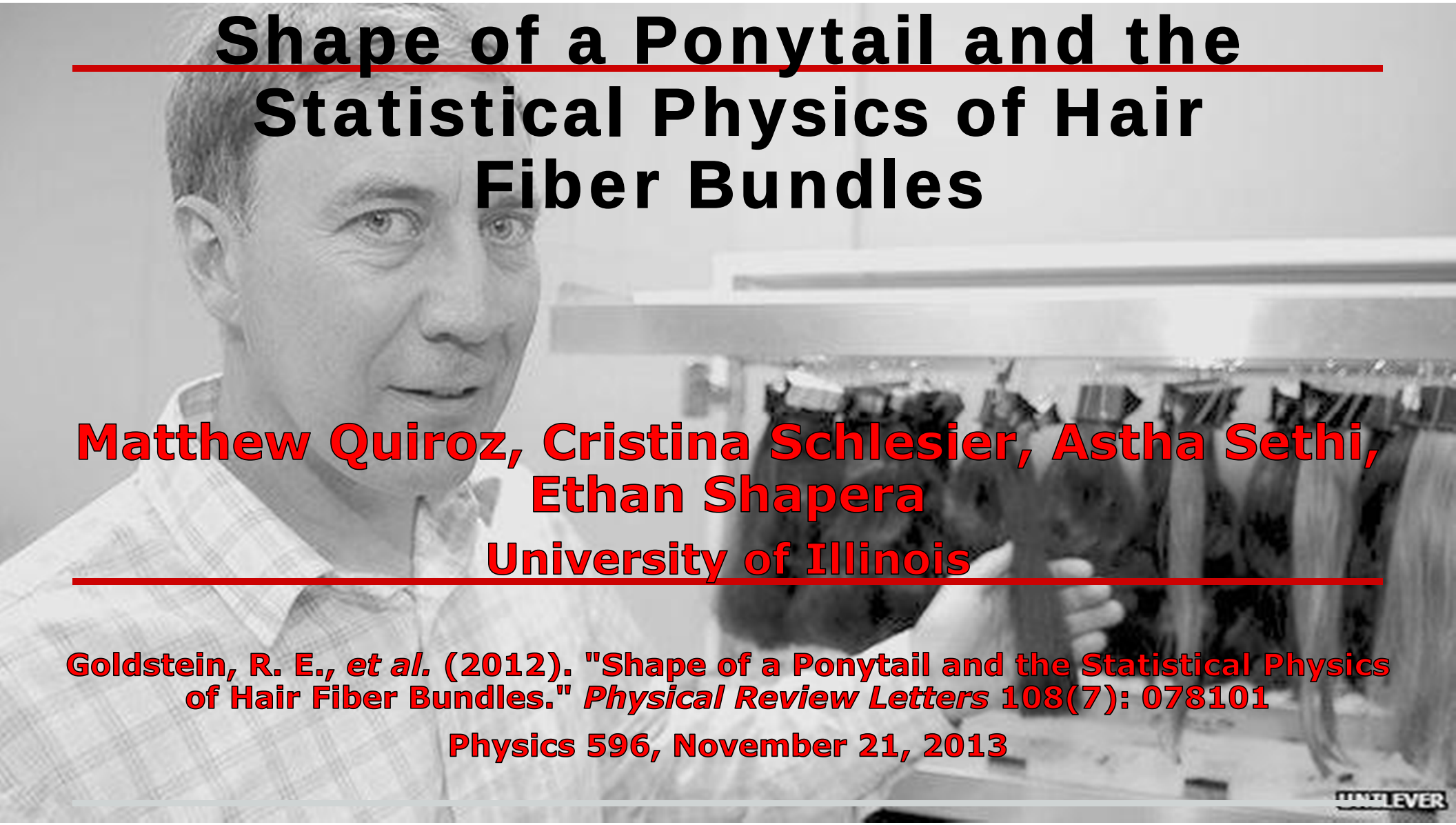

Shape of a Ponytail and the Statistical Physics of Hair Fiber Bundles

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Goldstein, R. E., *et al.* (2012). "Shape of a Ponytail and the Statistical Physics of Hair Fiber Bundles." *Physical Review Letters* 108(7): 078101

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Outline

- Background and Motivation
- Overview
- Previous work
- Methods and Results
- Critique
- Conclusion

Why study the shape of hair?

