# Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

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# Outline of Presentation

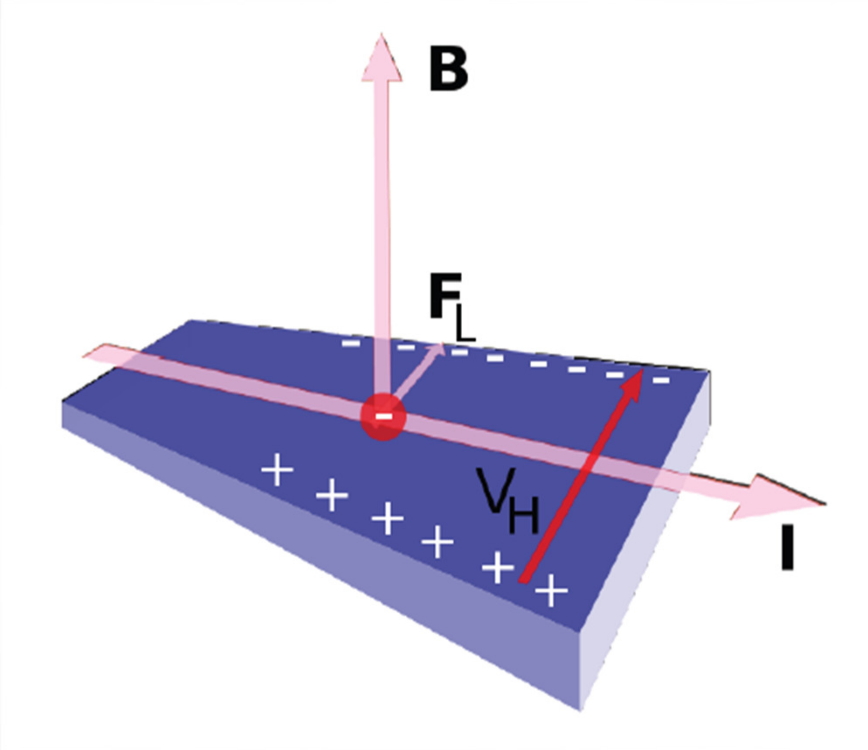
- What is the quantum spin hall effect?
- Why is this paper important?
- What are the results of this paper?
- What are the repercussions of this paper?

# The Quantum Spin Hall Effect

We will first introduce the classical Hall effect and then the Quantum Hall effect as motivation for the Quantum Spin Hall Effect

# Classical Hall Effect

- Occurs when an electric current I in a conductor is exposed to a perpendicular magnetic field.
- The Lorentz force causes a drift current perpendicular to I, causing a build up of charge on the sides.

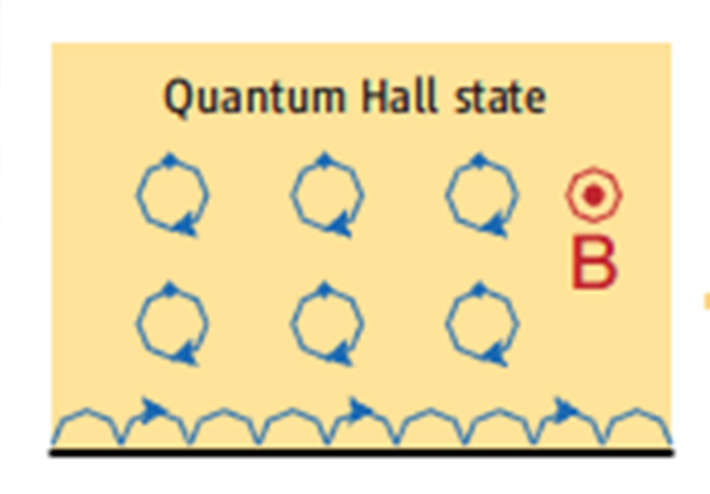



$$R_H = \frac{V_H}{I} = -\frac{1}{ne}$$ (Hall coefficient)

Where n is the charge carrier density.

# Quantizing the Hall Effect

Consider an electron gas confined to two dimensions (x-y direction) subjected to a strong magnetic field in the z-direction.




- The gas experiences a quantization of its energy into "Landau Levels" given by:

$$\epsilon_{n,k} = \hbar\omega_c\left(n+\frac{1}{2}\right), n = 0,1,2,\ldots$$

Where n is q quantum number , k the 2-d wave vector number and $\omega_c$ is the cyclotron frequency given by $\frac{eB}{m_*}$ with $m_*$ the reduced mass of the electron.

# Energy Levels in the Quantum Hall Effect

- Landau levels are degenerate with degeneracy given by:

$$d = \frac{L_x L_y}{2\pi l_B^2} = \frac{L_x L_y e B}{2\pi\hbar}, \qquad l_B = \sqrt{\frac{\hbar}{eB}}$$

In particular, the degeneracy of the Landau level is proportional to B. Hence at high enough B, electrons conglomerate into a finite number of energy states.

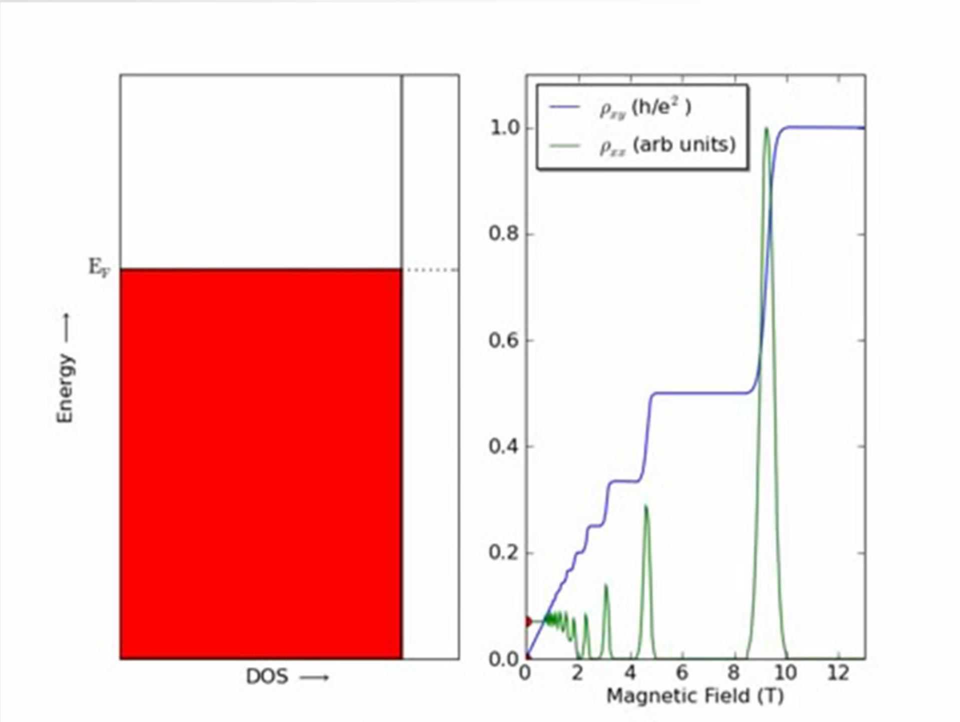
- The number of Landau levels filled is given by:

$$\nu = \frac{N}{\frac{d}{L_x L_y}} = \frac{Nh}{eB}$$

Where N is the number of free valence electrons. Note that $\nu$ is a small for high B.

