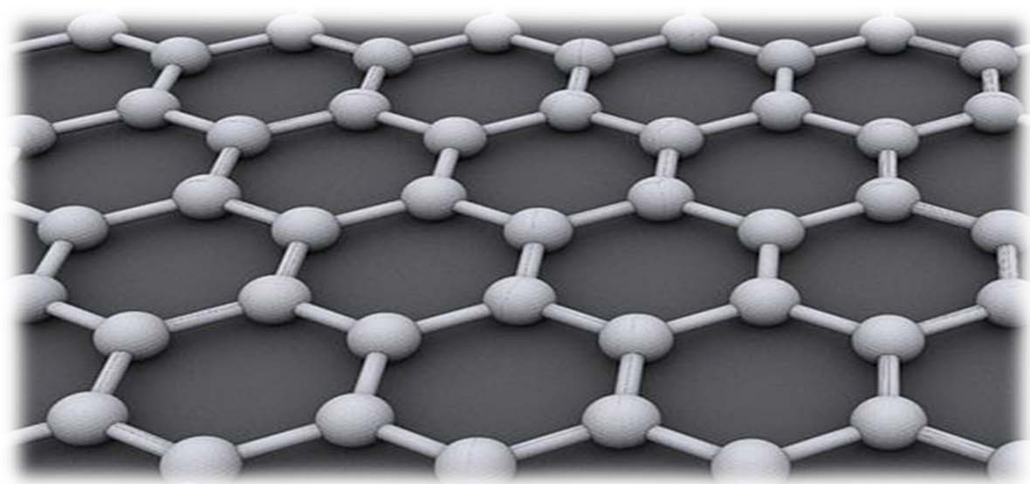
Two-dimensional gas of massless Dirac fermions in graphene

Novoselov, K. S., A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, 2005, *Nature* **438**, 197



Team 7

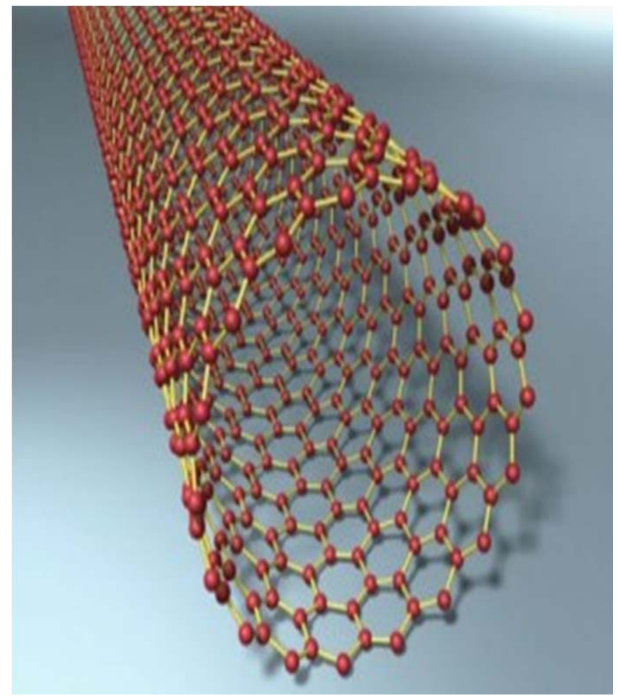
Donovan Myers, Thomas Neulinger, Jungsik Park, Gabriel Petrica

Outline

- Brief background
- Summary of the paper
- Comparison with relevant work
- Theoretical discussion
- Conclusions
- Impact of the paper

Motivation of the Research

- Low-dimensional systems are interesting as they demonstrate quantum mechanical phenomena.
- Graphene has not been thoroughly studied in an experimental setting.



