$$
$$m_c \simeq 1.6 \text{ GeV}$$
$$m_b \simeq 4.6 \text{ GeV}$$

Fundamental Interactions

Quarks and leptons experience four known interactions:

strong, electromagnetic, weak, and gravitational

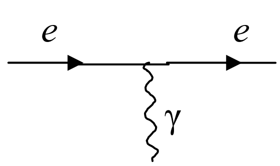
In quantum field theories, the interactions between these fundamental fermions are mediated by bosons. These bosons couple with the fermions with a wide range of coupling strengths, and the effective ranges of the interactions also differ:

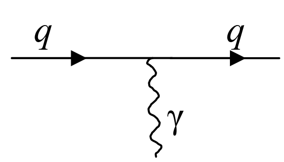
Interaction	Coupling Strength (α)	Range	Mediator
strong	~ 1	$\sim$1 fm (10^{-15}m)	gluons (8 of them, $m_g = 0$)
electromagnetic	$\sim 10^{-2}$	∞	photon ($m_\gamma = 0$)
weak	$\sim 10^{-6}$	$\sim$ 0.01 fm	w^+, w^-, z^0 ($m_w = 80.4$ GeV, $m_z = 91.2$ GeV)
gravitational	$\sim 10^{-39}$	∞	graviton

Gravitation plays no role in our current understanding of particles and nuclei, and will be ignored from now on.

The coupling between the lepton and vector boson is represented by the lepton-lepton-boson vertex such as

QED

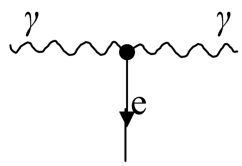




for QED

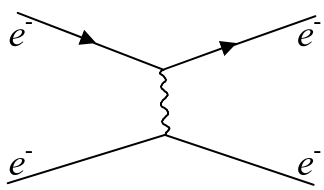
→ time

Note that one cannot have a vertex such as

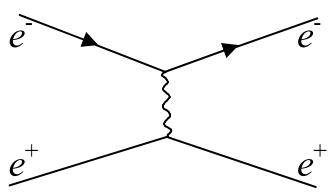


By combining two or more ‘primitive vertices’ together, we obtain the ‘Feynman diagram’ which can describe complicated processes.

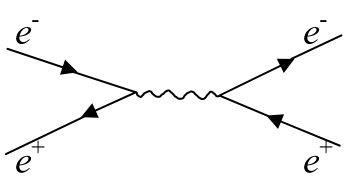
For example:



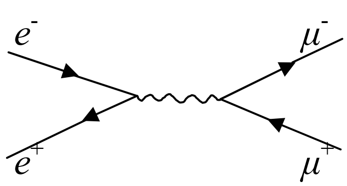
$e^-e^- \rightarrow e^-e^-$ Møller Scattering



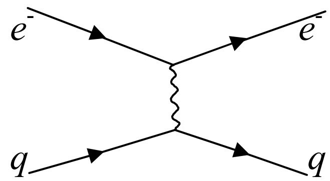
$e^-e^+ \rightarrow e^-e^+$ Bhabha Scattering



Another diagram for Bhabha Scattering

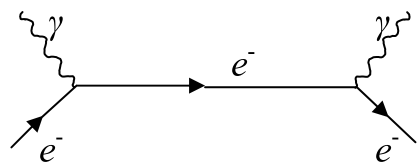


$e^-e^+ \rightarrow \mu^-\mu^+$

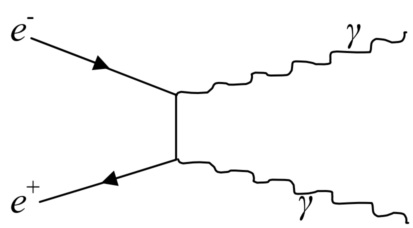


$e^- q \rightarrow e^- q$ "Deep Inelastic Scattering"

While all of the above diagrams have fermions as the 'external' lines and photons as the 'internal' line, one can also have diagrams where the external lines contain photons and internal lines contain fermions:

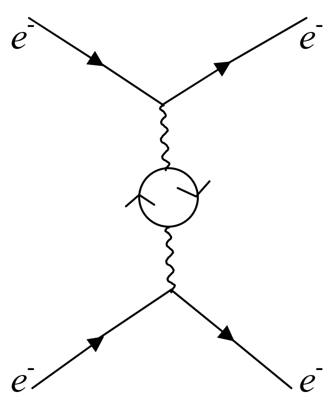


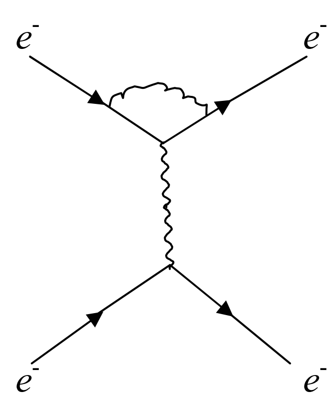
$\gamma e^- \rightarrow \gamma e^-$ Compton Scattering

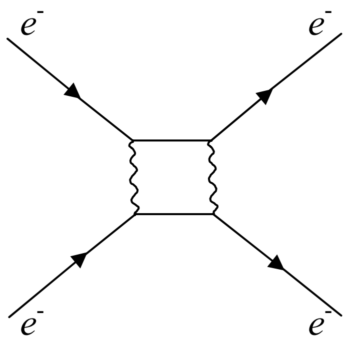


$e^- e^+ \rightarrow \gamma\gamma$ Pair-annihilation

One can also have 'higher order' diagrams containing more vertices



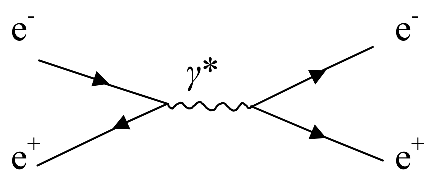




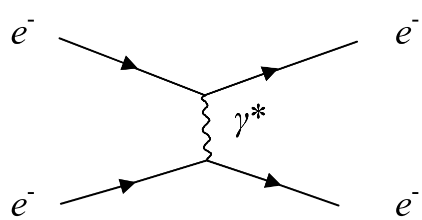
All these diagrams describe
$e^-e^- \rightarrow e^-e^-$ Møller scattering

Note that energy-momentum conservation is imposed at each vertex. However, the internal line does not correspond to real free particles. They are called virtual particles since their effective masses are different from their true masses (rest masses).

As an example, it is easy to show γ^* in this diagram has $(\text{mass})^2 > 0$ (time-like virtual photon)



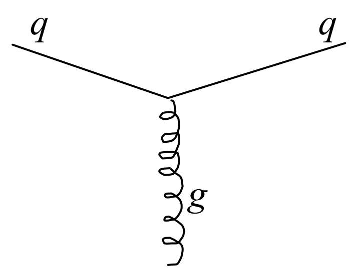
while in this diagram γ^* has $(\text{mass})^2 < 0$ (space-like virtual photon).



(One can calculate the $\left(m_{\gamma^*}\right)^2$ by evaluating $P_{\gamma^*} \bullet P_{\gamma^*}$ in the lab or c.m. frame of e, e pair. The algebra simplifies in these frames.)

QCD

In strong interaction, the fermion-fermion-boson vertex becomes



The 'color' charge in QCD plays the role of electric charge in QED. Since leptons do not have 'color', they do not participate in strong interaction.

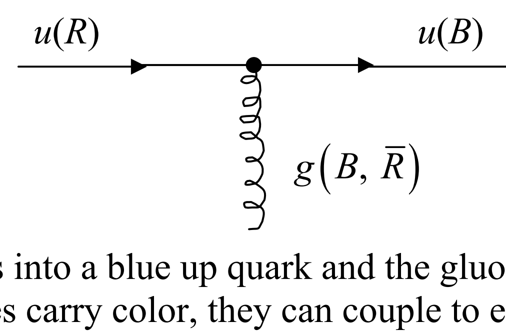
The q-q scattering via a gluon exchange is very much analogous to e^--e^- scattering, as demonstrated in high-energy hadron collider experiments. However, there are several important differences between QCD and QED.

1) Only quarks participate in strong interaction, while both quark and charged leptons participate in electromagnetic interaction.

