Quantum-Field Tomography

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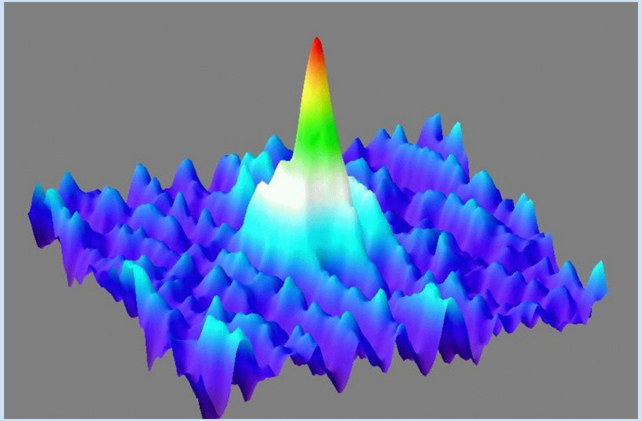

"Towards experimental quantum-field tomography with ultracold atoms"

A.Steffans, M. Friesdorf, T. Langen, B. Rauer, T. Schweigler, R. Hubener, J. Schmiedmayer, C.A. Riofrio & J. Eisert. *Nature Communications.* **6:7663,** (2015)

Outline

- The Experiment - Quantum Field Tomography
- Theoretical Analysis
- Motivation and Additional Tomography Methods
- Future Work and Overall Considerations

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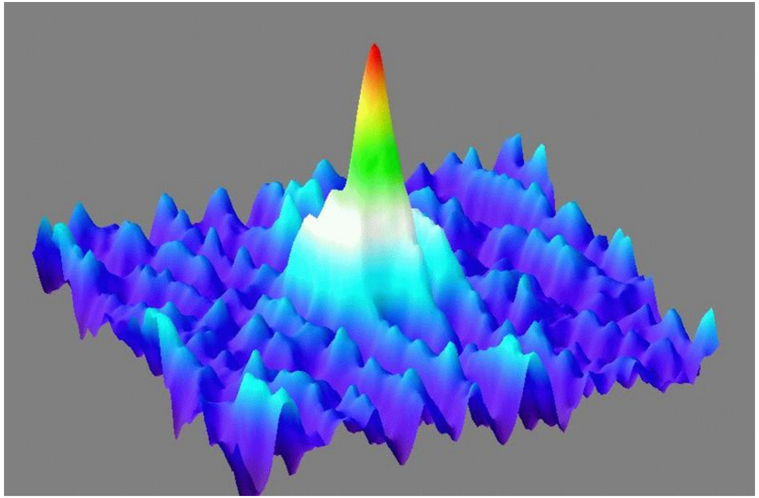
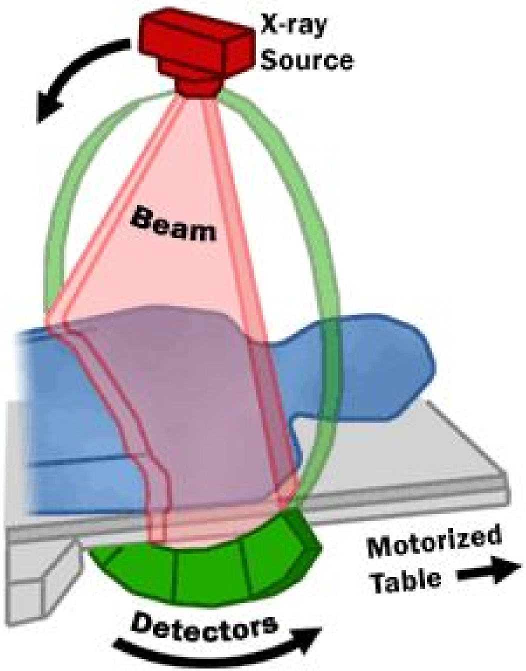
The recent experimental realization of continuous quantum AMO systems

- AMO systems can now be made larger than ever before (~1000s of atoms) while still maintaining precise control over individual constituents.
- Because such systems are essentially continuous, this motivates the use of quantum field theory.



What is Tomography?

