

# Is the proton as “big” as we thought?

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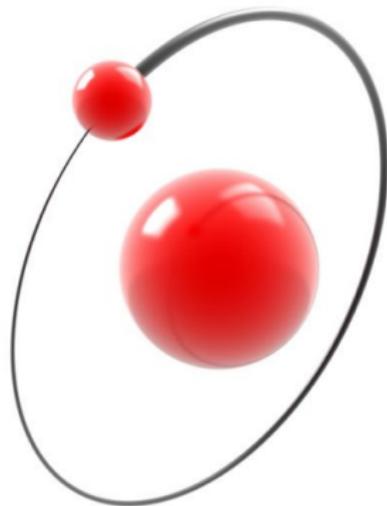
Journal Club Presentation  
PHYS 596

“Proton Structure from the Measurement of 2S-2P Transition Frequencies of Muonic Hydrogen”

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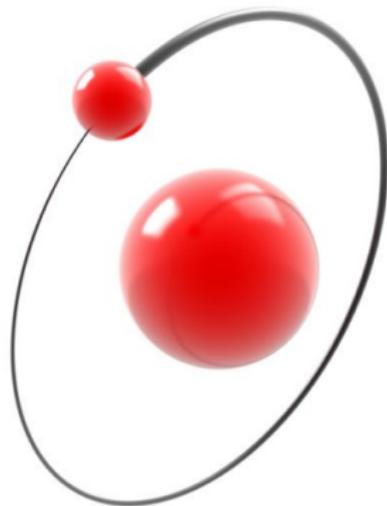
# The proton-size puzzle

- “Radius” of the proton can be *defined* and *determined* in various ways



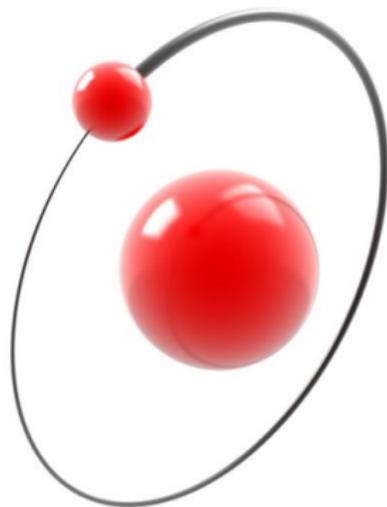
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- One attempt to do so runs into trouble - it seems inconsistent with everything else!