# Quantum Hall Effect

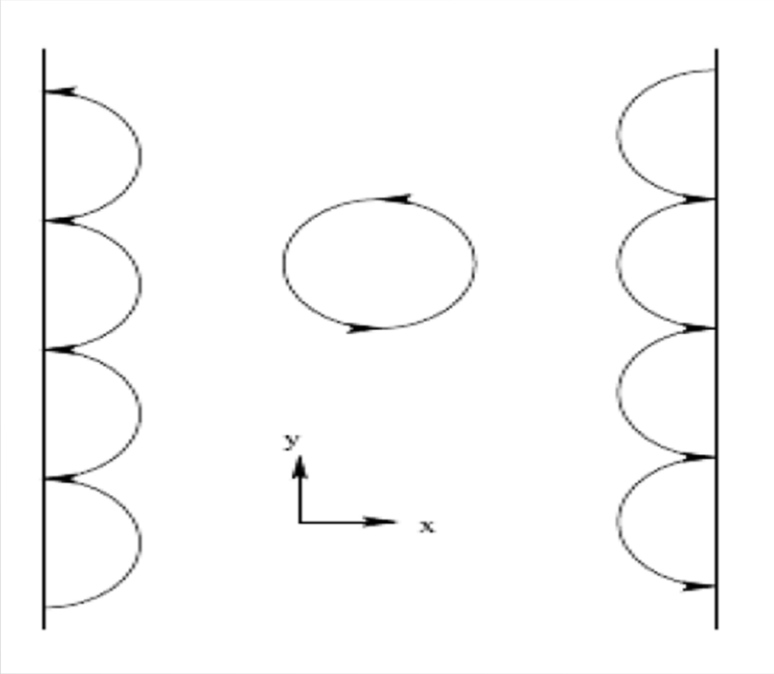
- Illustration of Landau levels filled as the magnetic field is increased.




The graph on the right shows the Hall resistivity and the diagonal resistivity with increasing B field

# Currents in the Quantum Hall Effect

- Geometrical picture of electron orbits in the 2-d gas.

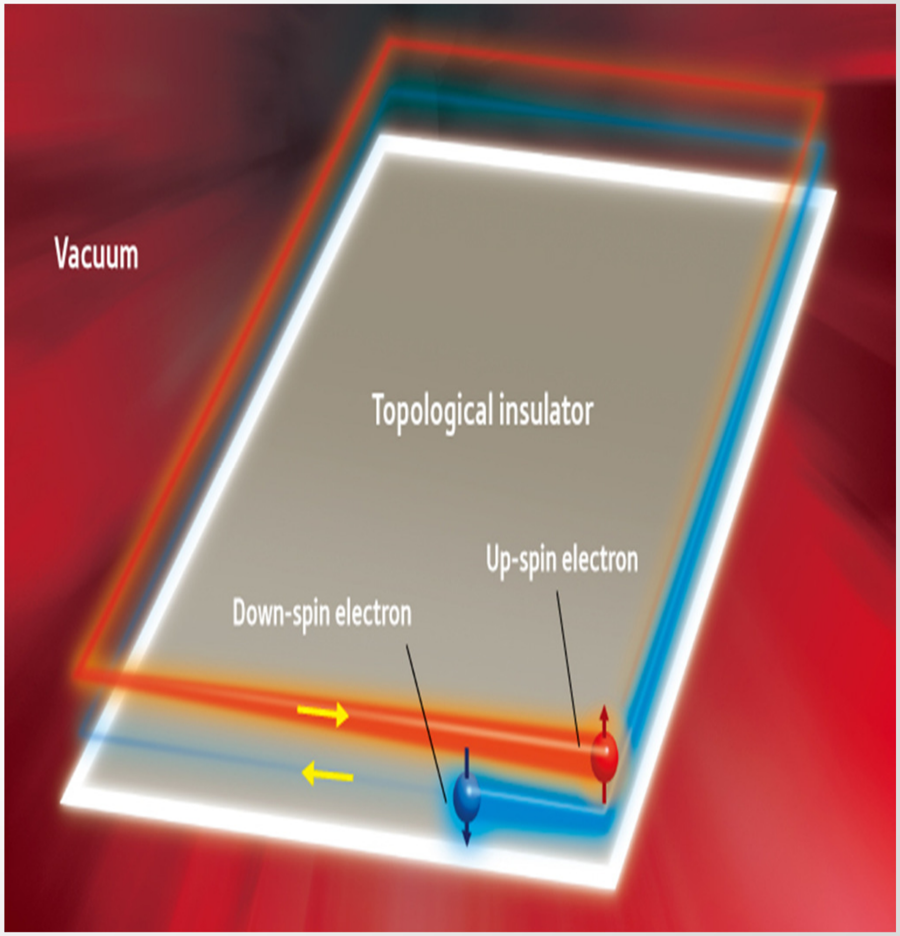



Edge currents of opposite direction form at the edges along the y-direction.