Carbon nanotube
Graham Templeton

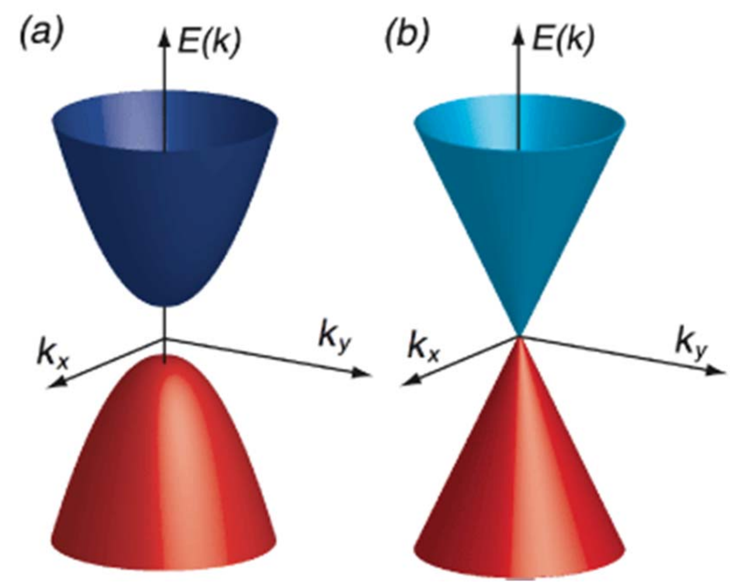
Graphene Timeline

- 1961: First named by Hanns-Peter Boehm
- 2004: Isolated by Novoselov *et al.*
- 2010: Geim and Novoselov win Nobel Prize in Physics
- 2013: Over 9000 patents have been filed for graphene (Wall Street Journal)

Nobel Prize
Wikimedia Commons

Charge Carriers are Massless in Graphene!

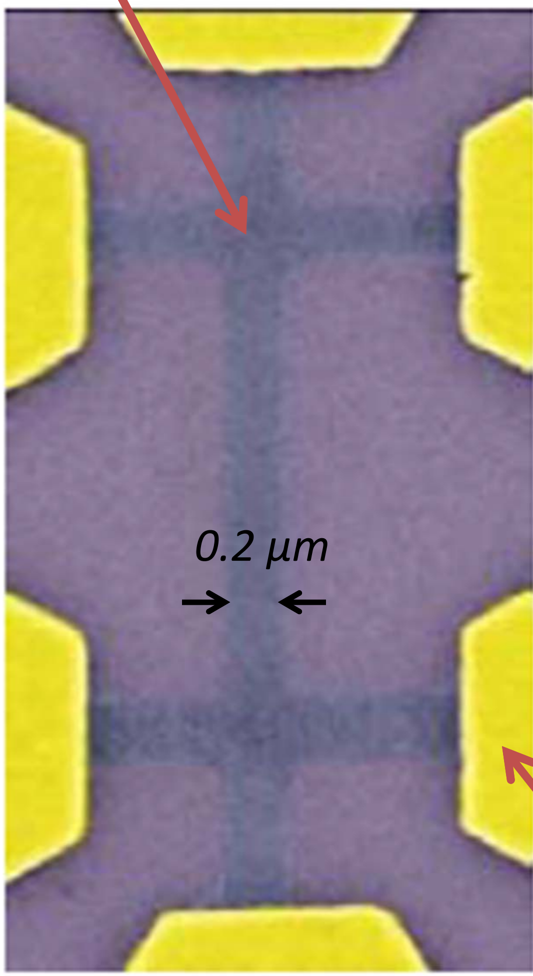
- Energy-momentum dispersion of typical semiconductor is parabolic.
- Energy-momentum dispersion of graphene is linear (Famous Dirac cone).



Energy dispersion relation for (a) conventional semiconductor; (b) graphene
Yasuhiro Hatsugai

t8

Sample Preparation and Well-Understood Measurement Technique



SEM image of device
Novoselov et al., *Nature* 2005

- Sample Processing:
  - Micromechanical cleavage of graphite (top-down approach)
    1. A “layered crystal [of graphene] was rubbed against another surface.”
    2. “Preliminary identification… was done in an optical microscope.”
    3. The next phase of selection of single-layer crystals was done using AFM.
- Transport measurements performed using six-terminal Hall bar device

