A Report on:
Measurement of the charge and current of magnetic monopoles in spin ice,
Bramwell *et al*., Nature 461, 956-959 (2009)

November 15th, 2013

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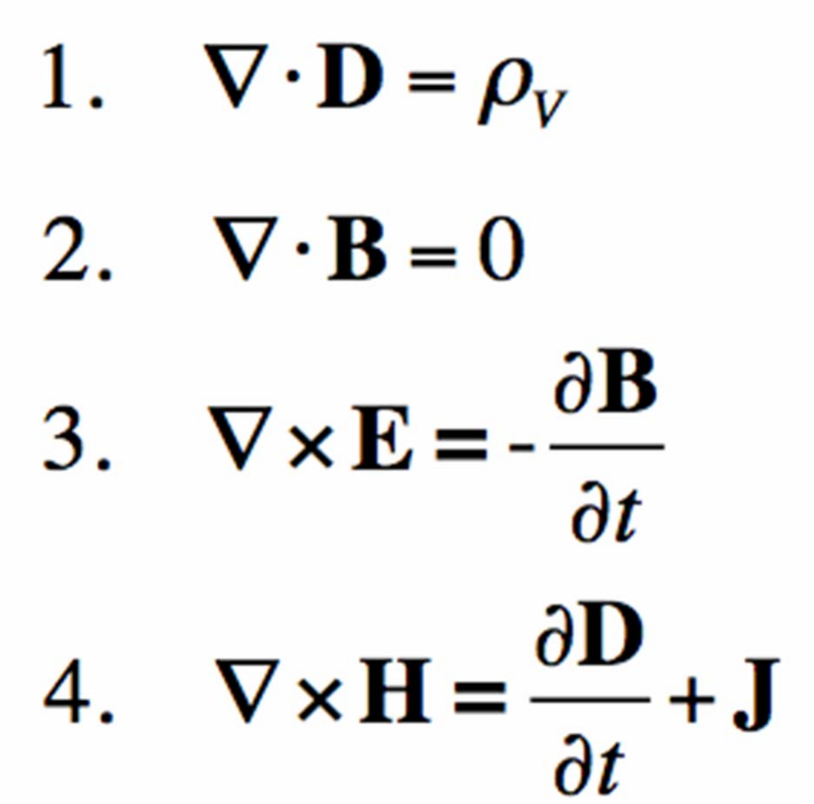
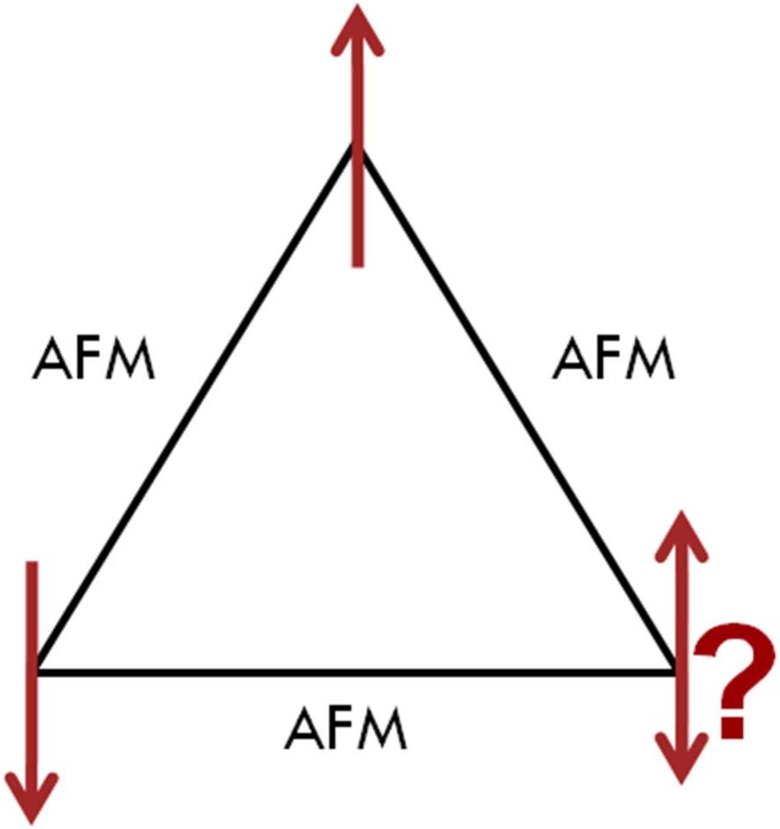
Magnetic monopoles would restore symmetry to Maxwell’s equations

- Elementary electric charge is abundant; what about elementary magnetic charge?
- Modern theories of physics predict magnetic monopoles



1. $\nabla \cdot \mathbf{D} = \rho_V$
2. $\nabla \cdot \mathbf{B} = 0$
3. $\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$
4. $\nabla \times \mathbf{H} = \dfrac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$