No net current exists unless an E-field is applied along the y-direction

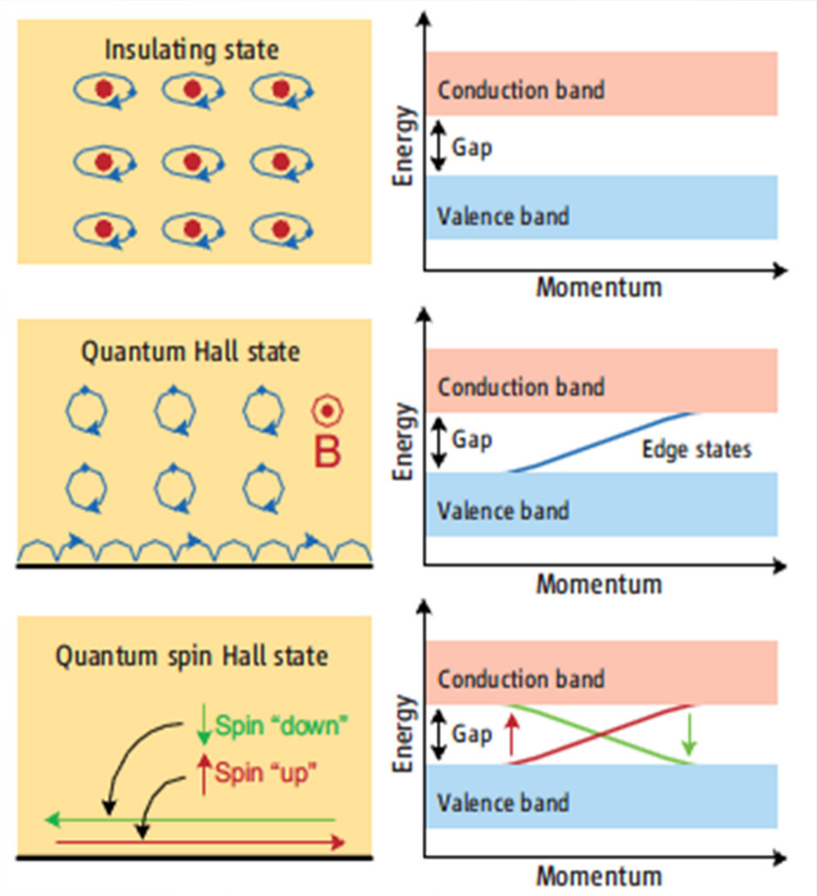
# Quantum Spin Hall Effect

- A special class of Topological Insulators that contain edge states that are spin filtered.
- That is one edge state made up of spin up carriers and another edge state made up of spin down carriers.
- Both carriers go in opposite directions.
- No external B field is needed. The spin currents and due to the internal electronic structure.



# Constructing the Quantum Spin Hall Effect

- The QSH can be explained intuitively by using the QH.
- The “acting” magnetic field is due to the nucleus B field from the strong spin-orbit coupling.




Kane, C. L., & Mele, E. J. (2006). A new spin on the insulating state. Science, 314(5806), 1692-1693.

# Currents in the Quantum Spin Hall Effect

- A spin up carrier will see an effective B-field going into the page due to the spin-orbit coupling.
- A spin down carrier will see an effective B-field going out of the page due to the spin-orbit coupling.
- This generates two opposing edge currents that are spin filtered.
- The currents are suffer no dissipation.

# Resistance in Quantum Spin Hall Effect

- The Spin up and Spin down edge currents give each a Hall resistance of:

$$R_H = \frac{h}{e^2}, \nu = 1$$

- Hence, the total Spin Hall Resistivity is:

$$R_H = \frac{2h}{e^2}$$

- And the total Spin Hall Conductivity is:

$$\rho_H = \frac{2e^2}{h}$$

- Since the edge currents have quantized Hall resistance, the Spin Hall Conductivity is quantized.

# Why This Paper is Important

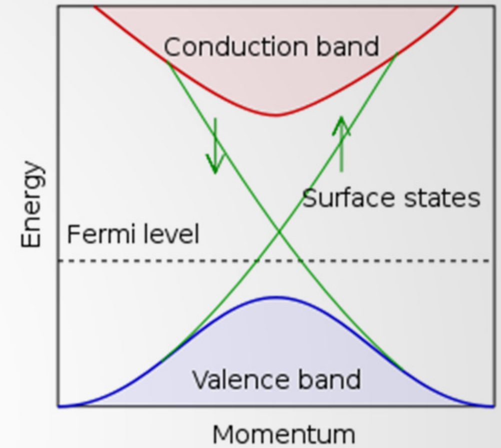
- Importance of Quantum Spin Hall effect
  - Provides new physics and new devices
- Previously published work
  - What lead to the theoretical prediction
  - What the previously proposed real world examples are
  - Why those proposed real world examples are unrealistic
- Importance of this paper
  - This paper fills the missing piece -> giving a real world example

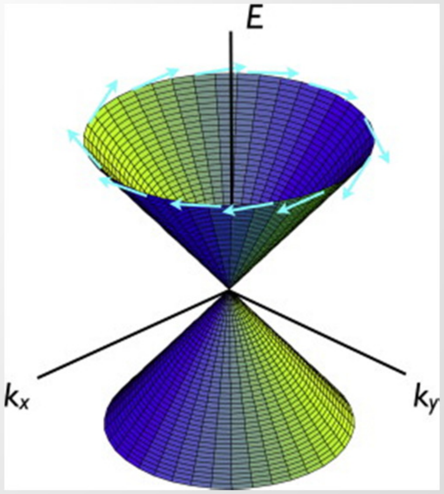
# Quantum spin hall effect has zero mass charge carriers

- Linear relationship between momentum and energy

$$E^2 = p^2c^2 + m^2c^4$$
$$E = pc$$

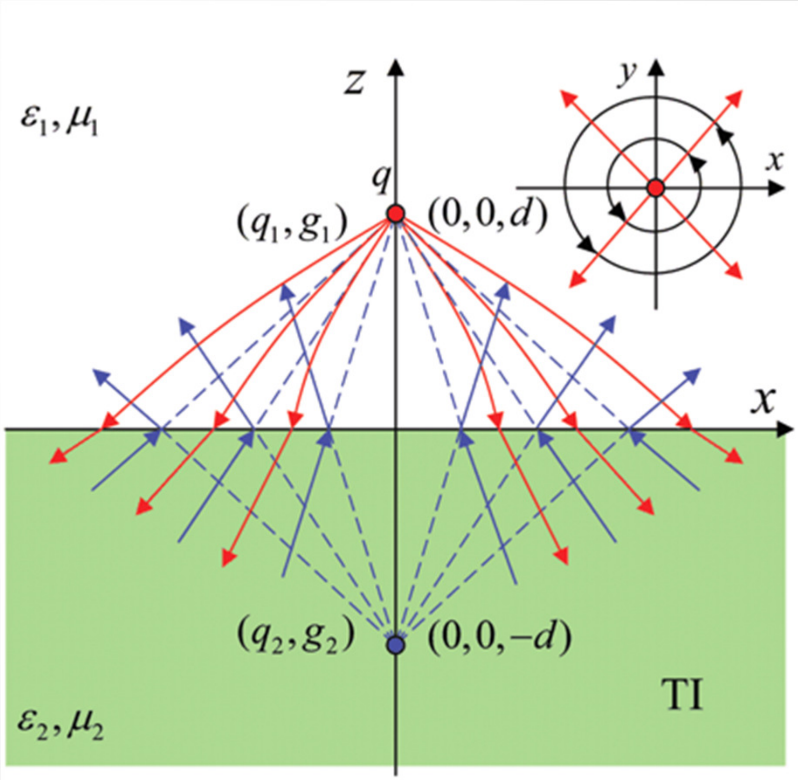
- Therefore the charge carriers behave like massless particles
- Velocity determined by slope
- This effect may have applications in high frequency electronics