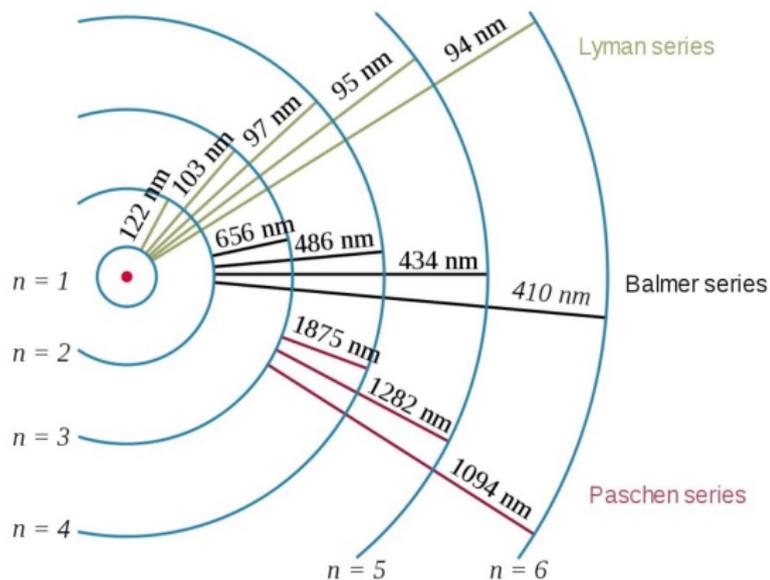


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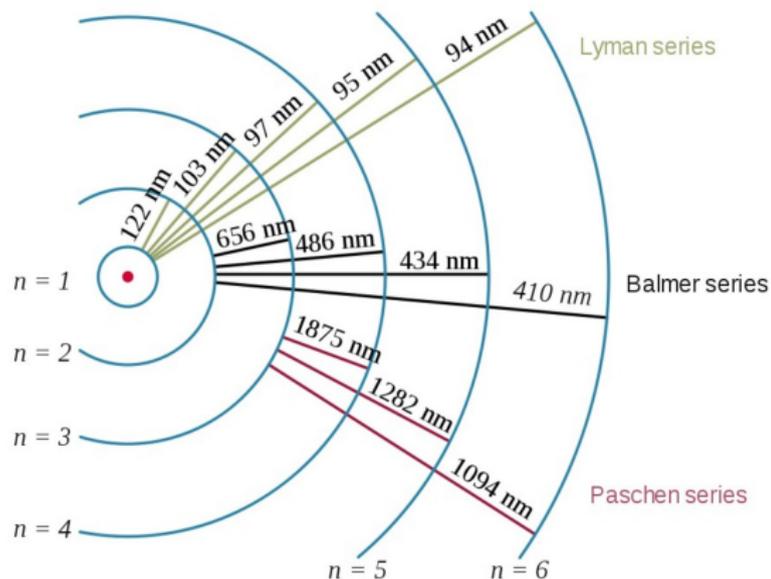
- “Radius” of the proton can be *defined* and *determined* in various ways
- One attempt to do so runs into trouble - it seems inconsistent with everything else!
- Is this a window for new physics, or another experimental goof-up?



# Preliminaries : how things got dirty!

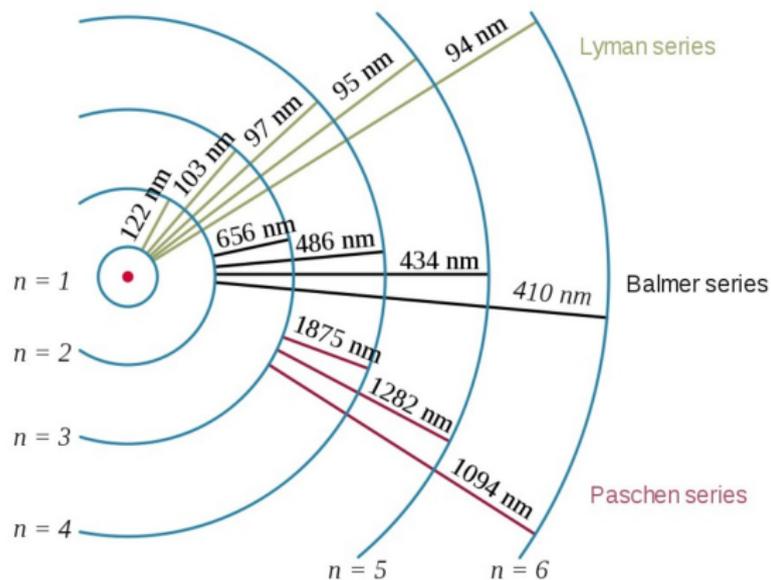


# Preliminaries : how things got dirty!



- ✓ Spin-orbit coupling
- ✓ Lamb<sup>†</sup> shift
- ✓ Hyperfine splitting

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† lamb = a small sheep ;)

## Digression : the spectroscopic notation

✓ Standard way to write down the angular momentum quantum numbers of a state.

$$N^{2S+1}L_J$$

$L$  : Orbital angular momentum (S, P, D, F ... for  $\ell = 0, 1, 2, 3, \dots$ )

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e.g. : single electron states (spin- $\frac{1}{2}$  system!), like in Hydrogen

$$1^2S_{\frac{1}{2}} \quad 2^2S_{\frac{1}{2}} \quad 2^2P_{\frac{3}{2}} \quad 2^2P_{\frac{1}{2}} \quad 3^2S_{\frac{1}{2}} \quad 3^2P_{\frac{3}{2}} \quad 3^2P_{\frac{1}{2}} \quad 3^2D_{\frac{5}{2}}$$