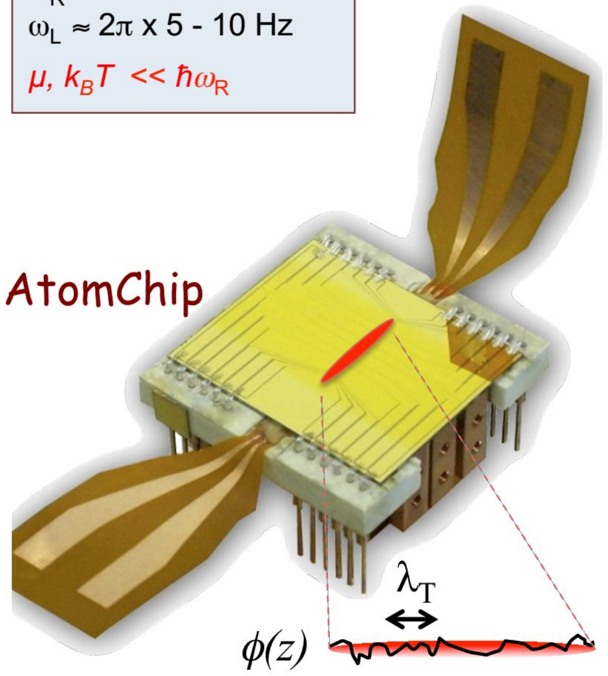


- Tomography probes a system with an experimental technique, thereby reconstructing some aspects of the system. It does not assume any theory to begin with.
- Computer-Aided Tomography (CAT) scans are an illustrative example of tomography in practice.
- This sort of logic is common in the history of science. The only novelty is finally being able to apply it to these continuous, quantum systems.

The use of matter-wave interferometry in probing the Bose condensate

- After “kicking” the system out of equilibrium (through a process known as transversal quenching), two separate, but correlated, quantum systems are created. The correlation is picked up as a phase variation throughout the condensate.
- Matter-wave interferometry allows us to resolve the separation between the two halves in order to determine the relative phase.

1000-10000 Rb atoms
T = 10-100 nK
$\omega_R \approx 2\pi \times 2 - 3$ kHz
$\omega_L \approx 2\pi \times 5 - 10$ Hz
$\mu, k_B T << \hbar\omega_R$



The notion of **sparsity** as taking simple data

- In principle, we could “kick” a system in any haphazard, arbitrary way we may like. But only a sufficiently simple “kick” will yield in easy to understand data. Hence, we probe the system in simple ways so as to excite only the basic structure.
- By probing the data in simple and symmetric ways, we also have the much needed luxury of making theoretically simplifying assumptions.

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Matrix Product State Representation

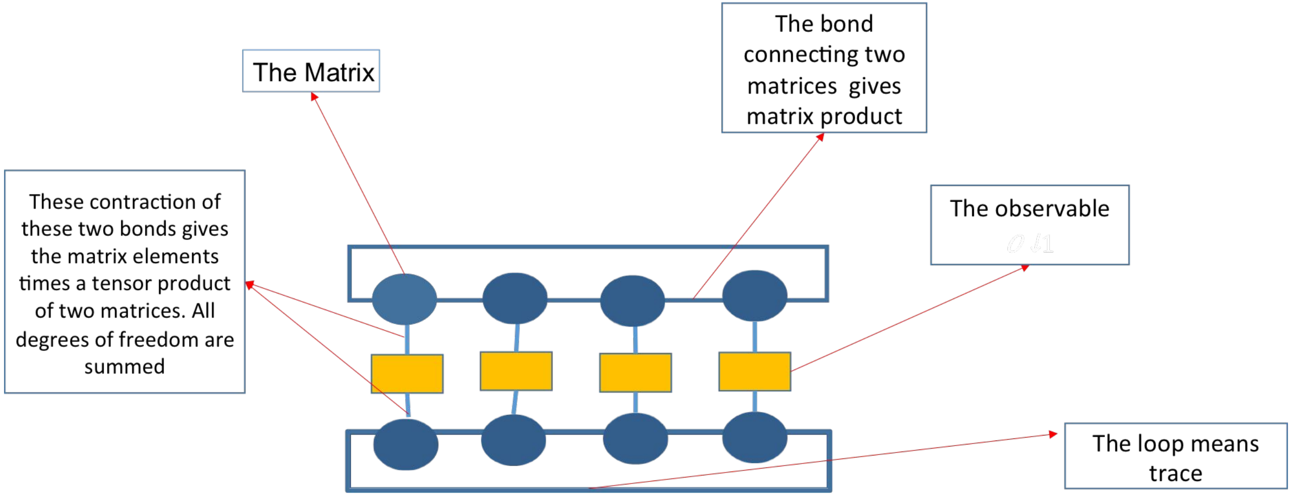
Every 1-dimensional quantum state $|\psi\rangle$ can be written as a product of matrices, called the matrix product state (MPS):

$$|\psi\rangle = \sum_{\boldsymbol{\sigma}} \mathrm{Tr}(M_{\sigma_1}^{[1]} M_{\sigma_1}^{[2]} \dots M_{\sigma_1}^{[N]}) |\boldsymbol{\sigma}\rangle$$