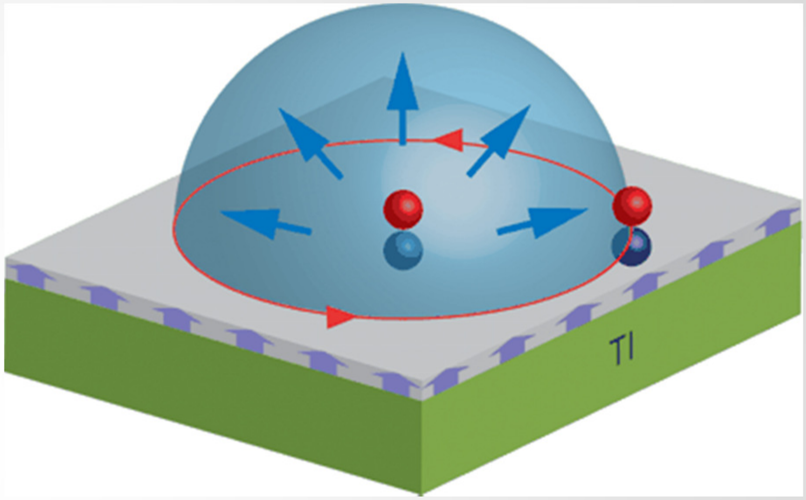
# Coupling in electric and magnetic images

- Similar to having an induced image charge on the surface of a conductor, the quantum spin hall effect will induce a **magnetic monopole** image when an **electric point charge** is outside




Red lines are electric fields, blue lines are magnetic field. This shows a point charge create an image magnetic monopole.

# Electric and Magnetic coupling



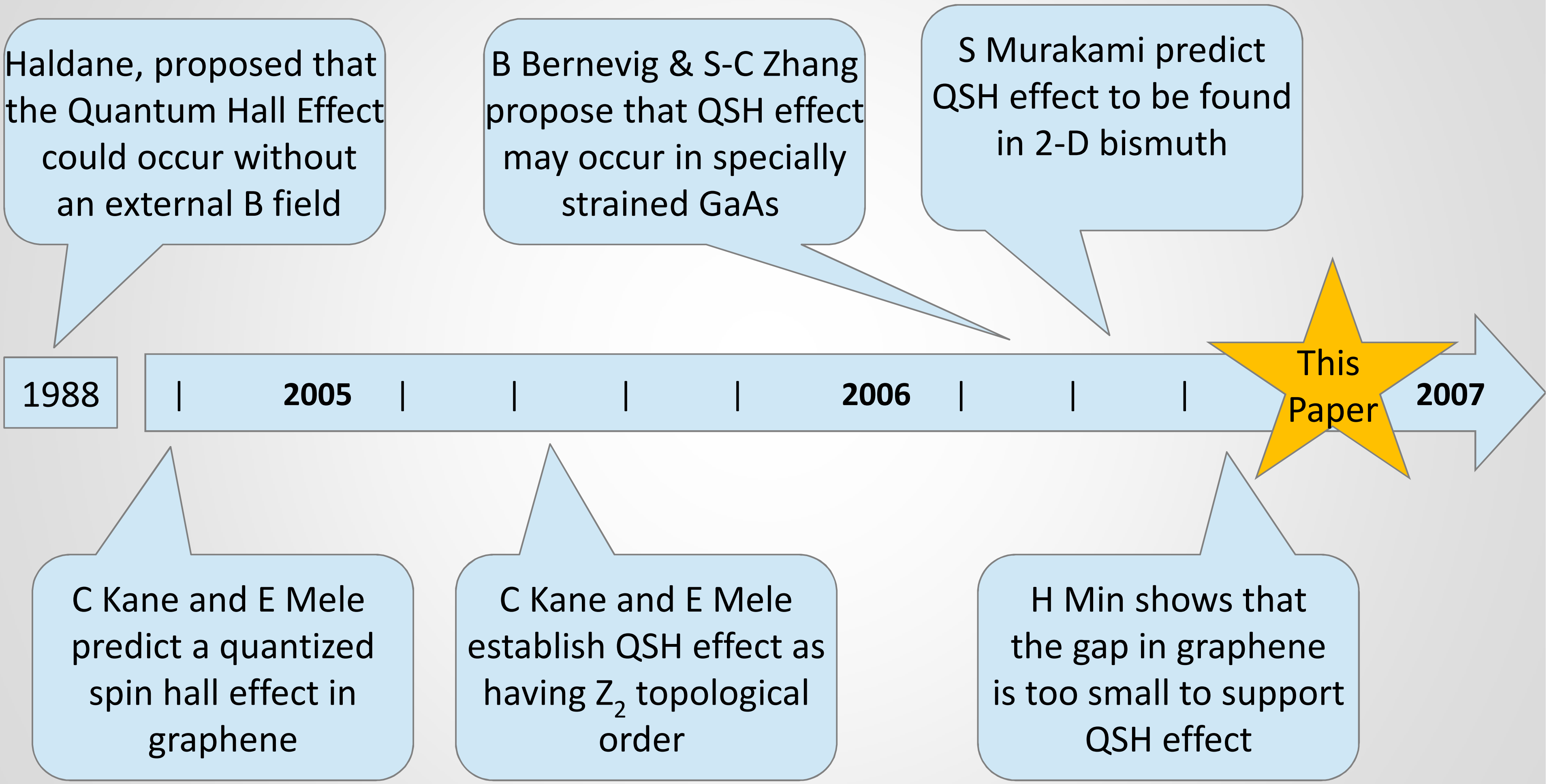

- The electric charge combined with the image magnetic monopole form a dyon; a particle with both magnetic and electric charge
- because of the Aharonov–Bohm effect, and the magnetic field of the monopole image, exchanging two charges will give a phase change