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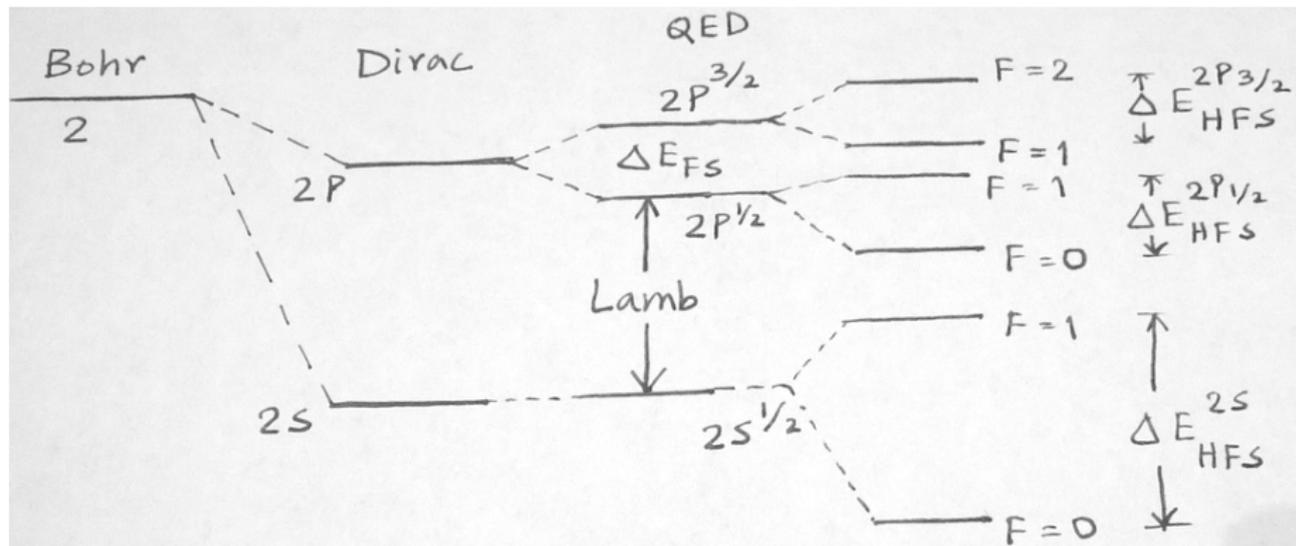


Fig: A subtle structure of the  $n=2$  level in hydrogen according to Bohr's, Dirac's and QED with Lamb Shift.

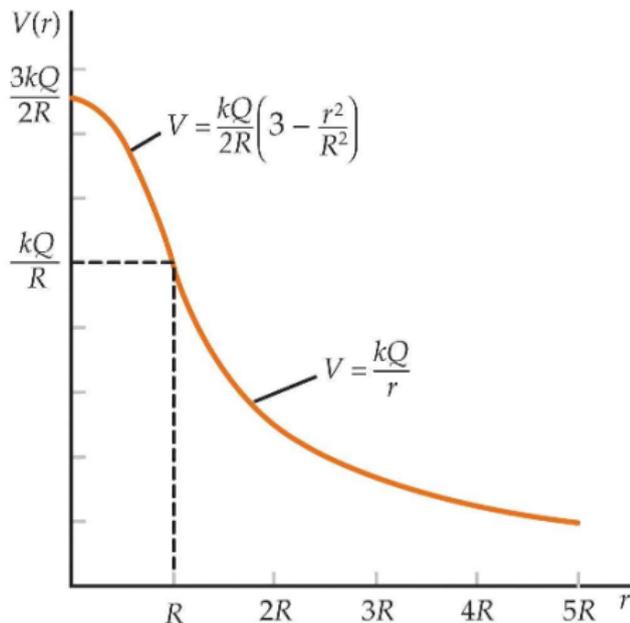
# Thou art not solid, proton!

- ✓ Finite probability of electron to be found inside the nucleus.

$$P(r \leq R) = \frac{1}{\pi a^3} \int_0^{2\pi} \int_0^\pi \int_0^R e^{-2r/a} r^2 \sin \theta \, dr d\theta d\phi \approx \frac{4}{3} \left( \frac{R}{a} \right)^3$$

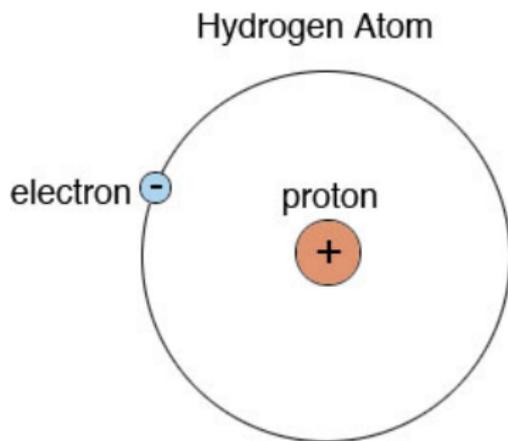
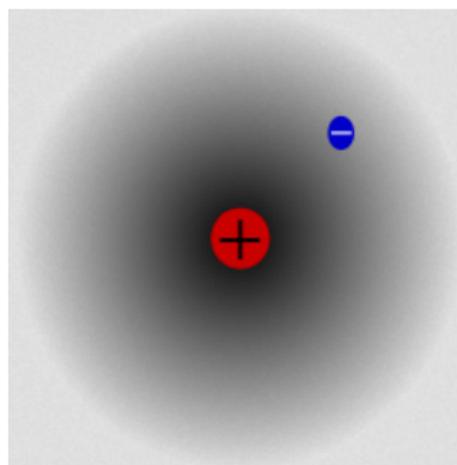
$V$  inside charge distribution is smaller than the corresponding field produced by a point charge.