Slide 6

t8 New title for this slide
thomas, 12/2/2013

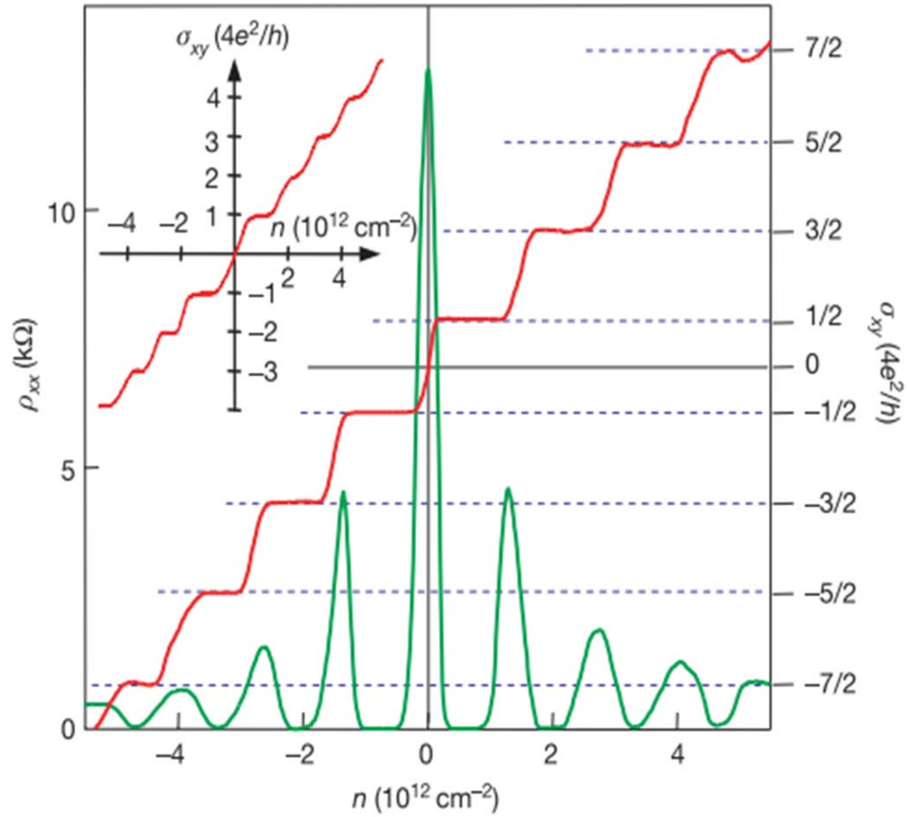
Single-Layer Graphene Shows Half-Integer Quantum Hall Effect

- If integer quantum Hall effect (QHE),

$$\sigma_{xy} = \frac{4e^2}{h} N$$

- Half-integer QHE in graphene

$$\sigma_{xy} = \frac{4e^2}{h} \left(N + \frac{1}{2} \right)$$



Quantum Hall effect of graphene
Novoselov et al., *Nature* 2005

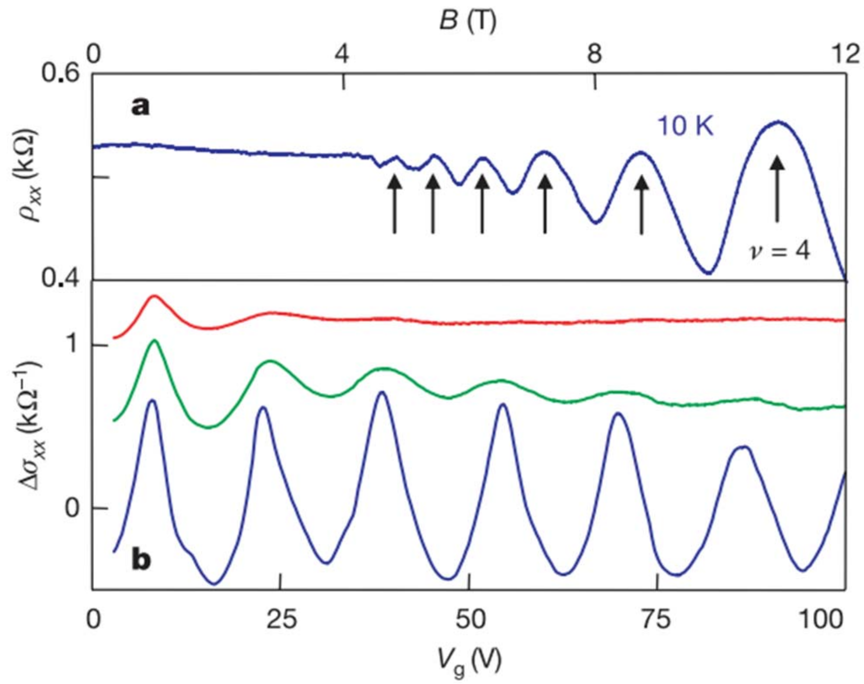
Shubnikov-de Haas Oscillations (SdHO) in Graphene

- Shubnikov-de Haas oscillations
  - Conductivity oscillations that occur at low temperatures in the presence of intense magnetic fields

- Lifshitz-Kosevich formula

$$\frac{A}{T} = \left[\sinh\left(\frac{am_cT}{B}\right)\right]^{-1}$$

  - *A*: amplitude, *T*: temperature, *a*: constant, m_c : cyclotron mass, *B*: magnetic field
  - m_c was calculated



SdHO in graphene

(a) SdHO at constant gate voltage -60V as a function of magnetic field B

(b) SdHO at constant magnetic field 12T as a function of gate voltage (blue, T=20K; green, T=80K; red, T=140K)

Massless Dirac Fermions Move at a Relativistic Speed!