$$|\psi_1\psi_2\rangle = e^{i\theta}|\psi_2\psi_1\rangle$$

this gives an entirely new set of statistics compared to bosons and fermions

$$|\psi_1\psi_2\rangle = \pm|\psi_2\psi_1\rangle$$

# Previous work

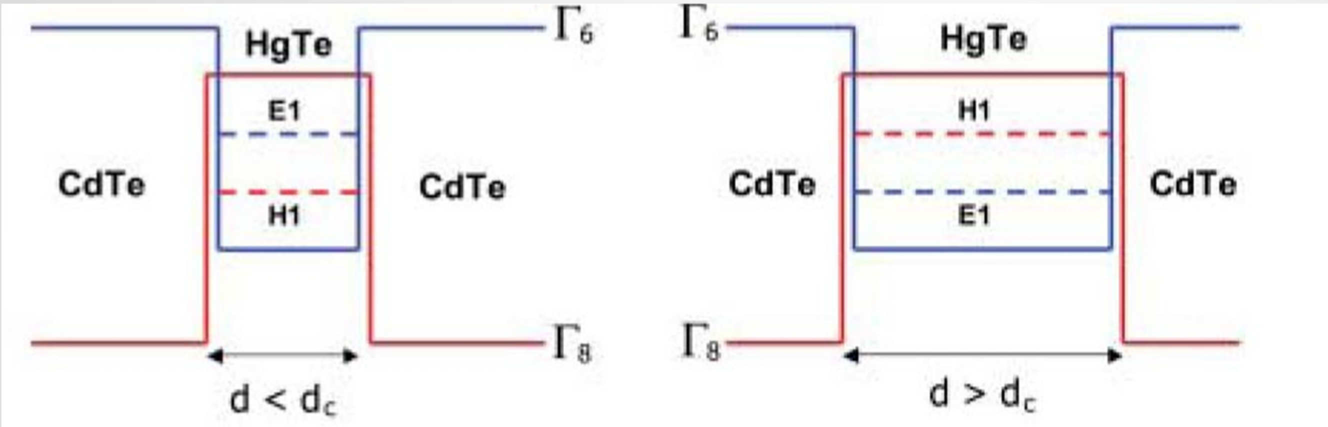


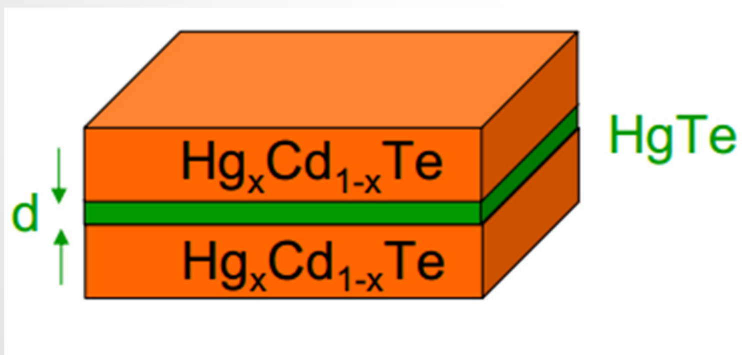
# Summary and Results of this Paper

## Quantum Spin Hall Effect and Topological Phase Transition in HgTe Quantum Wells

B. Andrei Bernevig Taylor L. Hughes Shou-Cheng Zhang

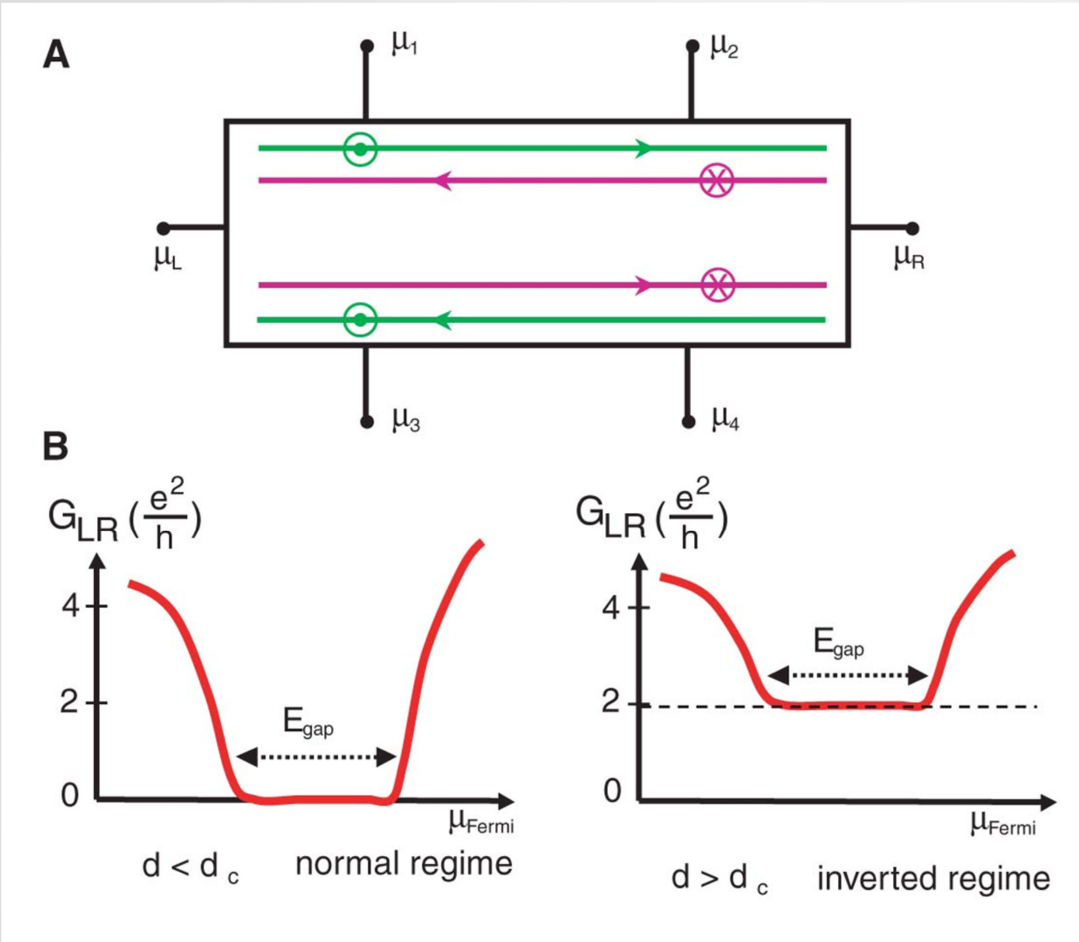
# Proposal of Experiment:




- HgTe between two pieces of CdTe.
- Thickness of Hg less or more than a critical point ($d_c$).

Bernevig, B. A., Hughes, T. L., & Zhang, S. -. (2006). Quantum spin hall effect and topological phase transition in HgTe quantum wells. *Science, 314*(5806), 1757-1761.




- ✓ Voltage drop is measured to see whether the conductance becomes zero or not.