- Hair has fascinated artists and scientists for centuries:
  - "Observe the motion of the surface of the water which resembles that of hair, and has two motions, of which one goes on with the flow of the surface, the other forms the lines of the eddies..." - Leonardo Da Vinci[1]
  - "Rapunzel, Rapunzel, let down your hair, so that I may climb the golden stair." - The Witch
- Studies on ponytail motion have modeled the ponytail as a pendulum or flexible string[2].<u>But a ponytail is not a piece of string.</u>

1.*The Notebooks of Leonardo da Vinci*, edited by J. P. Richter (Dover, London, 1989).
2. J. B. Keller, SIAM J. Appl. Math. 70, 2667 (2010).
http://imgc.allpostersimages.com/images/P-473-488-90/62/6291/CVV5100Z/posters/leonardo-da-vinci-head-hair-and-costume-studies-for-leda-art-poster-print.jpg

How do individual fibers determine the shape of a ponytail?

- This is a problem of statistical mechanics: There are $O(10^5)$ hairs…

Answering the Question

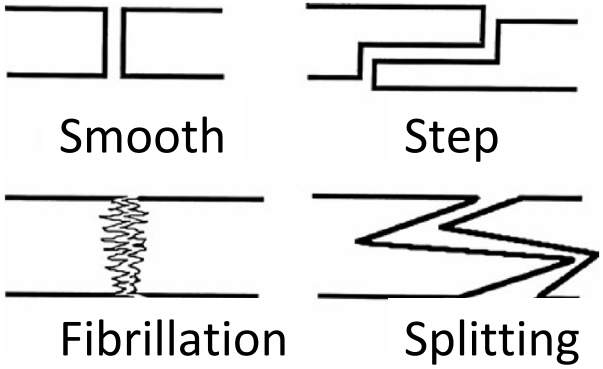
- General continuum theory for the distribution of hairs in a bundle
- Consider envelope rather than individual hairs
- Consider combined effects of gravity, elasticity, orientational disorder and tension
- Experimentally determine an equation of state

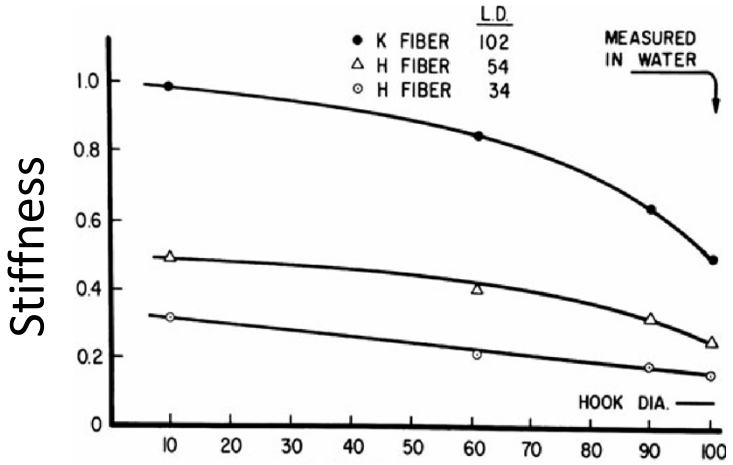
Defining Features of a Ponytail

- Body or volume of a ponytail
  - arises from stiffness and shapes of individual fibers
  - individual paths
    - collisions with other hairs
    - intrinsic waviness

Mechanical Properties

- Stress-strain versus relative humidity for single hair
- Fracture patterns
- Stiffness versus relative humidity
- Effect of chemical treatment
- Considered only single hair fiber