- Semi-classical calculation:

  $c_* \approx 10^6$ m/s

- Allows access to physics of quantum electrodynamics in a “bench-top” experiment

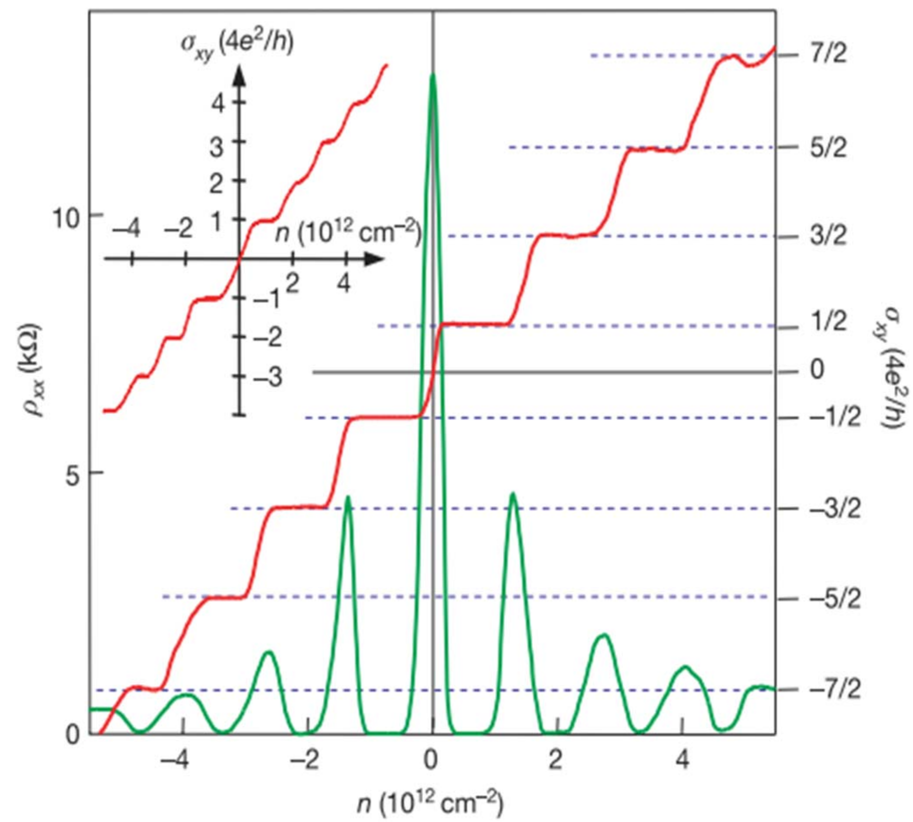
LHC at CERN
Maximilien Brice, © CERN

Electrical Conductivity Has a Minimum

- Note conductivity, not conductance
- Result of adherence to Dirac equation
- Mean free path of charge carriers never goes below minimum (wavelength)
  - Applies in the absence of localization
  - Noted by Mott

Shubnikov-de Haas Oscillations Again

- Note maximum of resistivity (green)
- Conductivity remains limited even as "concentrations of charge carriers tend to zero."



Quantum Hall effect of graphene
Novoselov et al., *Nature* 2005

Minimum Conductivity Predictions: Close, But Not Quite

- Predicted minimum conductivity near $e^2/\pi h$
- Measured minimum found at e^2/h

  - Compare to:

Gusynin, V. P., & Sharapov, S. G. (2005). Unconventional Integer Quantum Hall Effect in Graphene. *Physical review letters*, *95*(14), 146801

Peres, N. M. R., Guinea, F., & Castro Neto, A. H. (2006). Electronic properties of two-dimensional carbon. *Annals of Physics*, *321*(7), 1559-1567.

Other Findings Agree with Theory

- Same papers above anticipate half-integer SdHO effect
- "Relativistic" speed $c_* \sim 10^6$ m/s fits with band structure calculations, example source below

Dresselhaus, M. S., & Dresselhaus, G. (2002). Intercalation compounds of graphite. *Advances in Physics*, *51*(1), 1-186.