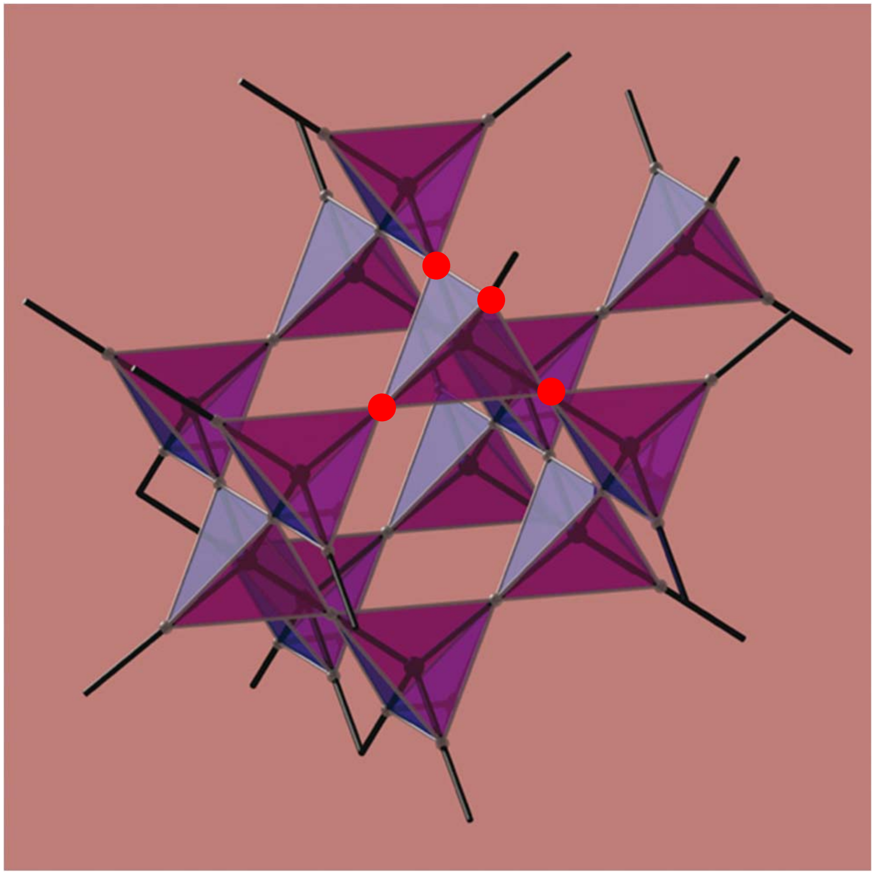
Geometric frustration leads to multiple ground state configurations



- Consider a triangle of electrons with spins connected by anti-ferromagnetic bonds
- Spins want to antialign
- Lower right spin wants to be in both the up state and down state

Spin ice is a class of geometrically frustrated magnets

- Notable spin ice: $Dy_2Ti_2O_7$
- Dy^{3+} ions are vertices of corner sharing tetrahedral lattice



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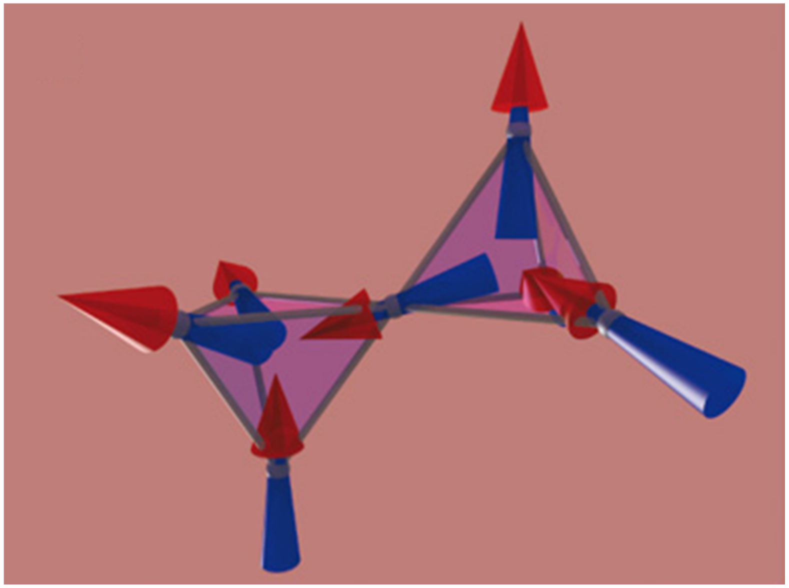
Dy^{3+} ions have magnetic moments

- Magnetic moments point toward or away from tetrahedra centers
- 2 moments pointing in and 2 moments pointing out of each tetrahedron is ground state configuration ("2-in 2-out")