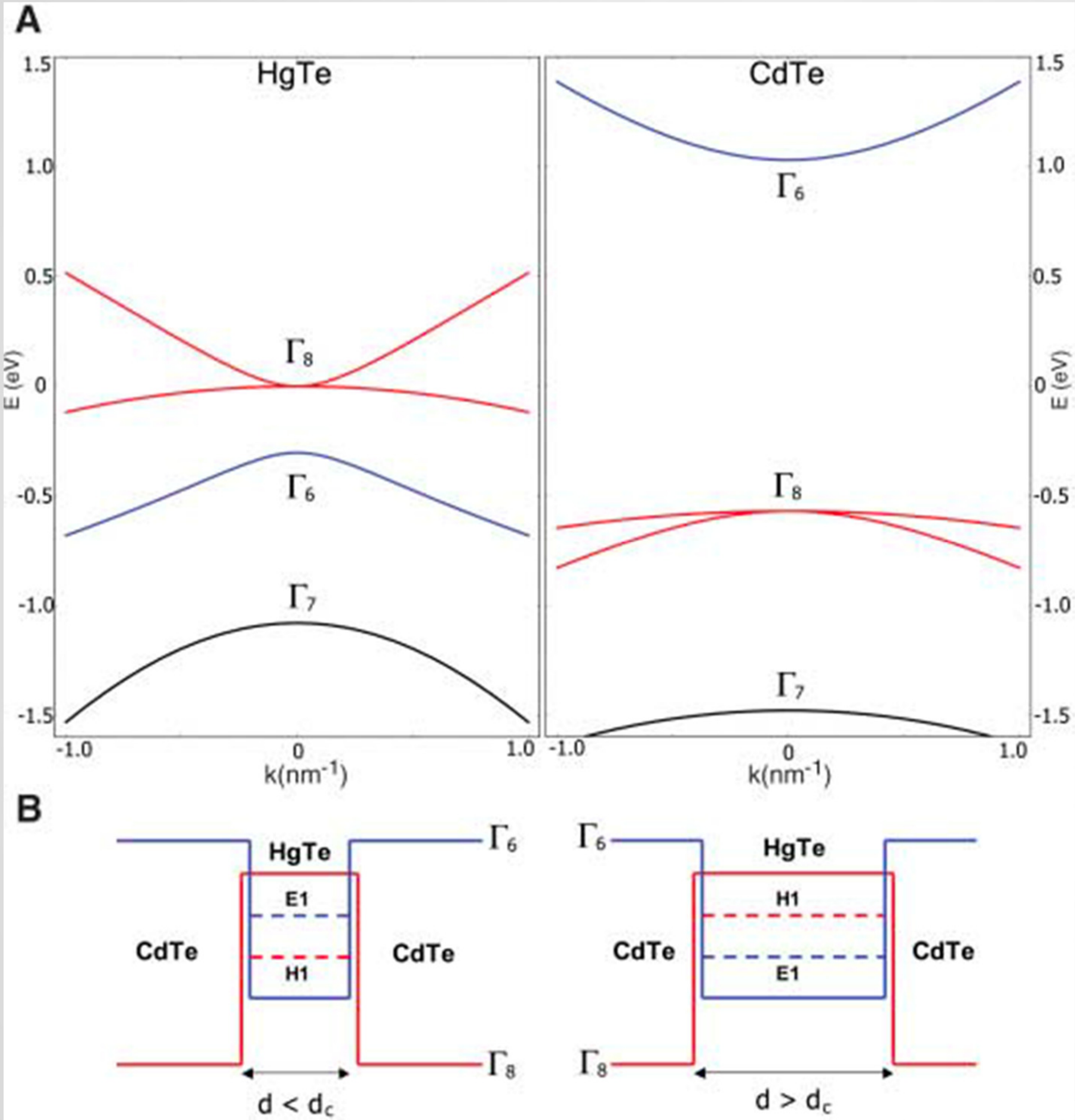
1. If conductance becomes zero ➔Normal Insulator.
2. If conductance is not zero➔Topological Insulator.

Bernevig, B. A., Hughes, T. L., & Zhang, S. -. (2006). Quantum spin hall effect and topological phase transition in HgTe quantum wells. *Science, 314*(5806), 1757-1761.

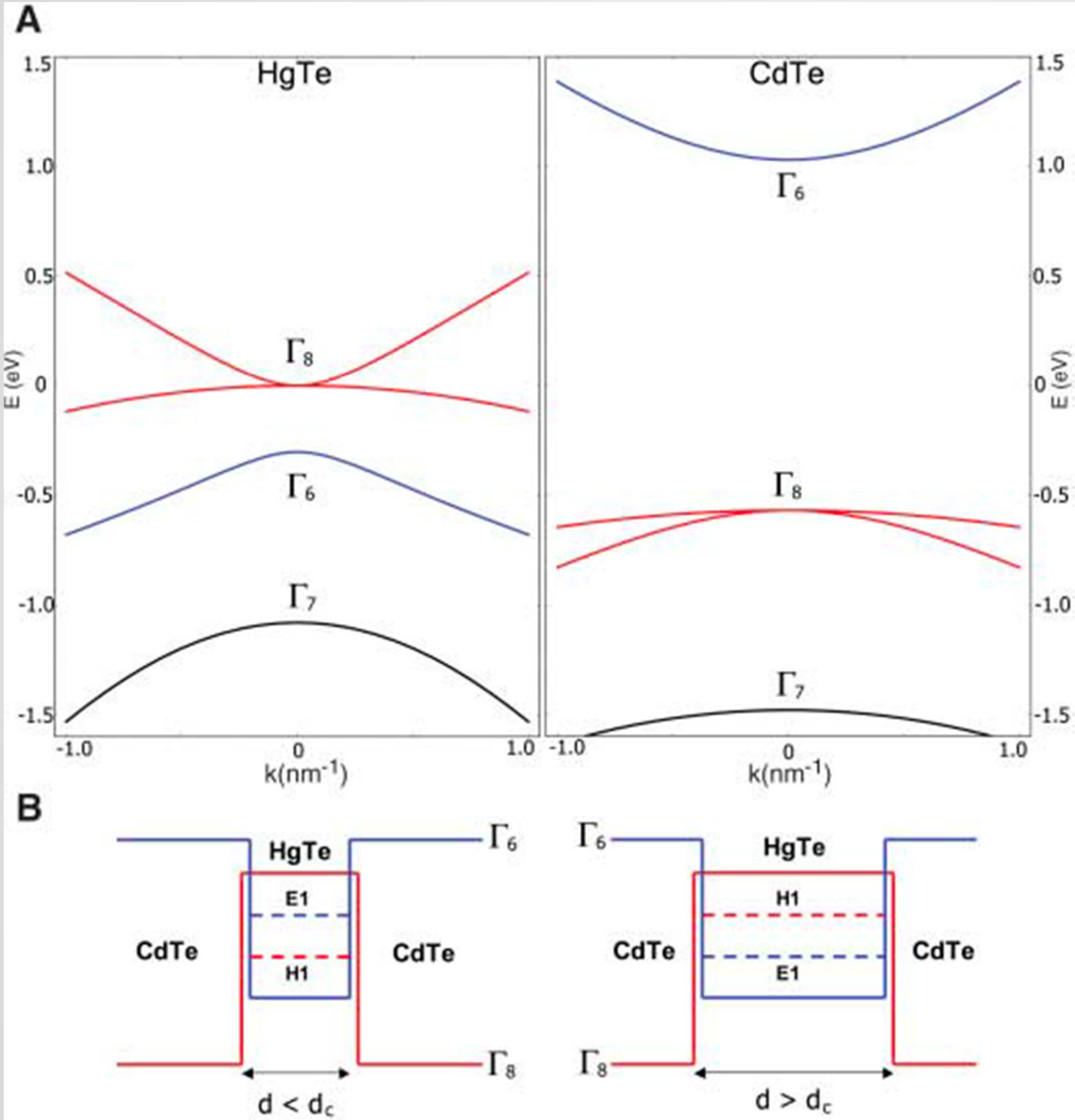
**There are six basic atomic states per unit cell** in HgTe and CdTe.

$$\Psi = \left(|\Gamma_6, \tfrac{1}{2}\rangle, |\Gamma_6, -\tfrac{1}{2}\rangle, |\Gamma_8, \tfrac{3}{2}\rangle, |\Gamma_8, \tfrac{1}{2}\rangle, |\Gamma_8, -\tfrac{1}{2}\rangle, |\Gamma_8, -\tfrac{3}{2}\rangle\right)$$

$\mathbf{\Gamma}_6$ is a s-type band $\mathbf{\Gamma}_8$ is a p-type band.