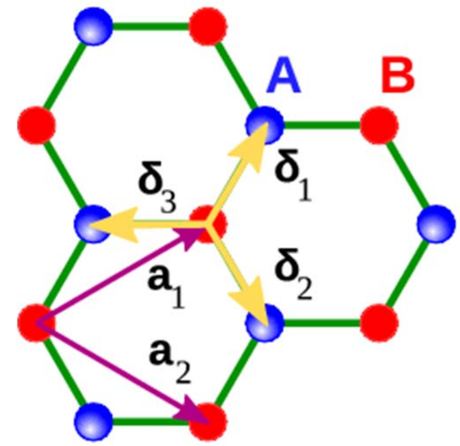
- We say relativistic because inertial mass is proportional to energy
- $m_c = E/c_*^2$ applies, where E can be assumed to equal kinetic energy

t5

Tight-Binding Model for Graphene

- Graphene is a 2D crystal; the carbon atoms form a honeycomb lattice (triangular Bravais lattice with 2 atom basis)

- Investigate electronic properties using a tight-binding model



$$\mathrm{H} = \sum_{\mathbf{k}\sigma} \left(a^\dagger_{A\sigma} a^\dagger_{B\sigma}\right) \begin{pmatrix} 0 & -t f(\mathbf{k}) \\ -t f^*(\mathbf{k}) & 0 \end{pmatrix} \left(a_{A\sigma} a_{B\sigma}\right)$$

$$f(\mathbf{k}) = \sum_{j=1}^{3} e^{i\mathbf{k}\cdot\delta_{\mathbf{j}}} \qquad \epsilon(\mathbf{k}) = \pm t \sqrt{1 + 4\cos\frac{k_y a}{2} + 4\cos\frac{k_y a}{2}\cos\frac{k_x\sqrt{3}a}{2}}$$

Slide 14

t5 play around with equation size

thomas, 12/3/2013

Effective Theory for Graphene is 2+1 QED!

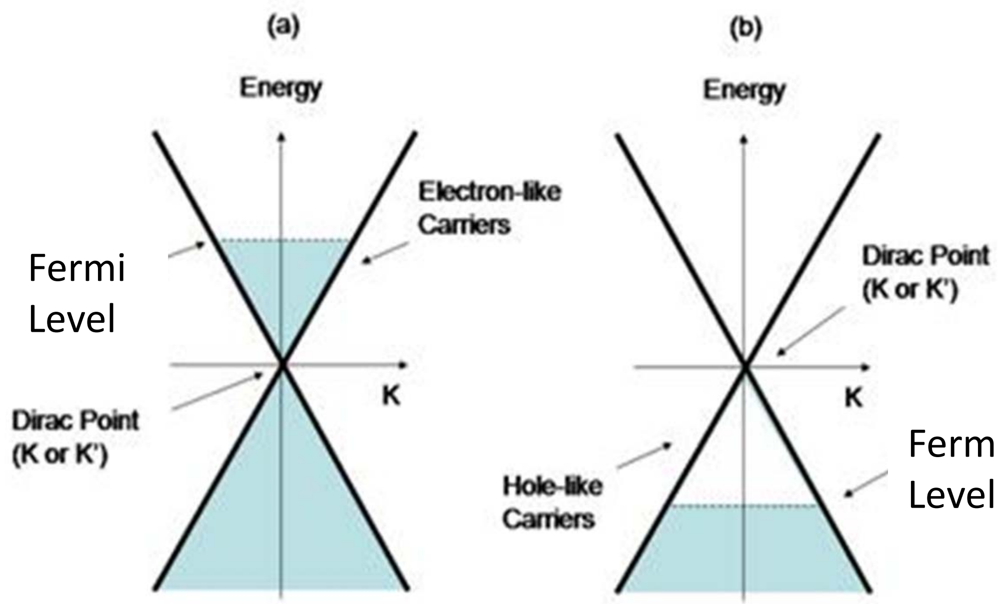
- Absent any doping/disorder Fermi level in graphene is at the Dirac points; thus graphene is a semi-metal
- At low T, the effective degrees of freedom in graphene are massless Dirac fermions; to see this expand Hamiltonian near the Dirac points

$$H(\mathbf{k} = \mathbf{K} + \mathbf{q}) \approx \mathbf{q} \cdot \nabla_{\mathbf{k}} H(\mathbf{k})$$