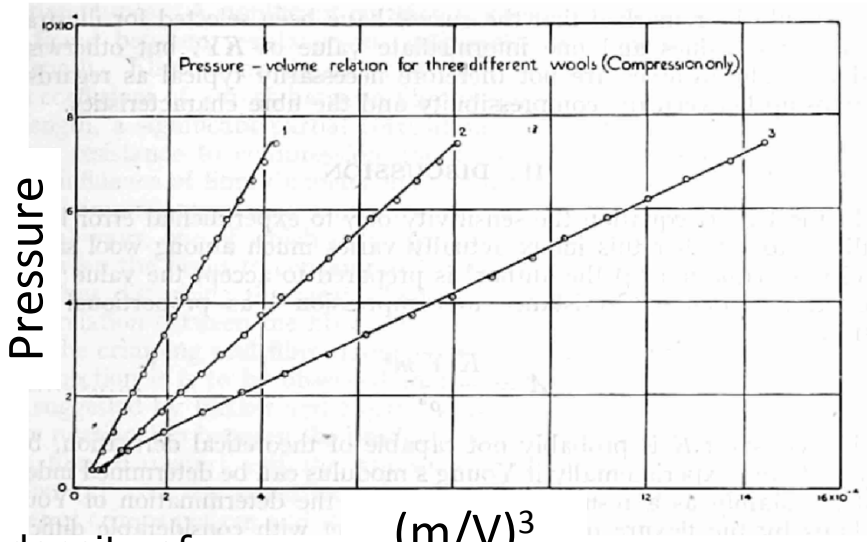
C. R. Robbins, Chemical and Physical Behavior of Human Hair (Springer-Verlag, New York, 2002).

An Earlier Model – Compressibility of Wool

- Considered wool as a system of rods with random orientation.
- Averaged over large number of rods to model as layered material.
- Equation of state:

$$p = \frac{K\,Y\,m^3}{\rho}\left(\frac{1}{V^3} - \frac{1}{V_0^3}\right)$$

C. M. vanWyk, J. Text. Inst. Trans. 37, T285 (1946).



Where: ρ = density of uncompressed wool

Y=Young's Modulus

m=mass of bundle

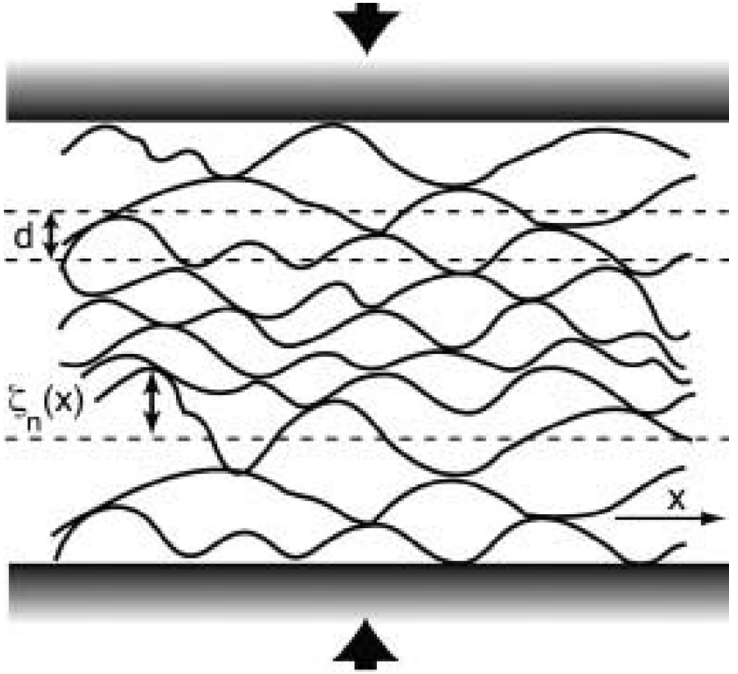
V_0=uncompressed volume

V=compressed volume

p=interaction pressure

Fiber Collision Model

- 2D bundles of large number of fibers
- Interaction energy due to collision between nearest neighbors
- Predictions of collective properties from individual fibers
- Generalizable to 3D