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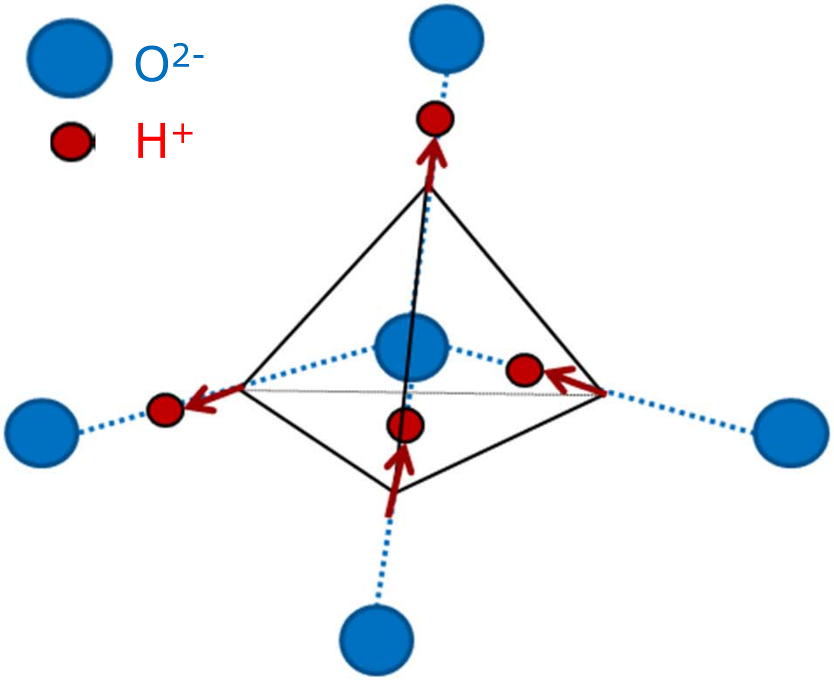
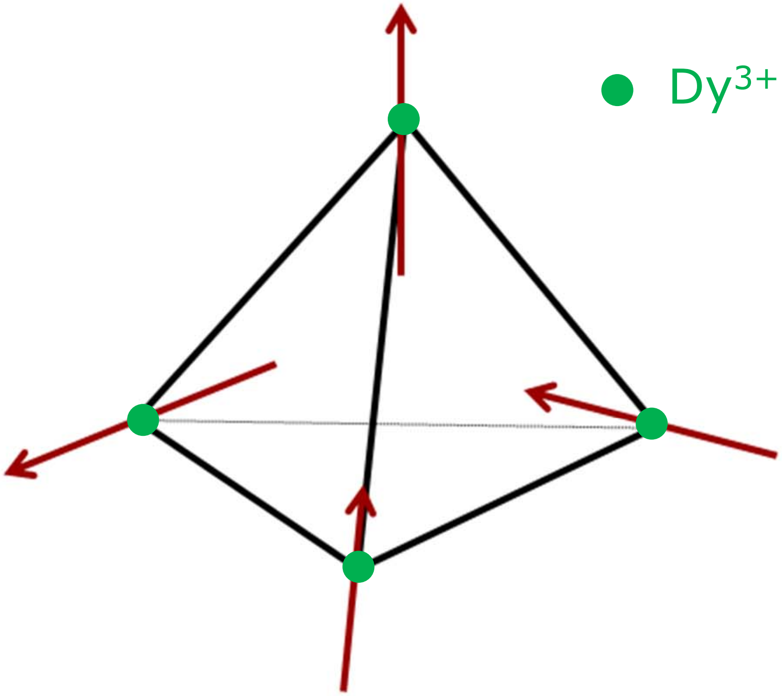
Spin “ice” is analogous to water ice

Water Ice

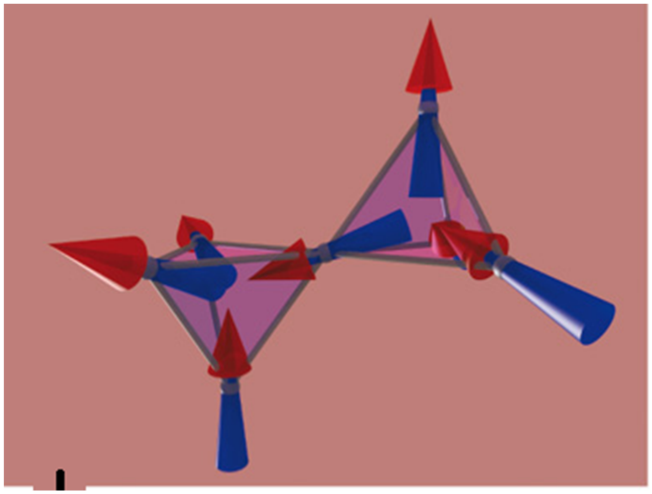


Spin Ice



Model magnetic moments as "dumbells" of magnetic monopoles

Spin-ice in its ground state. All tetrahedrons in "2-in 2-out" configuration



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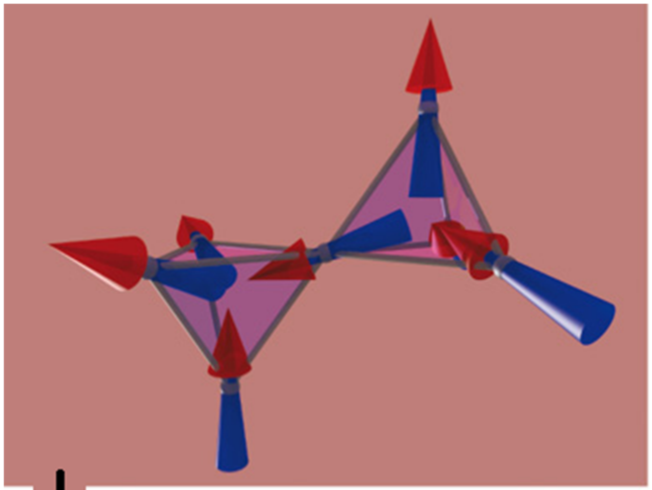
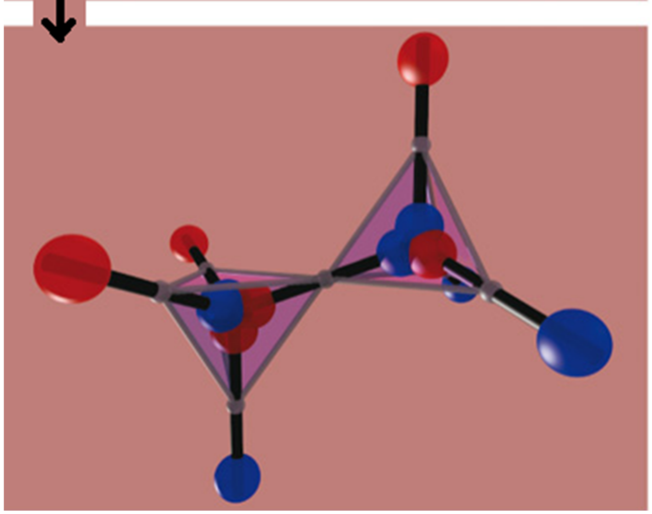