$$H(\mathbf{q}) = \hbar v_F \begin{pmatrix} 0 & q_x - iq_y \\ q_x + iq_y & 0 \end{pmatrix} = \hbar v_F \mathbf{q} \cdot \sigma$$

$$v_F = \frac{3t}{2a} \approx \frac{c}{300}$$

- Graphene can be doped using electric field effect



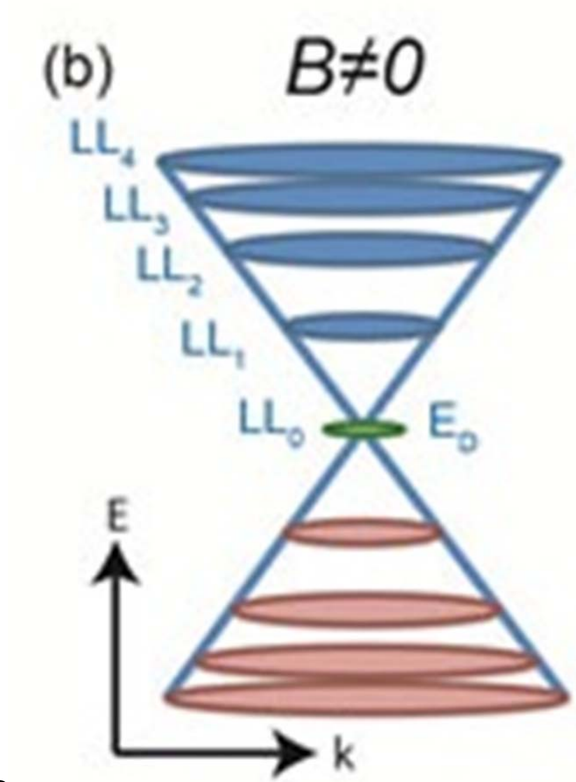
Zero-Energy Landau Level in Graphene

- To study Dirac fermions in magnetic fields -> Peierls substitution $\mathbf{\Pi} = \hbar\mathbf{k} = \mathbf{p} + e\mathbf{A}$ with $\nabla \times \mathbf{A} = B\hat{z}$

- Diagonalize the Hamiltonian (in some gauge) $H(\mathbf{p}, \mathbf{A}) = \hbar v_F \sigma \cdot (\mathbf{p} + e\mathbf{A})$

  to obtain relativistic Landau levels

$$\epsilon_n = \pm\hbar v_F \sqrt{2\frac{e}{\hbar}Bn}$$



- Degeneracy of Landau levels

$$N = \underbrace{2}_{\text{valleys}} \times \underbrace{2}_{\text{spin}} \times \underbrace{\frac{\phi}{\phi_0}}_{\text{\# flux quanta}}$$

- Degeneracy of lowest Landau level is N/2; only one valley contributes (next slides)

Semi-Classical Quantization Gives Exact Results!

- Provides very good intuition and correct (surprisingly) transport results for Dirac fermions; very easy compared to full theoretical quantum treatment
- Semi-classical transport $\hbar\dot{\mathbf{k}} = -e\dot{\mathbf{r}} \times \mathbf{B}$
- Semi-classical quantization $\oint \mathbf{p}\cdot\mathbf{r} = \oint \hbar\mathbf{k}\cdot\mathbf{r} - e\oint \mathbf{A}\cdot = e\Phi_n = 2\pi\hbar(n+\gamma)$

  where γ is the quantum mechanical phase accumulated

$$\epsilon = \pm\hbar v_F|\mathbf{k}|$$

$$\Phi_n = BS_n(\mathbf{r}) = BS_n(\mathbf{k})\left(\frac{\hbar}{eB}\right)^2$$

$$\epsilon_n = \pm\hbar v_F\sqrt{\frac{2e}{\hbar}B(n+\gamma)}$$

- **Turns out γ = ½ for electrons in regular metals and γ = 0 for Dirac electrons; semi-classical quantization recovers exact result! (next slide)**

Phase Shift and Berry's Phase

- The phase shift accumulated as the momentum varies on a loop in k-space
$$\gamma = \frac{1}{2} - \frac{\Gamma}{2\pi}$$

- The factor of ½ comes from failure of semi-classical quantization near classical turning points (e.g., WKB for harmonic oscillator)