Contracting the Matrix Product State Representation

- The expectation value of an observable $\hat{O} = \otimes_{k=1}^{k=N} \hat{O}_k$ can be computed: $\langle\psi|\otimes_{k=1}^{k=N}\hat{O}_k|\psi\rangle = \mathrm{tr}[\prod_{k=1}^{k=N} E_{O_k}^{[k]}]$, where $E_{O_k}^{[k]} = \Sigma_{i,j=1}^{f}\langle i|\hat{O}_k|j\rangle\, \bar{M}_i^{[k]} \otimes M_j^{[k]}$

For a state written in the MPS representation, the expectation value of an observable can be calculated and written graphically:



The quantum state described by a continuous MPS

- For a continuous 1-D translationally invariant bosonic system, a quantum many-body state can be written as

$$|\psi\rangle = \mathrm{Tr}\left\{P\exp\left[\int_0^L dx(Q \otimes I + R \otimes \psi^+(x)\right]\right\}|\Omega\rangle$$

- $Q, R \in C^{d\times d}$ are matrices acting on the auxiliary d-dimensional virtual space and serves as variational parameters which can be determined from experimental data.

State Reconstruction from Phase Correlation Function

- The n-point correlation is define as the phase correlation function:

$$G^n(x_1, x_2, \dots x_n) = \mathrm{Re}\left\langle e^{i(\theta_{x_1} - \theta_{x_2} + \theta_3 - \cdots)} \right\rangle$$

- It can be computed in the thermodynamic limit:

$$G^n(x_1, x_2, \dots x_n) = \mathrm{Re}\left\langle \hat{n}(x_1)^{\frac{-1}{2}} \hat{\psi}(x_1)^{+} \hat{\psi}(x_2) \hat{n}(x_2)^{\frac{-1}{2}} \dots \right\rangle$$

$$= \mathrm{Tr}\left[e^{T(L - x_n)} \dots e^{T(x_3 - x_2)} (\bar{R}^{\frac{1}{2}} \otimes R^{-\frac{1}{2}}) e^{T(x_2 - x_1)} (\bar{R}^{-\frac{1}{2}} \otimes R^{\frac{1}{2}}) \right]$$

$$T = \bar{Q} \otimes I + I \otimes Q + \bar{R} \otimes R$$

State Reconstruction from Phase Correlation Function

- By diagonizing the matrix T and taking the thermal dynamic limit, the correlation function takes the form $\sum_{\{k_j\}=1}^{d^2} \rho_{k_{1,} k_{2,} \dots k_{N-1}} e^{\lambda_{k_1}\tau_1} \dots e^{\lambda_{k_{N-1}}\tau_{N-1}}$.
- λ_k are eigenvalues of T and $\rho_{k_{1,} k_{2,} \dots k_{N-1}}$ is equal to $M^{-1}_{1,k_{N-1}} \dots . M^{-1}_{k_2,k_1} M_{k_1,1}$ with $X^{-1}TX$ in diagonal form and $M = X^{-1}(\overline{R}^{-\frac{1}{2}} \otimes R^{\frac{1}{2}})X$.
- In the Laplace transformation, λ_k are poles and the pre-factors $\rho_{k_{1,} k_{2,} \dots k_{N-1}}$ are referred to as residues.

Construction of Many-Body State

1. The poles are extracted and the dimension of the matrix product state is determined by using the experimental data of two-point correlation function.
2. The matrix *M* is constructed from the data’s four-point and two-point correlation functions.
3. Having the poles and matrix *M,* we can predict all n-point functions.
4. Once the poles and the *M* are determined, *R* can be known. In this paper, *R* is chosen to be diagonal and then *Q* is expressed.
5. The quantum many-body state is obtained if *R* and *Q* are known.

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What can quantum field tomography be used for?

Creating new methods for state identification of continuous quantum many-body systems.

