Extreme Quantum Entanglement in a Superposition of Macroscopically Distinct States

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Quantum Conflicts with Locality

- Quantum entanglement

  - Correlated spins between separate particles

- Local hidden variable predictions diverge with quantum probabilities

- Experimental analysis of spatially separate particle spin states

Einstein, Podolsky and Rosen Believe Quantum Mechanics Incomplete

- Entanglement requires either

  - Interactions between separated particles

  - Measurement outcomes encoded before separation

- Einstein rejects the first option in favor of locality

- Later, local hidden variables proposed to make a deterministic theory without entanglement

Bell's Theorem Distinguishes Hidden Variable Theories and Quantum Mechanics

- In 1964, John Bell described a measurement which distinguishes quantum mechanics from hidden variable theory

- Typically shown with two particle entanglement
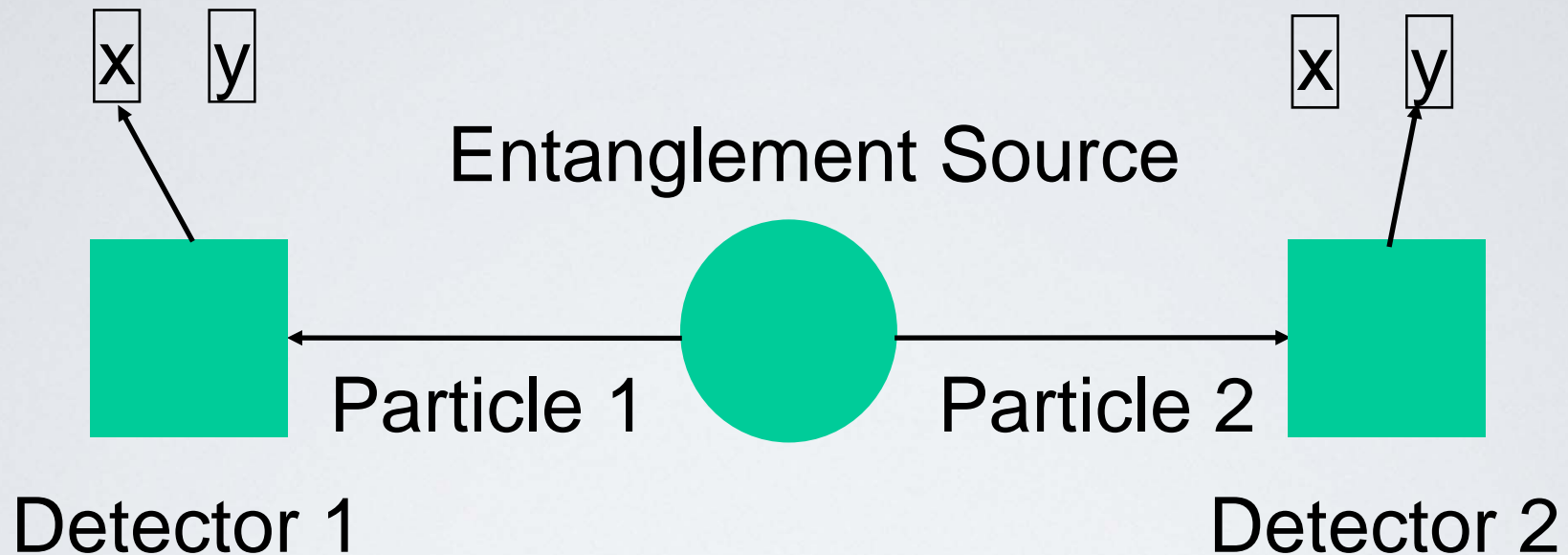
- This difference is statistical in nature (1/3 vs. 1/4)

No variable is hidden from John Bell's gaze

Model Experiment for Testing Bell's Inequality

Measurement Axis

Measurement Axis

x y

x y

Entanglement Source

Particle 1

Particle 2

Detector 1

Detector 2

A Greenberger-Horne-Zeilinger State consists of n entangled particles

- Each particle is a two-level system, such as photon polarization or spin states

- For n=3, with $|0>$ the first state and $|1>$ the second state,

$$|\phi>_{GHZ} = \frac{|000> + i|111>}{\sqrt{2}}$$

- Key feature of GHZ state: measurement of any particle leaves the system unentangled

- Mermin's paper focuses on applying hidden variables to this state

GHZ States & Classical Conflict

- GHZ spin states create "all-or-nothing" locality test

  - Need ideal detectors...