- Γ is known as Berry's phase $$\Gamma(C) = i \oint_C \mathrm{d}\mathbf{k} \langle u_{n,\mathbf{k}} | \nabla_{\mathbf{k}} | u_{n,\mathbf{k}} \rangle$$
where u(n, **k**) are Bloch wave-functions

- Non trivial Berry's phase because of band degeneracy at Dirac points

- On a contour *C* around the **K** point, y = 0; on a contour *C'* around **K'**, y = 1

ONLY ONE VALLEY CONTRIBUTES TO LOWEST LEVEL LANDAU AT E = 0

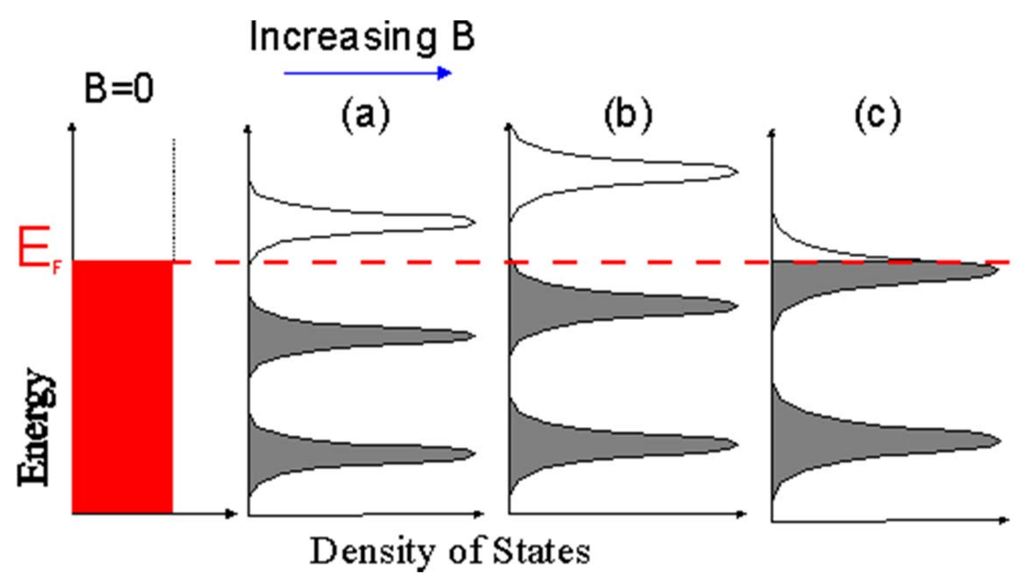
SdHO Peaks at Integer Landau Filling Factor

- Landau levels broaden in the presence of scattering mechanism (disorder)

$$\rho_{xx}(B) \sim \text{density of states}$$

- Density of states (and hence magnetoresistance) is maximum when highest Landau level (LL) is half-filled

- The presence of E = 0 LL in graphene, with half-degeneracy of the other LLs, implies magnetoresistance is maximum at **integer** Landau filling factor (next slide for definition)
- Even though rest-mass is zero, cyclotron mass (semi-classical expression) is not



$$m_c = \frac{\hbar^2}{2\pi}\frac{\partial S(E)}{\partial E} \qquad E = m_c v_F^2$$

Graphene Exhibits Anomalous QHE

- Hall conductivity as a topological invariant (TKNN)

spin x valleys

$$\sigma_{xy} = 4 \times \frac{e^2}{\hbar} \times \nu$$

Landau filling factor

Quantum unit of resistance

- In traditional quantum hall effect, the Landau filling factor is # filled Landau levels (also, no valleys)
- In graphene, because the degeneracy in lowest LL is only **half** of the other LLs

$$\sigma_{xy} = 4\frac{e^2}{\hbar}(n + \frac{1}{2})$$

t6

Dirac Equation Describes Electronic Properties of Graphene

- Main points:
  1. Graphene's conductivity never falls below a minimum, even when concentrations of charge carriers tend toward zero.

  2. Half-integer QHE in graphene related to properties of massless Dirac fermions.

  3. The cyclotron mass of massless carriers in graphene is described by $E = mc_*^2$

Slide 21

t6 possibly delete conductivity comment

thomas, 12/3/2013

t7

Our Thoughts

- Main points:
  1. Good:
     a. Good take-away message for the non-expert
     b. Arguments supported by figures
  2. Not so good:
     a. Why are localization effects suppressed?
     b. Minor details:
        – Poor distribution of images
        – Typos
        – Mislabeled figures
- Overall great paper!