When magnetic moments are modeled as monopole dumbells, each tetrahedron has zero magnetic charge at center



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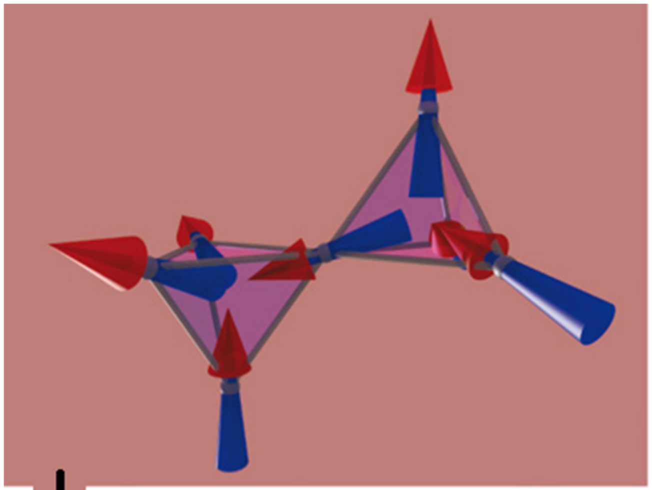
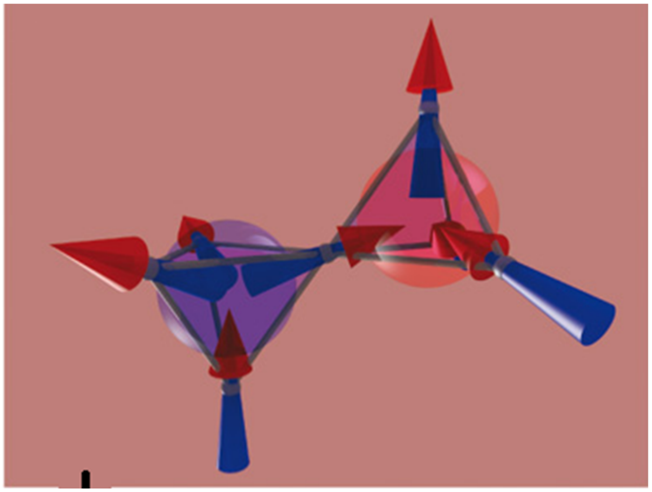


Excited spin-ice, with “3-in 1-out” tetrahedron next to “1-in 3-out” tetrahedron



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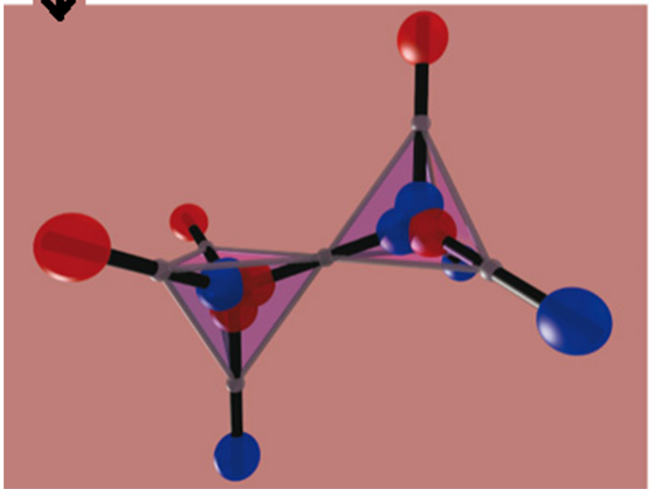

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Excited spin-ice, with “3-in 1-out” tetrahedron next to “1-in 3-out” tetrahedron