- ...but GHZ states permit arbitrarily-large AM/locality deviation

- "Cooking-up" appropriate n-spin operators shows this explicitly

Compute Traditional Quantum Expectation Values

- Imagine operator $\hat{A}$ with spin eigenstates $|\Phi\rangle$ such that, for n particles:

$$\hat{A} = \frac{1}{2i}\left[\prod_{j=1}^{n}(\sigma_x^j + i\sigma_y^j) - \prod_{j=1}^{n}(\sigma_x^j - i\sigma_y^j)\right]$$

$$\langle\Phi|\hat{A}|\Phi\rangle = 2^{n-1}$$

- Correlation measurements make this expectation value experimentally accessible

Results with Local Variables Theory

- If we have a set of local variables λ then our eigenvalues for operator $\hat{A}$ are:

$$\langle \hat{A} \rangle = 2^{n/2} \quad \text{(even n)}$$

$$\langle \hat{A} \rangle = 2^{(n-1)/2} \quad \text{(odd n)}$$

- With imperfect detectors, these become inequalities:

$$\langle \hat{A} \rangle \leq 2^{n/2}, \quad \langle \hat{A} \rangle \leq 2^{(n-1)/2}$$

No Limit to Quantum/Local Variables Theory Disagreement

- Exponential divergence between QM and local variables formulation

- "No limit" to the amount of possible disagreement

- Overall state is "macroscopically-distinct" - definitely spin-up or spin-down

How has the paper impacted the physics community?
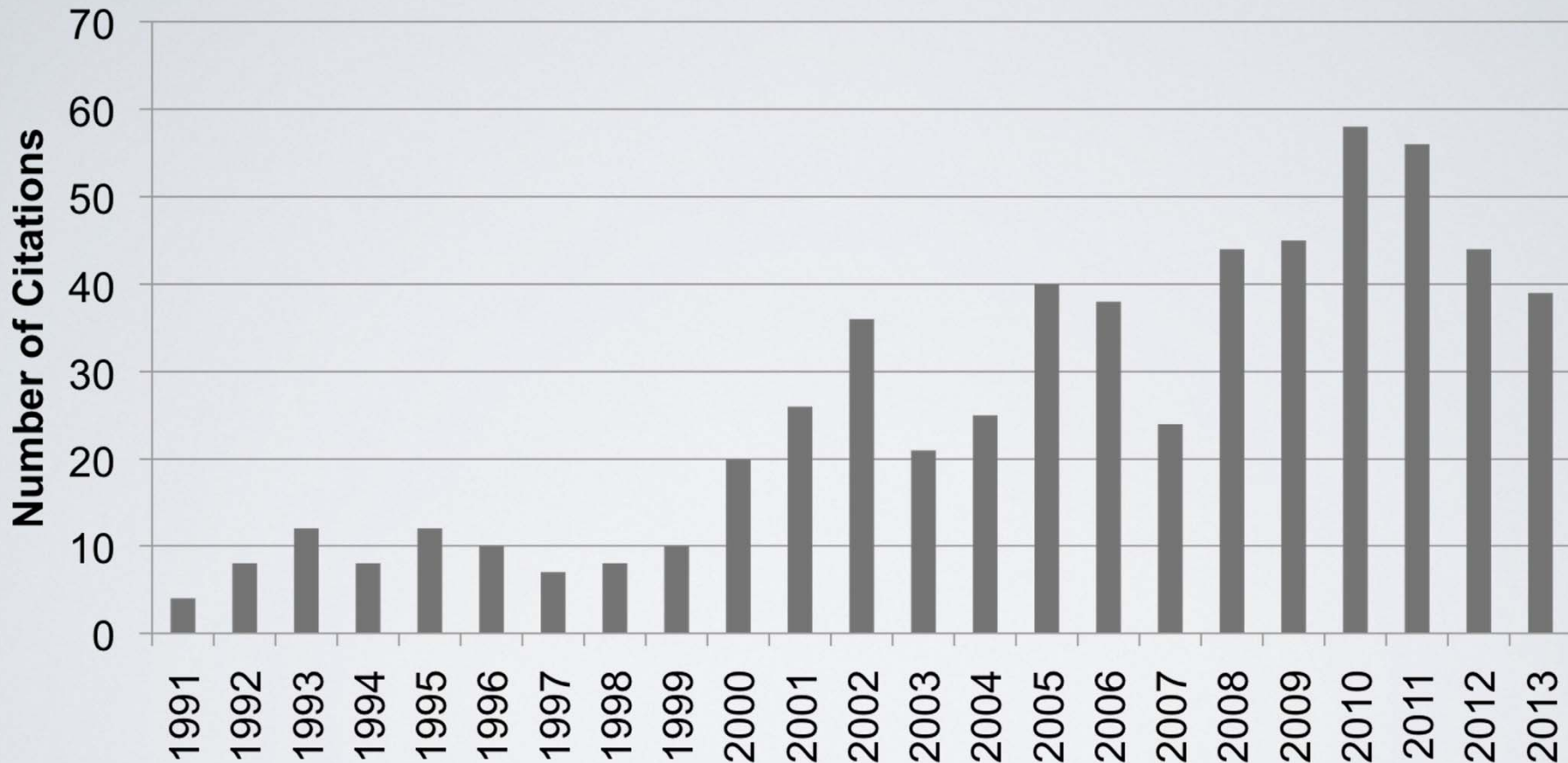
The paper has been cited ~570 times