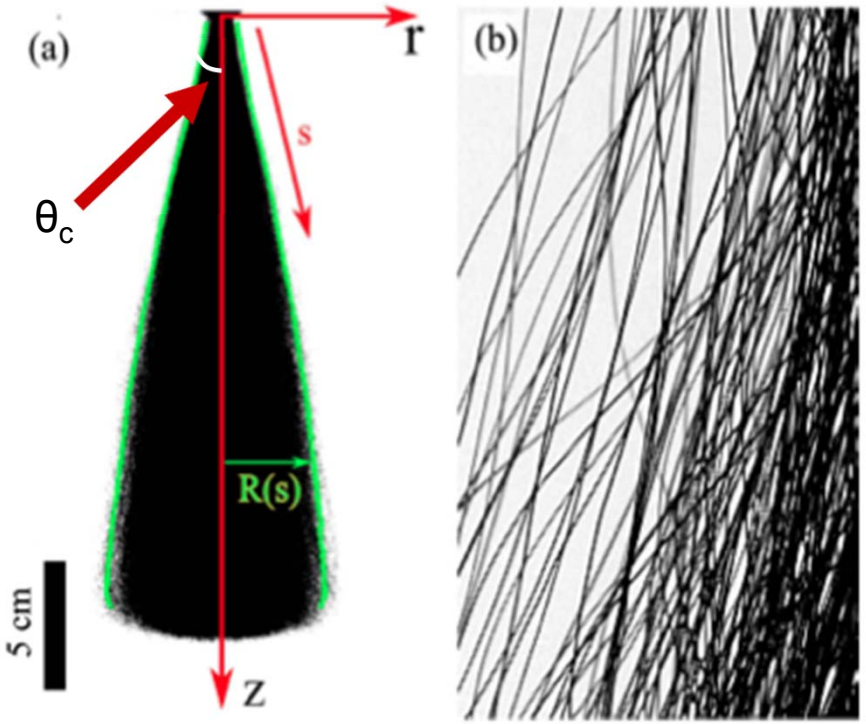


Colliding fibers subject to applied pressure

P. Beckrich, G. Weick, C. M. Marques, and T. Charitat, Europhys. Lett. 64, 647 (2003).

Overview of Methods

- Equation of state
- Exploit predominantly vertical alignment of hairs and symmetry
- Reduces the many body problem to a one body problem for a "ponytail envelope"



Energy Equation

Gravitational potential

Energy of fiber bundle : $\varepsilon[\rho, \boldsymbol{t}] = \int d^3\mathbf{r}\, \rho(\frac{1}{2}A\kappa^2 + \varphi(\mathbf{r}) + <u>)$

Elastic energy of curvature

Disorder, contact, Natural curvature etc.