Spin-ice in its ground state. All tetrahedrons in “2-in 2-out” configuration

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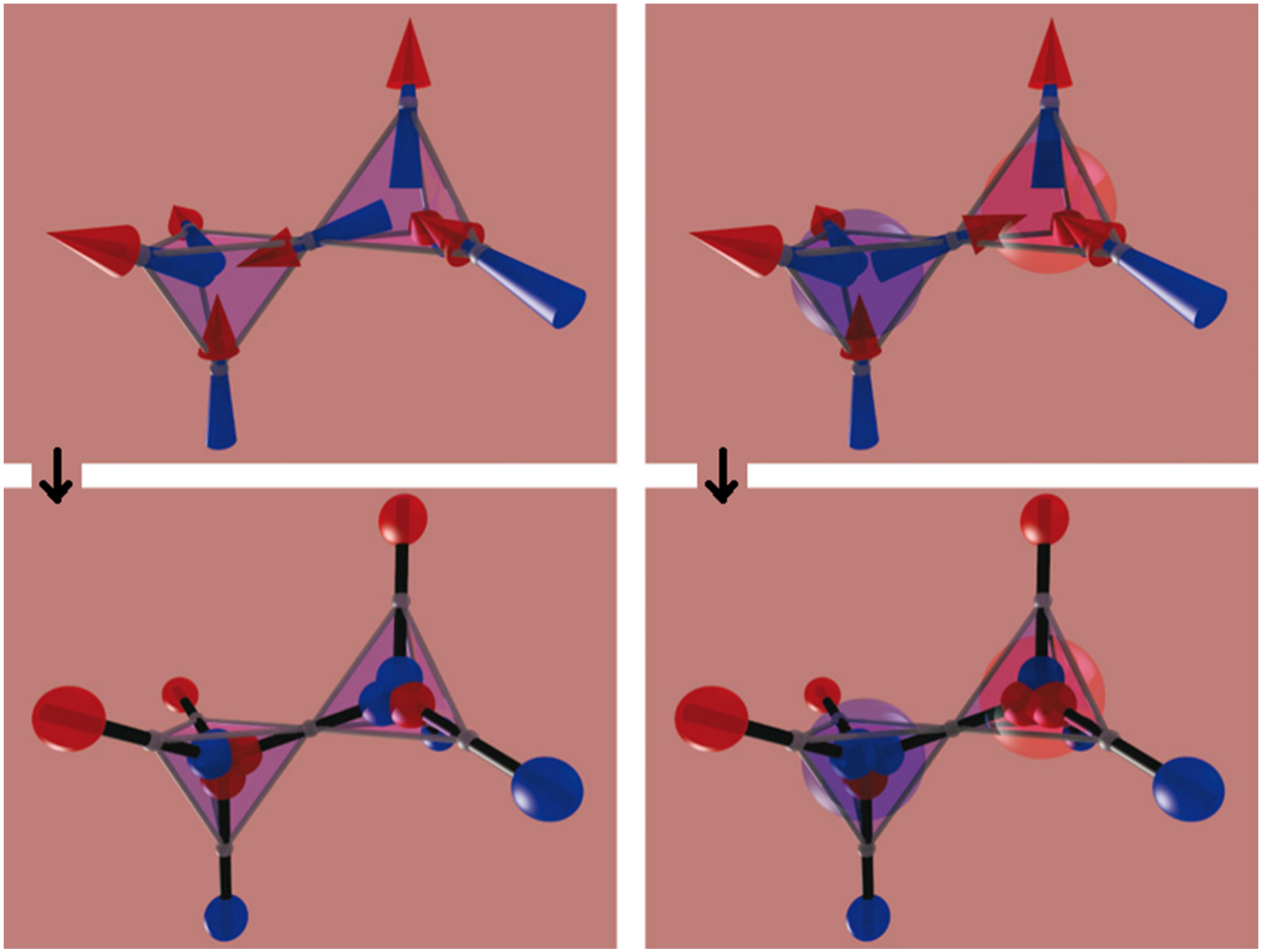
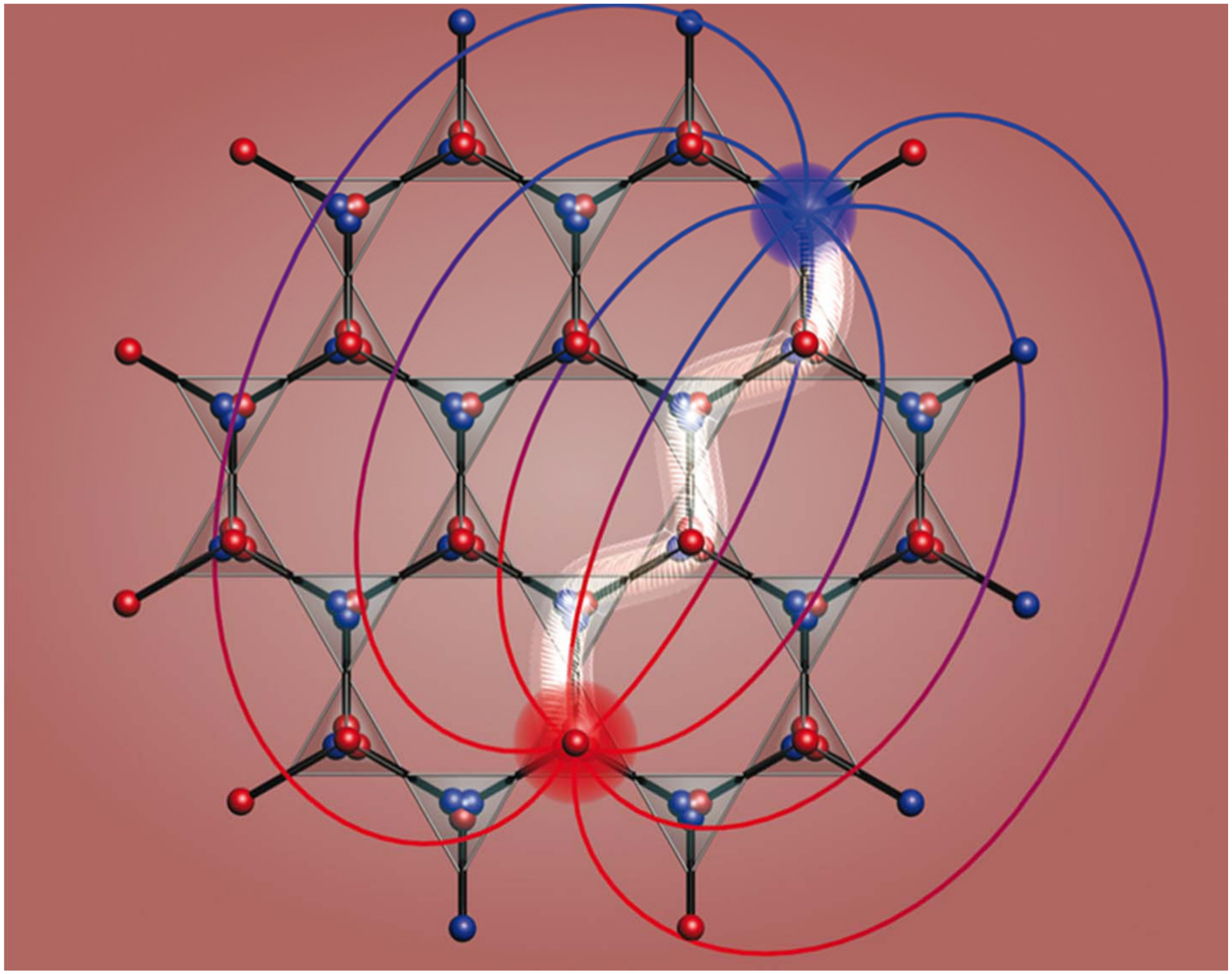