$\implies$  measured transition frequencies depend on proton size!



## Two ways of defining the “radius” of the proton

- ✓ Neither atoms nor their nuclei have definite boundaries - we define a
- **Charge radius** ( $r_E$ , based on the distribution of charge) and a
  - **Zemach radius** ( $r_Z$ , reflects the spatial distribution of  $\vec{\mu}$  smeared out by  $\rho(\vec{r})$ ).



# Working with exotic atoms

- ✓ Historically,  $r_E$  and  $r_Z$  were determined using measurements of the **differential cross section** in elastic e-p scattering.
- ✓ A more accurate measurement is expected from **laser spectroscopy** of “Muonic Hydrogen”. (Why?)

Muon ( $e^-$ 's heavier twin) orbiting the proton instead of electron.

$$m_\mu = 207 m_e$$

$$r_\mu = \frac{1}{186} r_e$$

0.511 MeV	105.7 MeV
-1	-1
$\frac{1}{2}$	$\frac{1}{2}$
e	$\mu$
electron	muon

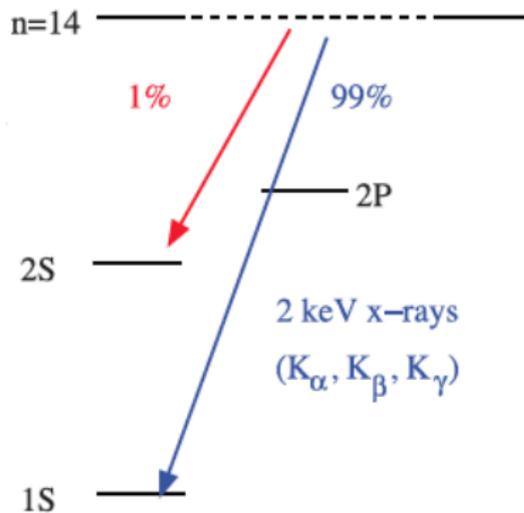
# The experiment

(We'll try not to make it boring!)



## Step 1/3: prepare muonic hydrogen in 2S state

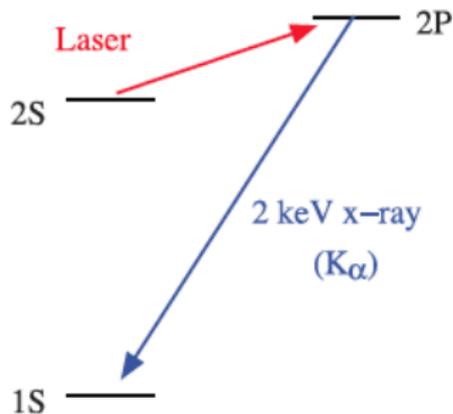
- ✓ Highly energetic  $\mu^-$  stopped in  $\text{H}_2$  gas
- ✓ Highly excited  $\mu$ -p atoms form ( $n \approx 14$ )
- ✓  $\sim 1\%$  populate long-lived 2S state



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## Step 2/3: induce $2S \rightarrow 2P$ transitions

- ✓ Laser pulse induces  $2S \rightarrow 2P$  transitions
- ✓ Immediately follows  $2P \rightarrow 1S$  de-excitation



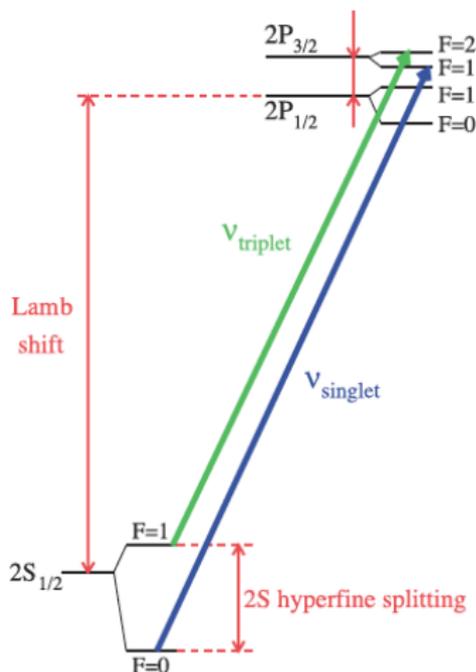
# Step 3/3: measure the transition frequencies, what else!