Where: ρ = density of fibers

A=bending modulus

$\kappa = |(\mathbf{t}.\nabla)\mathbf{t}|$ = curvature field

$\varphi(\mathbf{r})$=external potential

<u>=fiber confinement energy per unit length

Euler-Lagrange Shape Equation

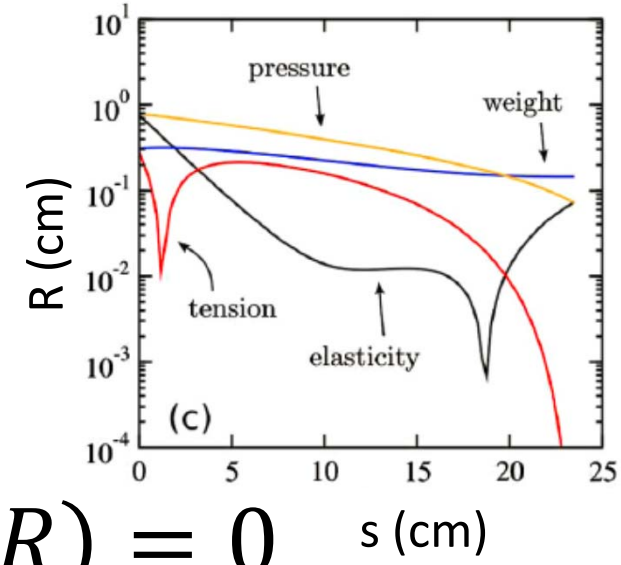
Elastic restoring force

Weight term

$$l^3 R_{ssss} - (L - s)R_{ss} + R_s - \prod(R) = 0$$

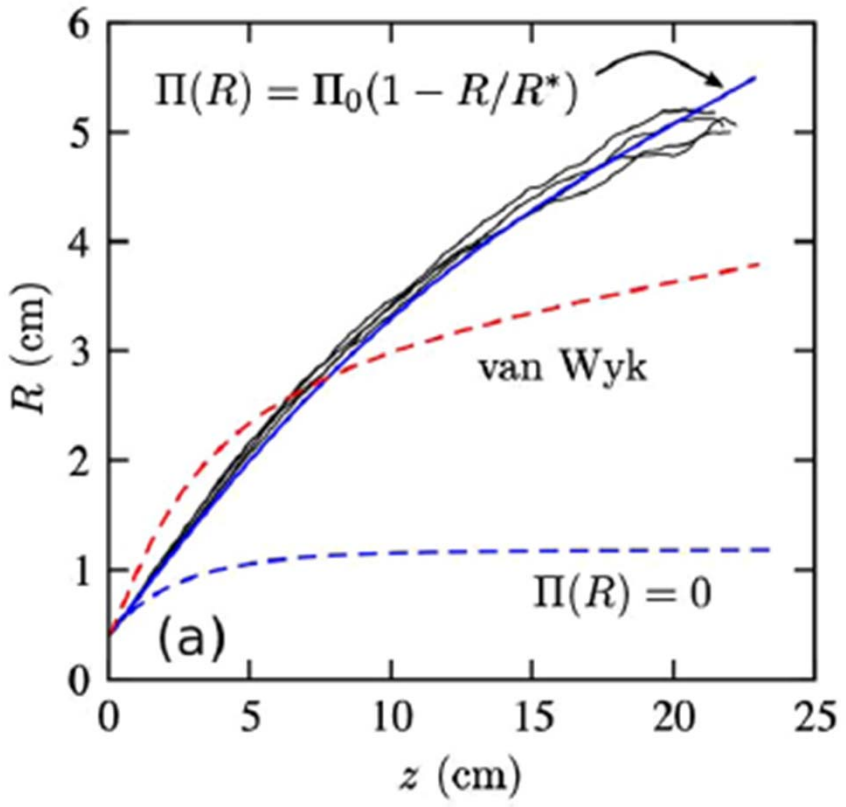
Tension term

Radial pressure

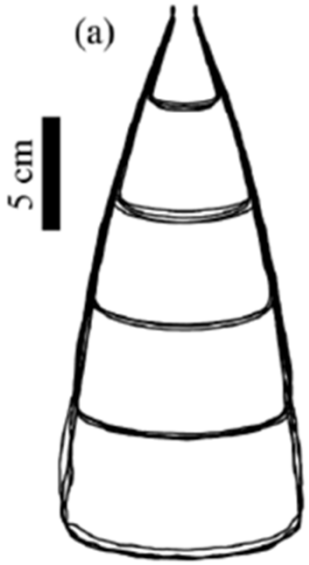


Match between the EOS and Experiment

- Obtain Π(R) by neglecting elastic restoring force
  Π_0 and R* are fitting parameters
- Plug Π(R) back into shape equation
- Predicted profiles of a ponytail:
  Dashed blue- No interaction
  Dashed red- vanWyk model
  Solid black- Measured profiles
  Solid blue- Fitted equation of state



Verification of the Theory for Varying Length



Hair Switches

$$\Pi(R) = \Pi_0(1 - \frac{R}{R^*})$$

$$\Pi(R) = \Pi_0\left(1 - \frac{R}{R^*}\right)(\frac{2\,s}{L^*})$$

Ponytail envelope versus length cut down from 25 cm in steps of 5 cm. A) Experimental envelope shape. B) Predicted envelope and C) with extra compactification.

CRITIQUE

- ✓ Unique approach
- ✓ Theory and Experimental data match

- ? Experiment carried out only for one particular clamping condition, $\theta_c = 17^{\circ}$
- ? Validity of neglecting elasticity
- ? Effect of chemical preparation

How did the field progress?

- 2012 IgNobel Prize
- "A Non-Linear Rod Model for Folded Elastic Strips"
  - Develops non-linear model of elastic annulus
  - Identifies buckling instabilities
- Elastic and Conformational Properties of Strictly Two-Dimensional Chains"
  - Characterizes thermal and conformal properties of polymer chains

Conclusions

- Statistical Mechanics approach
- Determining interactions is key to finding the shape of a ponytail
- Combined analytic and experimental methods to find shape equation and equation of state ponytail
- Effectively combine experimental and theoretical approaches