Now dumbell model gives net magnetic charge at center of each tetrahedron

Successive magnetic moment flipping produces magnetic monopole behavior

If you successively flip the magnetic moments along the white line, the magnetic charges separate further



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Magnetic Analogue to Onsager's Theory

Onsager's Theory states that the dissociation constant of a weak electrolyte (K) increases with increasing applied E-field.

$$\frac{K(E)}{K(0)} = 1 + \frac{q_e^3 E}{\epsilon_0 8\pi k_b^2 T^2}$$

$$q_e \rightarrow q_b$$

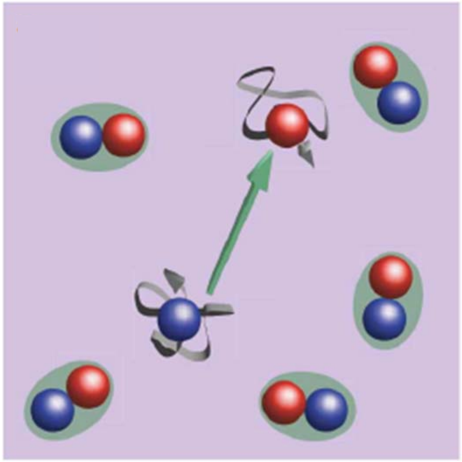
$$\epsilon_0 \rightarrow 1/\mu_0$$

$$E \rightarrow B$$

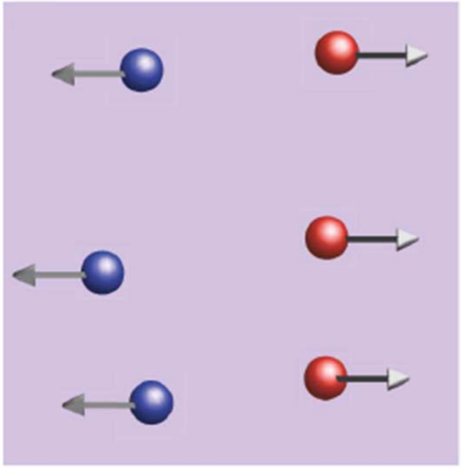
$$\boxed{\frac{K(B)}{K(0)} = 1 + \frac{\mu_0 q_b^3 B}{8\pi k_b^2 T^2}}$$

Applied B-Field Separates Monopoles

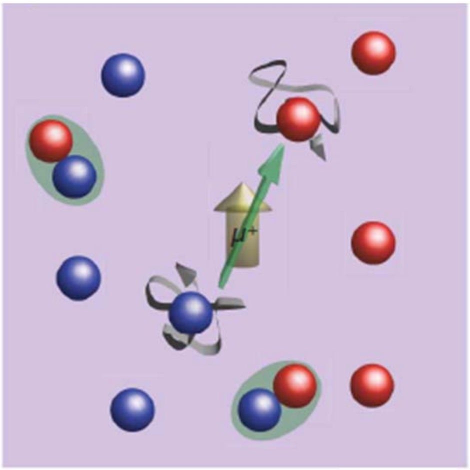
A magnetic field is applied briefly to spin ice, and then the system is allowed to relax back to equilibrium.



Applying a B-field separates magentic dipoles.



Monopoles move around (“conduct”) and recombine.



The amount of dissociation and relaxation rate depends on strength of applied field.

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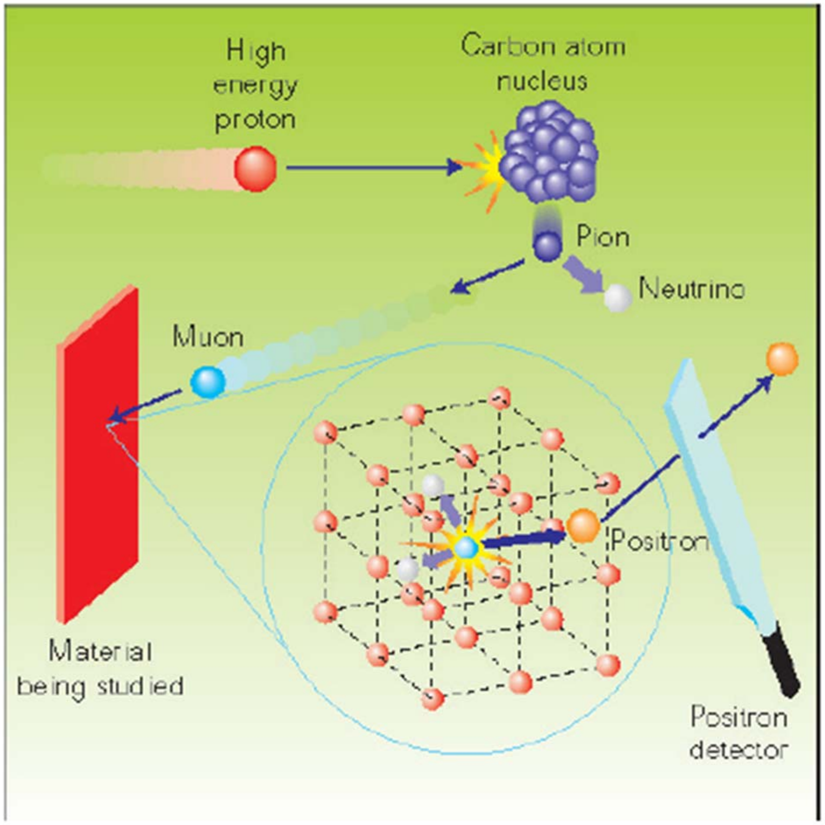
Experimental Method: Muon Spin Spectroscopy