Slide 22

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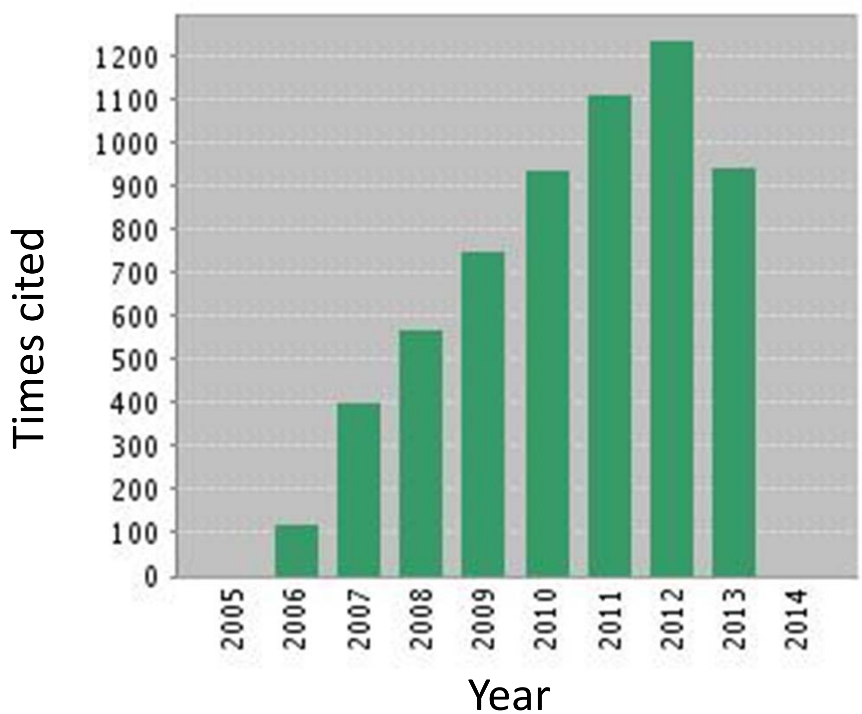
t7 lack of explanation
thomas, 12/3/2013

Paper quickly becomes highly-cited

- Number of citations:
  - 6170 (Web of Knowledge)
  - 6331 (SCOPUS)
  - Most cited in 2012 (1306 times)

Times cited by year



Citation analysis graphic from Web of Knowledge

Contributes to explosion in research

- 2010 Nobel Prize in Physics!
- "The rise of graphene," Nature Materials (8393 citations)
  - Same authors describe "new paradigm of 'relativistic' condensed-matter physics."
- "Observation of electron-hole puddles in graphene using a scanning single-electron transistor," Nature Physics (569 citations)
  - "Density of states can be quantitatively accounted for by considering non-interacting electrons and holes [unlike non-relativistic particles].

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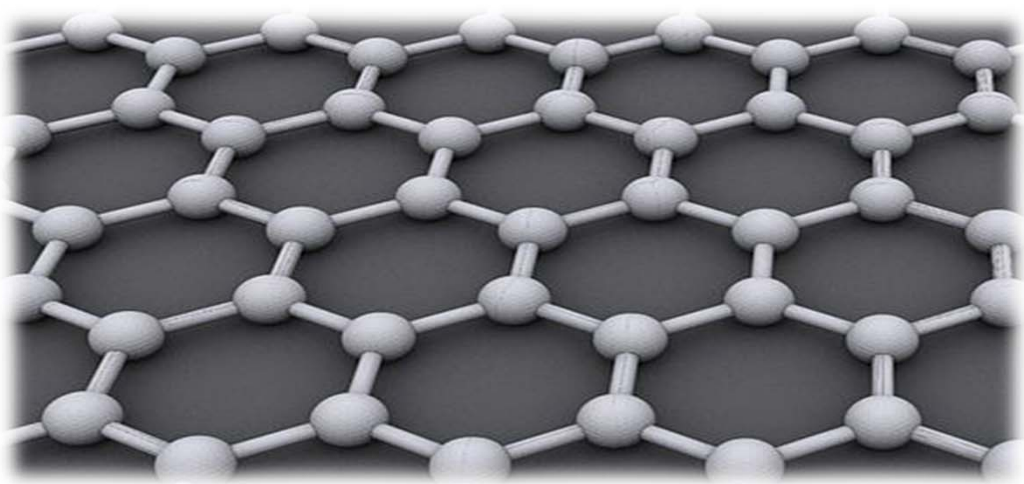

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