Two  $2S \rightarrow 2P$  transitions measured

$$\checkmark \quad \nu_s \equiv \nu(2S_{1/2}^{F=0} \rightarrow 2P_{3/2}^{F=1})^\dagger$$

$$\checkmark \quad \nu_t \equiv \nu(2S_{1/2}^{F=1} \rightarrow 2P_{3/2}^{F=2})$$

$$\dagger \quad N^{2S+1}L_J \quad F = J + I$$



## Do they conform with known data?

- ✓ Finite proton size significantly affects **Lamb shift** and **2S HFS**
- ✓ Some linear combinations of  $\hbar\nu_s, \hbar\nu_t, \Delta E_{\text{FS}}^{2P}, \Delta E_{\text{HFS}}^{2P}$  yield **Lamb shift** and **2S HFS**

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- ✓ From **2S hyperfine splitting**, we get  $r_Z$ , the **Zemach radius**.

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$$r_Z = 1.082(37) \text{ fm}, \quad r_Z^{\text{Friar}} = 1.086(12) \text{ fm}, \quad r_Z^{\text{Distler}} = 1.045(40) \text{ fm}$$

Experimental limit for measuring  $r_Z \sim 3.4 \%$

✓ So far, so good!

## Do they conform with known data?

- ✓ From **Lamb shift**, we get the **charge radius**.

$$\Delta E_L^{\text{th}} = \dots + (\dots) r_E^2 + (\dots)$$

$$r_E = 0.84087(39) \text{ fm}, \quad r_E^{\text{CODATA}} = 0.8775(51) \text{ fm}$$

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At  $7 \sigma$  variance with CODATA!

## Did we miss something?

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! Maybe we should try a different  $\rho(\mathbf{r})$ ?  $\implies$  changes  $r_E$  by less than the quoted uncertainty! (more on a later slide.)

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- ! Other possibilities (spectroscopy of  $pp\mu$  or  $\mu pe^-$  instead of  $\mu$ -p) can be excluded.
- ! Yet other recent e-p scattering measurements support the CODATA value.

## For the thinking layman . . .

- ? Is the  $\mu$ -p interaction different (in what way?) from the e-p interaction?
- ? Is this a window (albeit possibly small) of new physics?
- ? Do new force carriers (MeV-mass) exist? (In conformity with other results)

## More serious attempts at resolution

- × Gorchtein: uses finite-energy sum rule to find the correction to the proton-polarizability of  $-(40 \pm 5) \mu\text{eV}$ . Not enough to explain the  $300 \mu\text{eV}$  difference.
- × Griffith: uses bound-state field theory on proton structure in  $\mu$ -p. No positive results yet.
- ? Moumni: Corrects for the noncommutativity in space-space and space-time versions. Says discrepancy is solved by the corrections depending on  $m^3$  giving shifts in the spectrum. New paper though, hasn't been reviewed yet.

# Citation Evaluation

No. of citations : 24

- **13** new proposals, theories, or experiments : Trying to explain the “proton radius puzzle” .
- **2** were self-citations (by someone among the authors)
- **9** were papers using the determined values or similar techniques as this experiment.



# From the reviewer's POV

What we thought was good :

- Attention to detail - sources of experimental errors; possible sources of discrepancy
- Clear flow of reasoning



What we thought wasn't :

- Talk about changing  $\rho(\mathbf{r})$  out of the blue  
[LOOK UP](#)
- No obvious reason for de-excitation into 1S and 2S only [LOOK UP](#)



# Questions?

... and apart from that, the world is still as beautiful.

[GO TO SUMMARY](#)

# Definitions of the radii

The charge radius is defined as

$$r_E^2 = \int_0^\infty r^2 \rho(r) dr$$

In general, of course, we could define it in any way, for e.g.

$$r_E = \left( \int_0^\infty r^n \rho(r) dr \right)^{1/n}$$

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# What we should retain!

- We aim to measure the **charge** and **Zemach** radii of the proton.
- Useful as inputs for tests of bound-state QED
- Earlier experiments with e-p scattering in H-atom - this time, laser spectroscopy of muonic Hydrogen
- From spectroscopy, we deduce **Lamb shift** and **2S hyperfine splitting** - and from them the **charge** and **Zemach** radii.
- Turn out to be significantly **smaller** than accepted proton radius
- Consequences? Window for new physics?

PROTON SIZE PUZZLE