- Muons injected into sample
- Muons precess about local B-fields, and decay into positrons
- Direction positrons get ejected tells us info about monopole movement



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Decay of Muon Precession Directly Related to Magnetic Charge

$$\boxed{\frac{\lambda(B)}{\lambda(0)} = \frac{K(B)}{K(0)} = 1 + \frac{\mu_0 q_b^3 B}{8\pi k_b^2 T^2}}$$

λ= relaxation rate of muon precession

K= dissociation constant

q_b= magnetic charge

Magnetic Charge Determined

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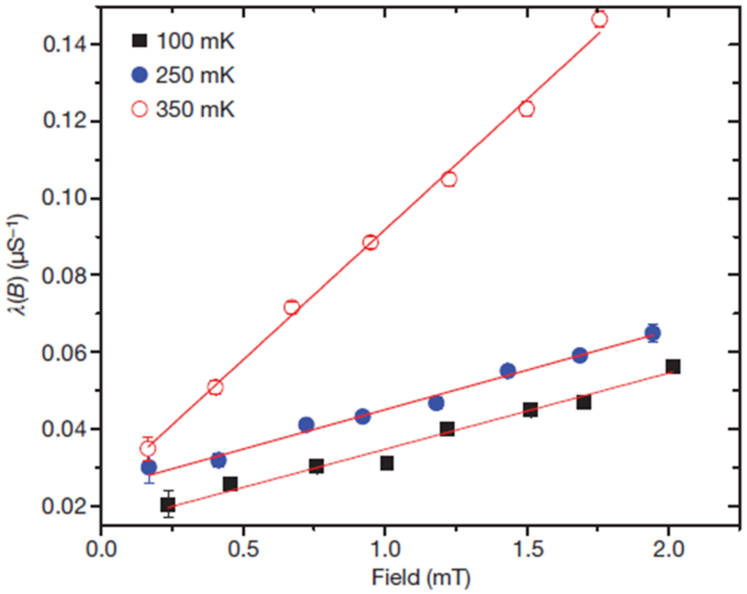
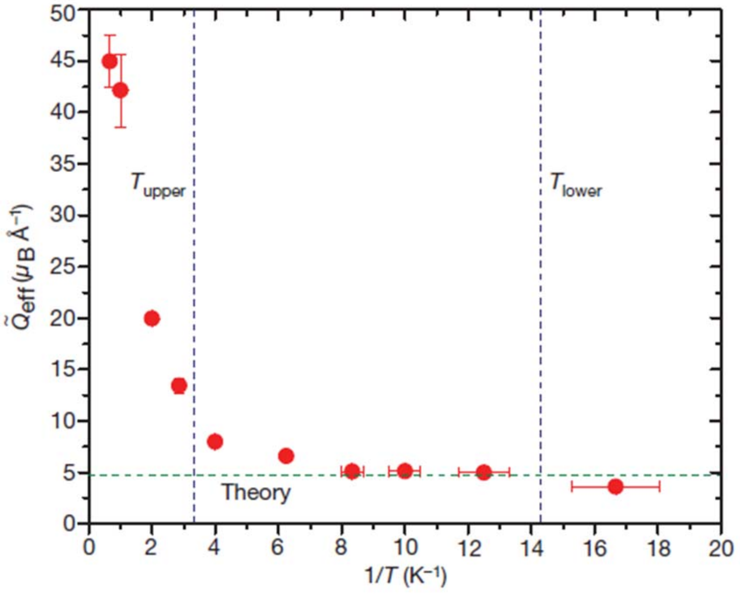
Bramwell et al. 2009





1. Magnetic conductivity depends on B-field strength

2. Using Onsager’s Theory, Q_B can be determined.

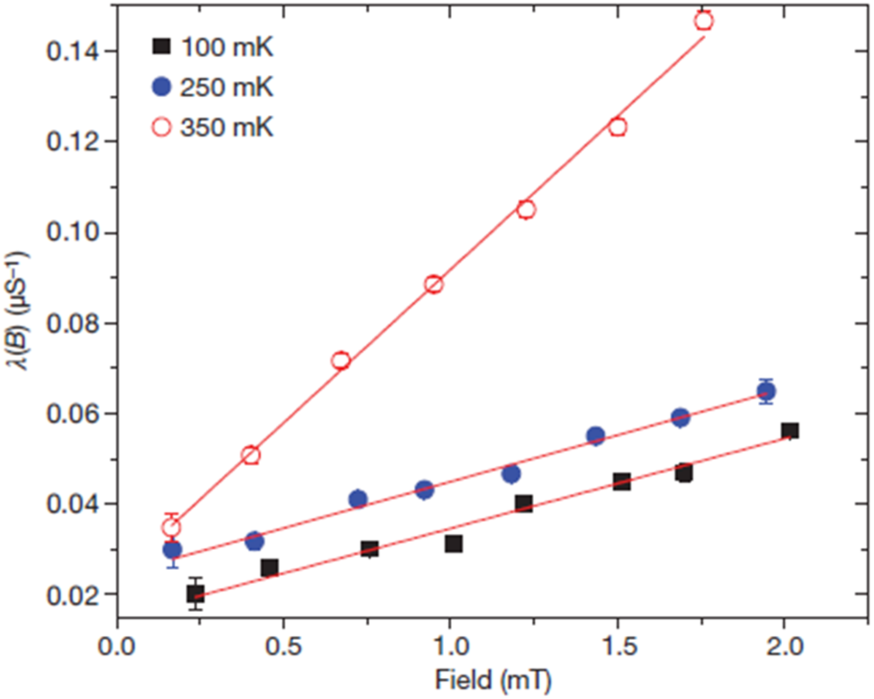
3. $Q_B \approx 4.6\ \mu_B/\text{Å}$

Double Checking Results: Reproducing Authors' Calculations

- $Q_B = 2.1223m^{1/3}T^{2/3}$
  - Q_B is the monopole charge
  - m = slope / intercept
  - T is the temperature