Significant Citations: Experimental Realization

- "Experimental test of quantum nonlocality in three-photon Greenberger-Horne-Zeilinger entanglement" by Pan et al. in *Nature,* **403** (2000)

  - Experimental confirmation of quantum predictions for Greenberger-Horne-Zeilinger states by measuring the polarization correlations between three entangled photons

- "Experimental entanglement of four particles" by Sackett et al. in *Nature,* **404** (2000)

  - Implemented an entanglement technique to generate entangled states of two and four trapped ions

  - Technique enabled multi-particle entangled states to be created with vastly greater stability and certainty than existing experimental methods.
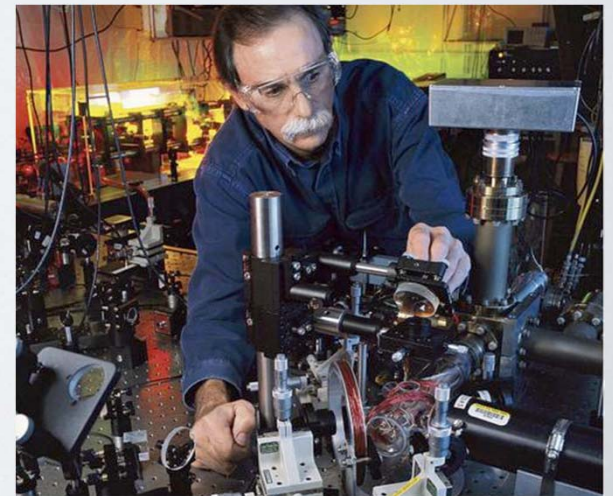
# What about developments in more recent years?

- "Preparation and measurement of three-qubit entanglement in a superconducting circuit" by DiCarlo et. al in *Nature*, **467** (2010)

  - Most cited paper since 2010 (148 times), the year with the most papers to cite Mermin's paper.

  - First to experimentally achieve entanglement in a superconducting circuit with more than two qubits (three in this case).

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  - Marked a new direction of research

- "Deterministic entanglement of superconducting qubits by parity measurement and feedback" by Ristè et al. in *Nature*, October 2013
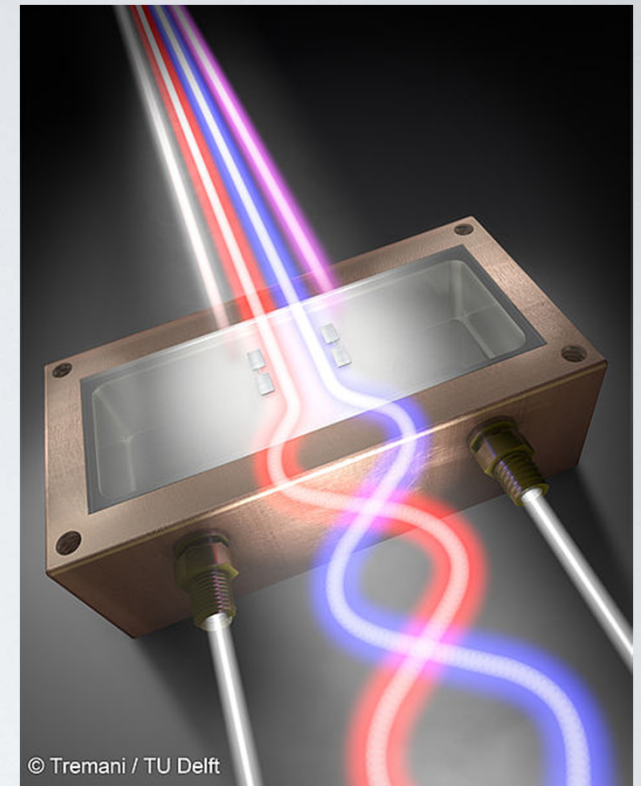
Critiques And Conclusions

- Critiques:

  - Ignores time dependence of local variables

- Conclusion: The prediction of hidden variable theory and quantum mechanics diverge exponentially with particle number.