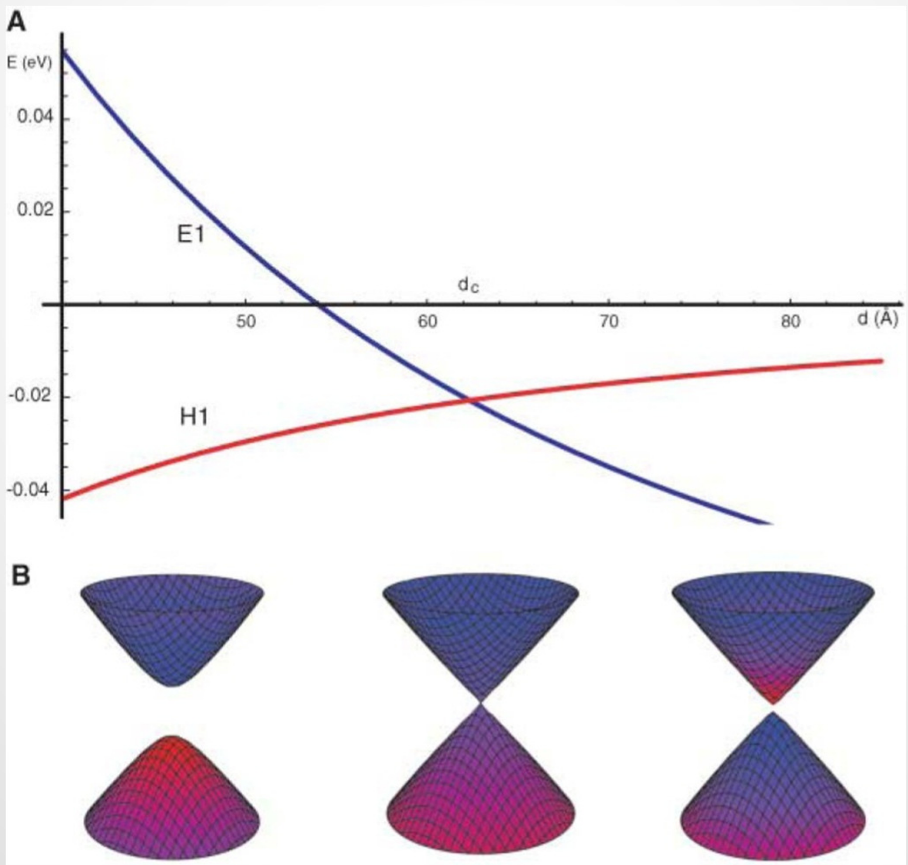


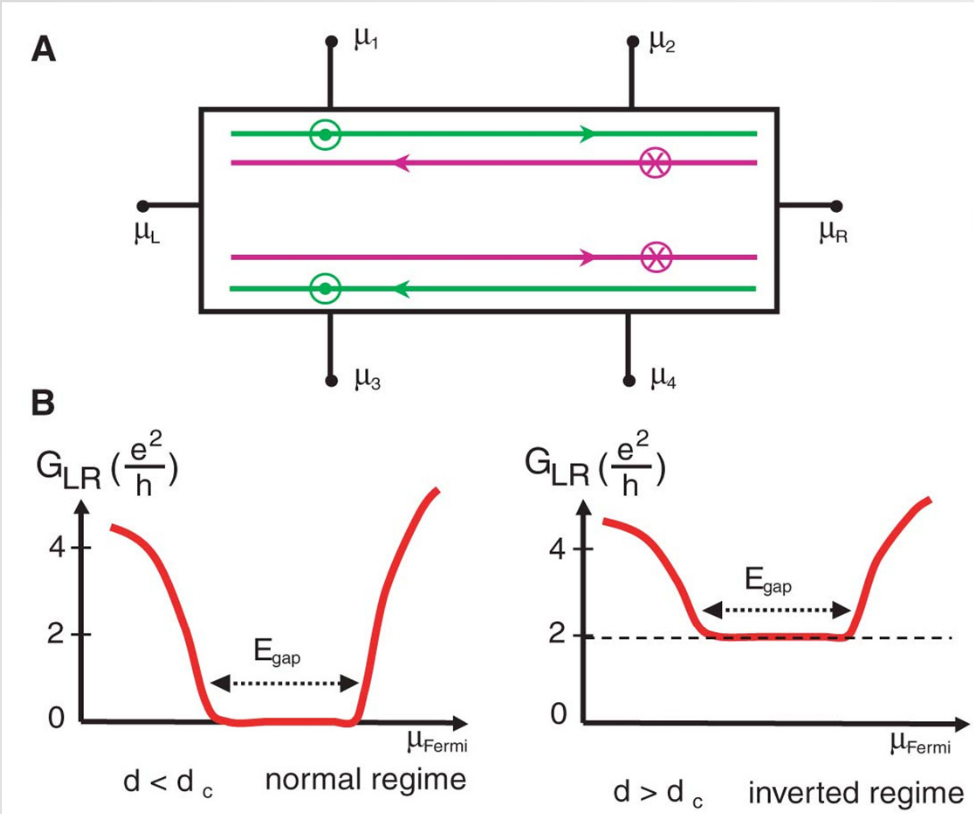

- ✓ **CdTe** has a **normal band progression** (s-type $\mathbf{\Gamma_6}$ band lying <u>above</u> the p-type $\mathbf{\Gamma_8}$ band).
- ✓ **HgTe** has **an inverted band progression** ($\mathbf{\Gamma_6}$ band lies <u>below</u> the $\mathbf{\Gamma_8}$ band).