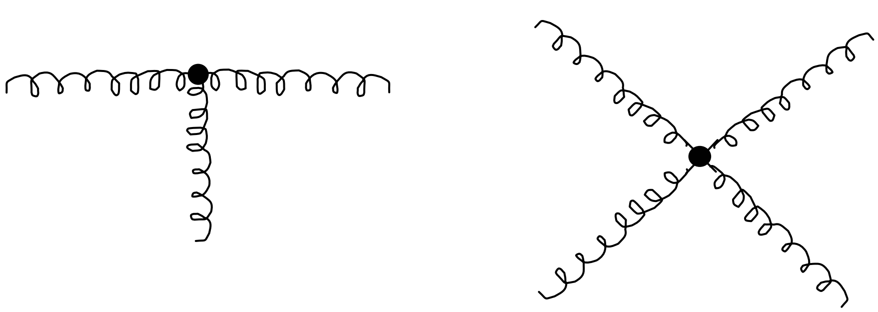
2) The strengths of the coupling are quite different:

$$\alpha_{\text{strong}} \sim 1 \text{ and } \alpha_{\text{EM}} \sim {}^{1}\!/_{137}$$

3) There are three kinds of color in QCD (i.e. red, green, and blue), while only one type of charge in QED. Each quark can carry one of the three types of color charge (R, G, B). The quark color may change in the $q \rightarrow q + g$ process:



A red up quark changes into a blue up quark and the gluon carries a color of $B\bar{R}$. Since gluons themselves carry color, they can couple to each other.



Such coupling between the bosons does not occur for QED, where photons do not carry charge and cannot couple to each other.

4) A charged particle can polarize the vacuum by creating electron-positron pairs. The effective charge increases at short distance. In QCD, the additional vertices of the *g*-*g*-*g* coupling causes an opposite effect such that the effective color charge decreases at short distance (or at high energy). One implication of the 'asymptotic freedom' of QCD is that one can apply perturbative technique to calculate high energy strong interaction processes, since the coupling strength decreases at high energy.

The different screening properties of QED and QCD also qualitatively explain why a free quark is never observed. As the distance between a quark and an antiquark increases, the color field lines get squeezed into a tube-like region due to the gluon-gluon interaction. If the color tube has a constant energy density per unit length, the potential energy between the quarks will increase with separation

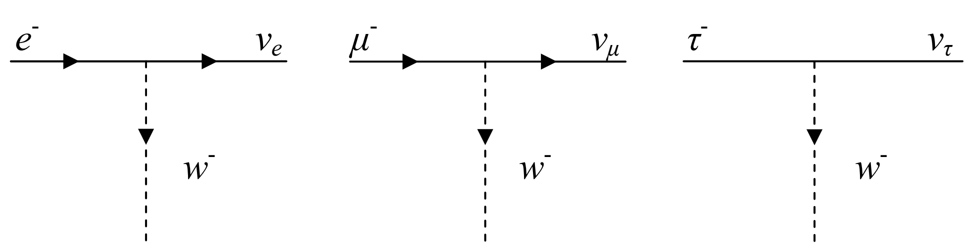
$$V(r) \sim \lambda r \qquad (\lambda \simeq 1 \text{ GeV/fm})$$

and the quarks and gluons can never escape.

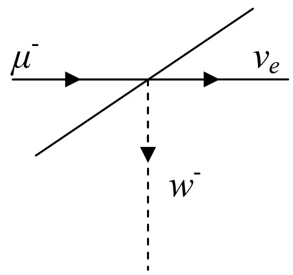
<u>Weak Interaction</u>

Unlike the QED and QCD, weak interaction applies to all leptons and quarks. There are two types of weak interactions: the charged current mediated by $w^{\pm}$ and the neutral current mediated by z^0.

First consider $w^{\pm}$ coupling to leptons:



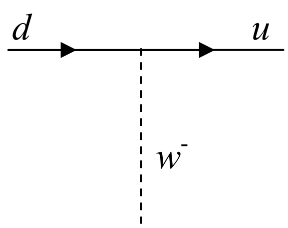
Note that

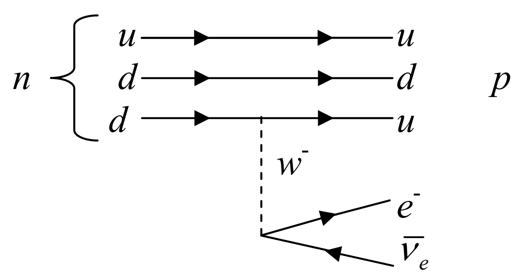


$w^{\pm}$ only couple to leptons within the same lepton generation

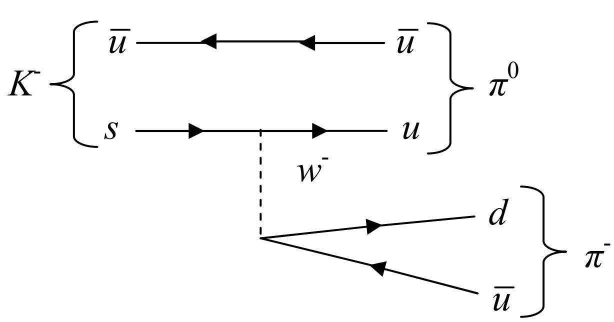
$$\begin{pmatrix} \nu_e \\ \downarrow\uparrow \\ e^- \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \downarrow\uparrow \\ \mu^- \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \downarrow\uparrow \\ \tau^- \end{pmatrix}$$