- For 100 mK curve:
  - $Q_B \approx 4.8\ \mu_B/\text{Å}$

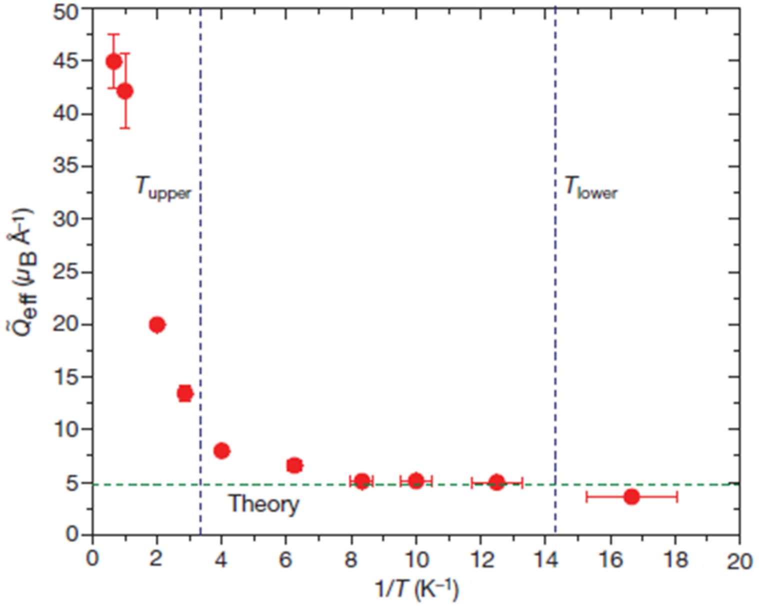
- For 200 mK curve:
  - $Q_B \approx 6.6\ \mu_B/\text{Å}$



Determined Monopole Charge Matches Theory in Certain Temperature Range

- Theoretical value of monopole charge:
  $Q_B = 4.6\ \mu_B/\text{Å}$

- Theory estimated to work in the temperature range $0.07\ K < T < 0.3\ K$, with these bounds somewhat arbitrarily determined by data

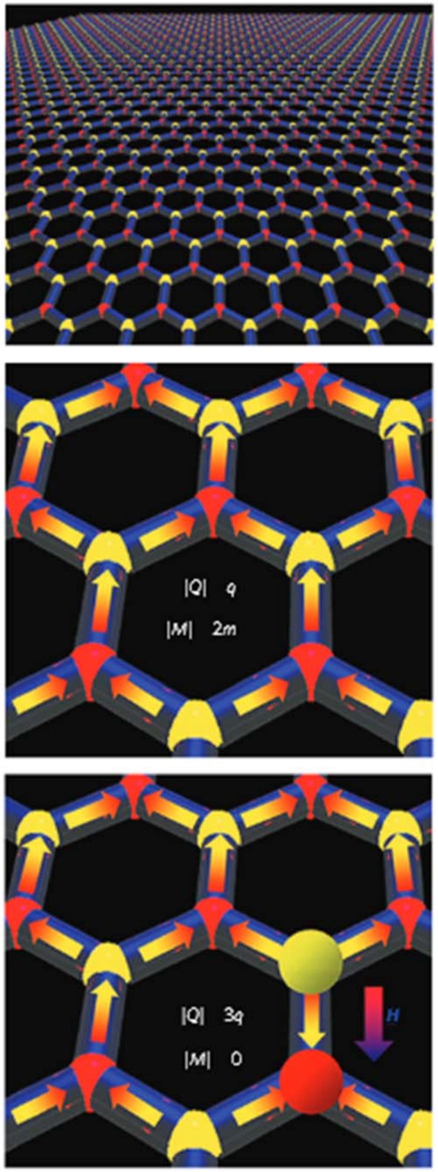


Are there alternative explanations to the data besides monopoles?

- Evidence of magnetic conductivity and thus magnetic charge, but are they actually monopoles? The authors do not discuss possible alternatives
- Monopoles in spin ice are quasiparticles, so not quite the monopoles budding physicists have been dreaming about since E&M

Field Progression

- 117 citations according to Scopus (including 36 in 2012 and 26 so far in 2013)
- Further probing nature of monopoles in spin ice; stray field effects of monopoles (Blundell 2012), flux quantization due to monopole currents (Chen *et al*. 2013), narrowing down mechanism of monopole motion (Bovo *et al*. 2013)
- Artificial spin ice is a mesoscopic material that mimics that properties of spin ice (Wang *et al*. 2006)
- Studies of monopole defects in artificial spin ice; Mol *et al*. 2010, Ladak *et al*. 2010



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Conclusions

- "Magnetic charges exist"
  - Interact by Coulomb's law
  - Accelerated by applied field
- "Monopole currents exist"
  - Alternating currents achievable?
- Demonstrate equivalence of electricity and magnetism
  - Comparing Onsager's theory with experimental results

Our Conclusions

- Method and theory are reasonable
- Open about approximations and assumptions
- Clear writing and flow of logic