Can be used for precise, model-independent quantum-state identification: “quantum engineering”

Applications in quantum metrology, quantum information, and quantum simulation

Previous Quantum Tomography Efficient for small subsets of the full Hilbert Space

- 2010: “Quantum state tomography via compressed sensing”: Focus their measurements on pure states, states with symmetry, and ground states to limit experimental data acquisition. Use compressed sensing to recover sparse vector (containing only a few non-zero entries in specified basis) from small number of measurements. [2]
- 2010: “Efficient Quantum State Tomography”: two different methods, one requiring unitary operations on a constant number of subsystems, and the other requiring only local measurements with more elaborate post-processing. These methods have a linear number of experimental operations (always good!) and polynomially-scaled post-processing (not as good). [3]
- “Efficient and feasible state tomography of quantum many-body systems”: Focuses on discrete systems (atoms in a lattice). They show that measuring a tomographically complete set of observables is not necessary if a single observable is measured after allowing the state to evolve under appropriately- chosen quantum circuits. Results can then be generalized. [4]

Summary of efficient tomography for *continuous* quantum systems

- The continuous system (BEC) has infinitely many degrees of freedom $\rightarrow$ Quantum Field Tomography
  - Overcome this using quenching to form two separate quantum systems
- Sparsity - can use assumptions in theoretical analysis to simplify the system (low-order behavior)
- Continuous matrix-product states (cMPS) allows the creation of a theoretical model based on the data

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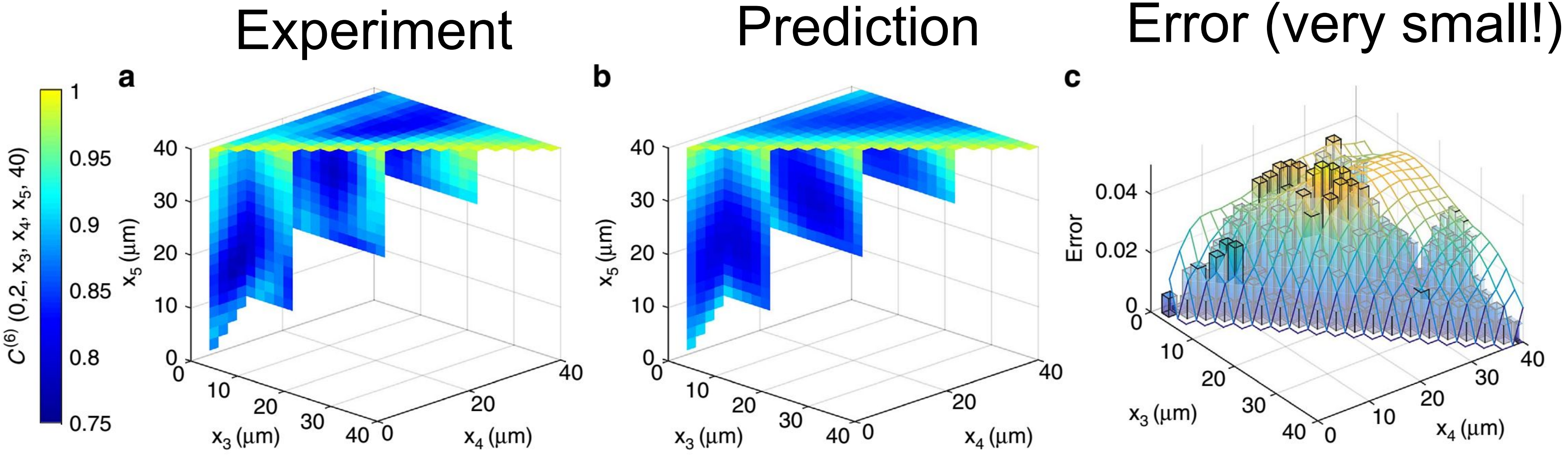
Data of Projected N-Point Function



First steps towards experimental quantum-field tomography!

What else can you do with this setup?

Experimental observation of a generalized Gibbs ensemble

(Science, Apr 2015) T. Langen, S. Erne, R. Geiger, B. Rauer, T. Schweigler, M. Kuhnert, W. Rohringer, I. E. Mazets, T. Gasenzer, J.Schmiedmayer

- Using correlation functions to probe statistical properties
- Non-equilibrium system being described with multiple temperature-like parameters

Our critiques and final thoughts about this work

Concerns

Accuracy?

Novelty?

Influence?

Accessibility?

Conclusions

- Probing quantum many-body systems using N-point correlation functions present the first steps towards efficient quantum-field tomography.
- Reconstructing the quantum many-body states as a quantum field, without the need of a specific model (Hamiltonian) at hand is important for many applications in quantum information, as well as for general knowledge of strongly correlated states and far-from-equilibrium states.
- The authors could have better described the experimental setup, instead of focusing on the theory.

References

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