The situation is different for $w^{\pm}$ coupling to quarks. While nuclear β-decay is understood as w coupling to u and d quarks:

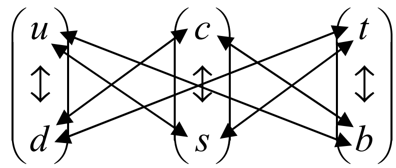




Observation of strangeness-changing weak decays such as $K^- \rightarrow \pi^-\pi^0$, $\Lambda \rightarrow p\pi^-$ showed that a strange quark can convert into up quark in weak interaction.



Indeed $w^{\pm}$ can couple any of the $Q = ⅔$ quarks to any of the $Q = -⅓$ quarks



The strength of the varions coupling is represented by the following CKM matrix

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

The matrix transforms $\begin{pmatrix} d \\ s \\ b \end{pmatrix}$ into $\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

and $w^{\pm}$ couples $u \leftrightarrow d'$, $c \leftrightarrow s'$, $t \leftrightarrow b'$.

The most recent values for the magnitudes of the CKM matrix elements are

$$\begin{pmatrix} 0.9741-0.9756 & 0.216-0.226 & 0.0025-0.0048 \\ 0.219-0.226 & 0.9732-0.9748 & 0.038-0.044 \\ 0.004-0.014 & 0.037-0.044 & 0.9990-0.9993 \end{pmatrix}$$

In terms of mixing angles, the CKM matrix corresponds to

$\theta_{12} = 12.9^o$ $\quad \theta_{23} = 2.4^o$

$\theta_{13} = 0.2^o$ $\quad \delta = 59^o \pm 13^o$

Finally, z^0 couples to all leptons and quarks. The flavor of the leptons and quarks is conserved. No evidence of flavor-changing neutral current such as $s \rightarrow dz^0$ has been found.

Here is a summary for the types of interaction various leptons/quarks can experience:

Particles	Strong	Electromagnetic	Weak
$e\pm$, $\mu\pm$, $\tau\pm$	no	yes	yes
ν_e, ν_μ, ν_τ $\left(\overline{\nu}_e,\ \overline{\nu}_\mu,\ \overline{\nu}_\tau\right)$	no	no	yes
u, d, s $\left(\overline{u},\ \overline{d},\ \overline{s}\right)$	yes	yes	yes

Conservation laws such as energy, momentum, angular momentum, baryon number, lepton number, CPT, etc. hold for all interactions. However, other conservation laws, which are respected in strong interaction, are violated in E.M. and weak interactions:

Conservation Laws

Interaction	Flavor Conservation (u, d, s, etc.)	Isospin	Parity	CP
strong	yes	yes	yes	yes
electromagnetic	yes	no	yes	yes
weak	no	no	no	no

How can one tell whether a reaction (or decay), if allowed by conservation laws, proceeds via strong, electromagnetic, or weak interaction?

Here are some tips:

1) Try to draw the Feynman diagram for the process
   - If the diagram requires $w^{\pm}$ or z^{0}, it is a weak interaction

- If γ^* is required, it is an E.M. interaction
- If only gluons are needed, it is a strong interaction

2) Check the types of particles involved in the process
   - If neutrino appears, it has to be a weak interaction (since neutrino only participates in weak interaction)
   - If γ (photon) spears, it is an electromagnetic interaction
   - If not ν, γ, $e^{\pm}$ (or other charged leptons) are involved, it is most likely a strong interaction. However, it can also be E.M. or weak interaction. (In this case, use criteria 1) to distinguish them.)

3) Check the lifetime of the decaying particles, or the cross-section of a reaction

	Strong	E.M.	Weak
Lifetime	$\sim 10^{-22}$ sec	$\sim 10^{-18}$ sec	$\geq 10^{-12}$ sec
Cross-section	$\sim$ *mb* (10^{-27} cm^2)	$\sim 10^{-31}$ cm^2	$\leq 10^{-40}$ cm^2