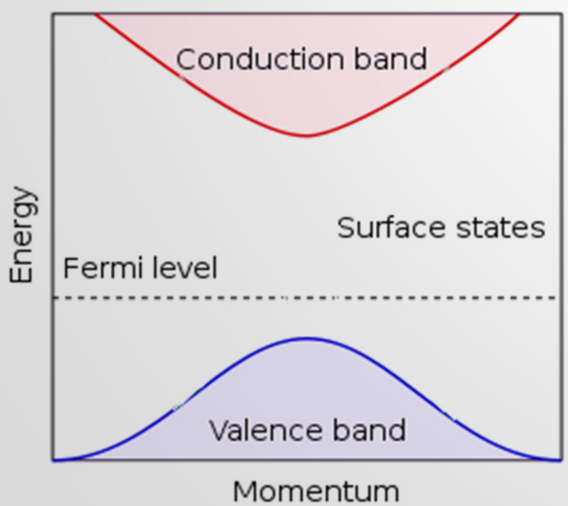


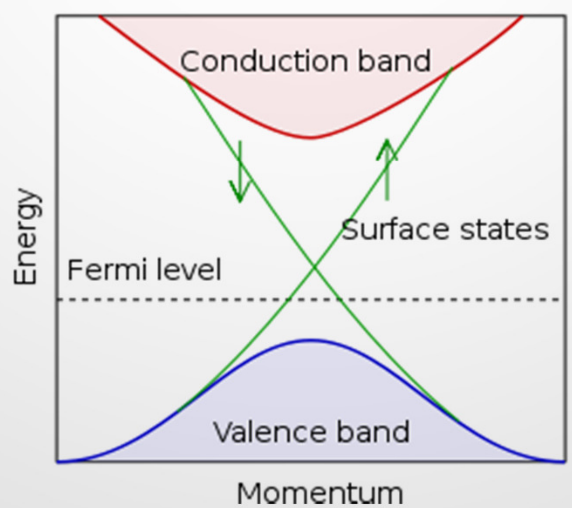

- ✓ |E1, mJ⟩ state is formed from the **combinations** of |Γ6, mJ= ±1/2⟩ & |Γ8, mJ = ±1/2⟩ states.
- ✓ |H1, mJ⟩ state is formed from the |Γ8, mJ = ±3/2⟩ states.

# $E_1$ and $H_1$ states flip when thickness of HgTe is more than $d_c$ .





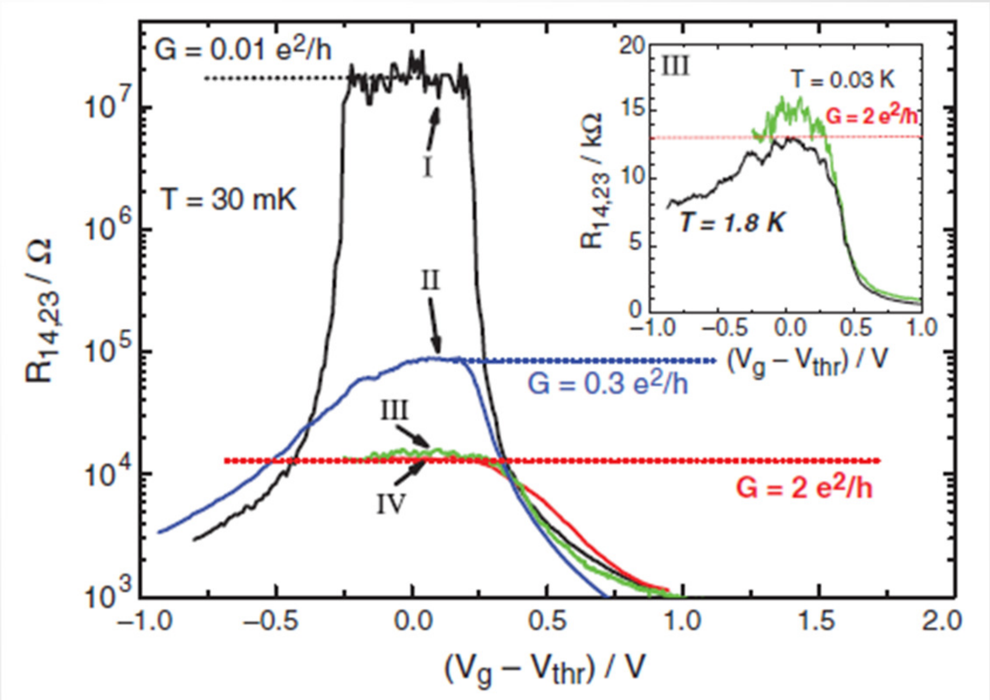




- In a **normal insulator** conductance **vanishes** while Fermi level is between valence and conductance band.
- In a **topological insulator** conductance is not zero between conductance and valence bands.
- ✓ In **a six-terminal measurement**, the QSH state would exhibit **electric voltage drop** between the terminals ($\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ and between $\boldsymbol{\mu}_3$ and $\boldsymbol{\mu}_4$), in the zero temperature

# Experimental verification

- König *et al.* confirmed the prediction an year later (2007)

König, M., Wiedmann, S., Brüne, C., Roth, A., Buhmann, H., Molenkamp, L. W., . . . Zhang, S. -. (2007). Quantum spin hall insulator state in HgTe quantum wells. *Science, 318*(5851), 766-770.



# Critical Analysis

- Overall we found this to be a very good paper with very important consequences in condensed matter experiment and theory
- However, effects of inversion symmetry breaking is not considered in the paper which the materials HgTe and CdTe actually have.
- In that case $S_z$ Conservation is also broken, and with no conservation of spin, the material does not show true Quantum Spin Hall effect

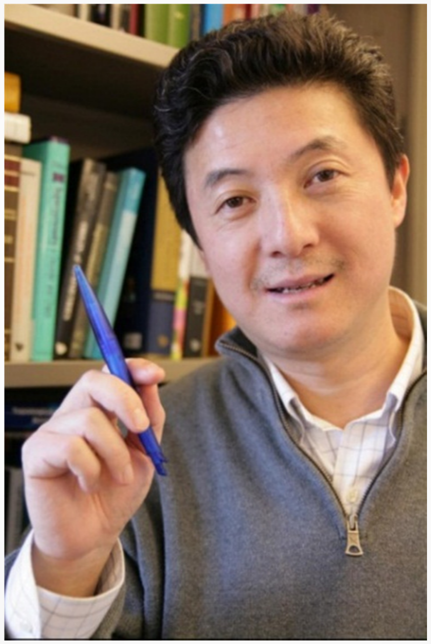
# Extended to 3D

- In 2007 Kane and Fu predicted that this idea can be extended to 3 D materials as well.
- $Bi_{1-x}Sb_x$ alloy can act as topological insulator.
- In 2008 by Zahid Hasan at Princeton University, observed topological surface states in that system.

# Citations

- Scopus - 773
- Web of Knowledge - 748

# 2012 Dirac Medal & 2010 Europhysics prize



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# Summary

- The quantum spin hall effect can be constructed as two separate quantum hall effect currents, one for spin up, one for spin down
- Materials with this behavior have quantized resistance, can carry spin currents. With applications in spintronics and quantum computing
- Bernevig, Hughes and Zhang were the first to propose a realistic method of observing this effect
- As a result of this discovery there is a huge amount of research in condensed matter theory and experiment

Acknowledgements:
Prof. Cooper
Prof. Hughes

# References


- Bernevig, B. A., Hughes, T. L., & Zhang, S. -. (2006). Quantum spin hall effect and topological phase transition in HgTe quantum wells. *Science, 314*(5806), 1757-1761
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- Kane, C. L., & Mele, E. J. (2006). A new spin on the insulating state. Science, 314(5806), 1692-1693