Topological Quantization in Units of the Fine Structure Constant

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Topological Quantization in Units of the Fine Structure Constant

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(Received 16 April 2010; published 12 October 2010)

Fundamental topological phenomena in condensed matter physics are associated with a quantized electromagnetic response in units of fundamental constants. Recently, it has been predicted theoretically that the time-reversal invariant topological insulator in three dimensions exhibits a topological magneto-electric effect quantized in units of the fine structure constant $\alpha = e^2/\hbar c$. In this Letter, we propose an optical experiment to directly measure this topological quantization phenomenon, independent of material details. Our proposal also provides a way to measure the half-quantized Hall conductances on the two surfaces of the topological insulator independently of each other.

Outline

- Introduction and Background
- Proposed Experiment
- Critique
- Citation Analysis

Topological Quantization in Condensed Matter Systems

- Superconductor(SC):

  magnetic flux is quantized in the units of flux quantum $\phi_0 \equiv \frac{h}{2e}$

- Quantum Hall effect(QHE):

  Hall conductance is quantized in the units of conductance quantum $G_0 \equiv \frac{e^2}{h}$

- Topological magnetoelectric effect(TME):

  a quantized coefficient in the units of fine structure constant $\alpha \equiv \frac{e^2}{\hbar c}$

} described by topological field theories in the low-energy limit with quantized coefficients

⇨ provide the most precise measurement of fundamental physical constants e, h and c

Topological Magnetoelectric Effect (TME)

For the time-reversal (T) invariant topological insulator (TI), the effective Lagrangian is

$$\mathcal{L} = \frac{1}{8\pi}\left(\varepsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \underbrace{\frac{\theta}{2\pi}\frac{\alpha}{2\pi}\mathbf{E}\cdot\mathbf{B}}_{\text{Described the topological magnetoelectric effect}},$$

Under T: $\mathrm{B} \longrightarrow -\mathrm{B}$, $\mathrm{E} \longrightarrow \mathrm{E}$

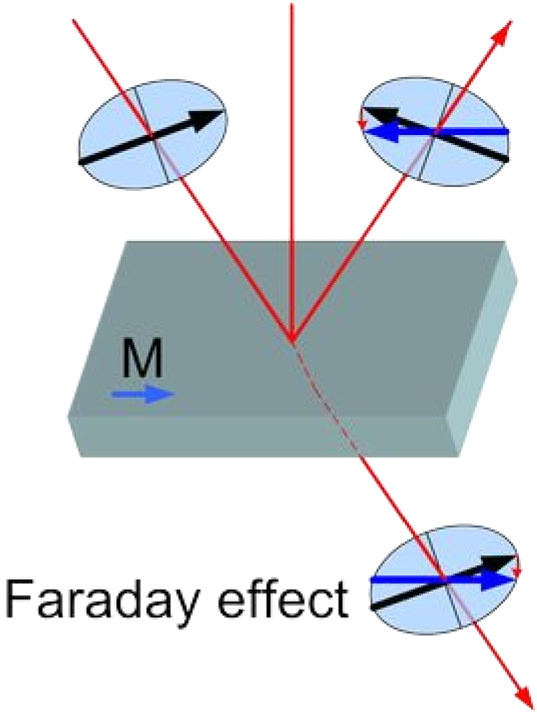
Described the topological magnetoelectric effect

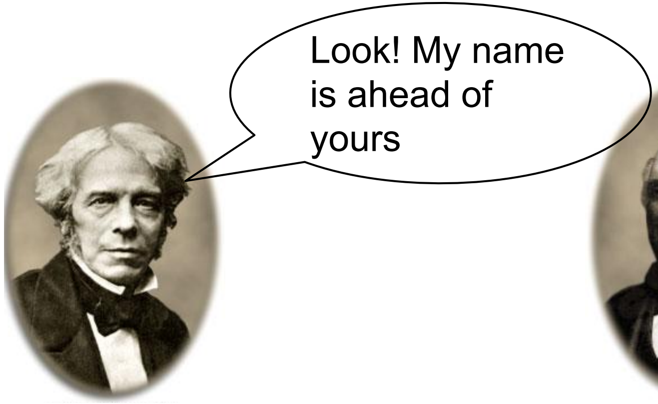
The system is invariant under shifts of θ by any multiple of 2π

The only allowed value of θ by time reversal symmetry is 0 or π

For topological insulators, $\theta=\pi$; trivial insulators, $\theta=0$.

Faraday Effect vs. Kerr Effect





The polarization of light is rotated when light is transmitted through (Faraday) or reflected from (Kerr) magnetized materials

$$\mathcal{L} = \frac{1}{8\pi}\left(\varepsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \frac{\theta}{2\pi}\frac{\alpha}{2\pi}\mathbf{E}\cdot\mathbf{B},$$

Purpose of the Paper

- Provide an optical experiment to measure the topological magnetoelectric effect

How? ⟹

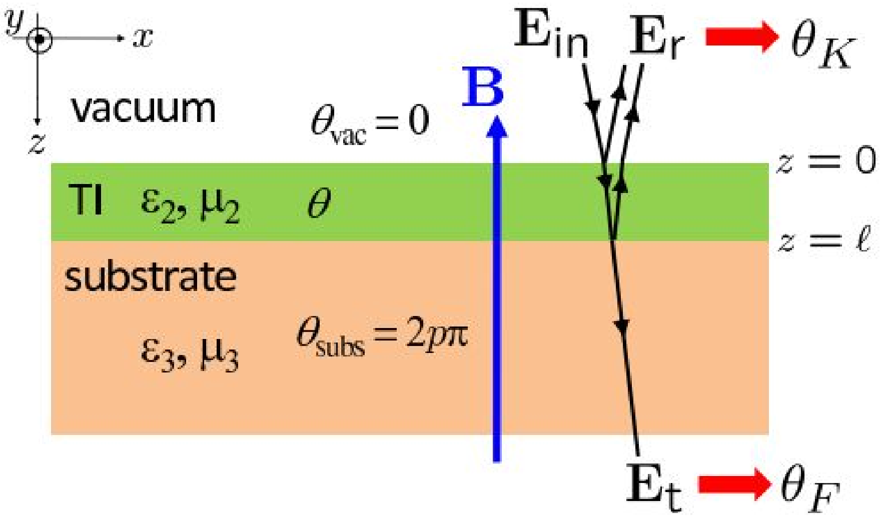
$$\mathcal{L} = \frac{1}{8\pi}\left(\varepsilon \mathbf{E}^2 - \frac{1}{\mu}\mathbf{B}^2\right) + \frac{\theta}{2\pi}\frac{\alpha}{2\pi}\mathbf{E}\cdot\mathbf{B},$$

$$\alpha = e^2/\hbar c.$$

- Exploit the Faraday & Kerr effects
- Measure Kerr and Faraday Angles (easy)
- Deduce the quantization of the parameter θ from the measured angles

Experimental Setup

1. A film of TI (topological insulator) of thickness ℓ and parameter θ
2. A topologically trivial substrate with parameter $\theta_{\text{subs}} = 2p\pi \text{ with } p \in \mathbb{Z}$
3. A magnetic field is applied in the z direction in order to have the Faraday-Kerr effects



Kerr and Faraday Angles

{ *what we can measure!* }

- Incident Monochromatic Light :

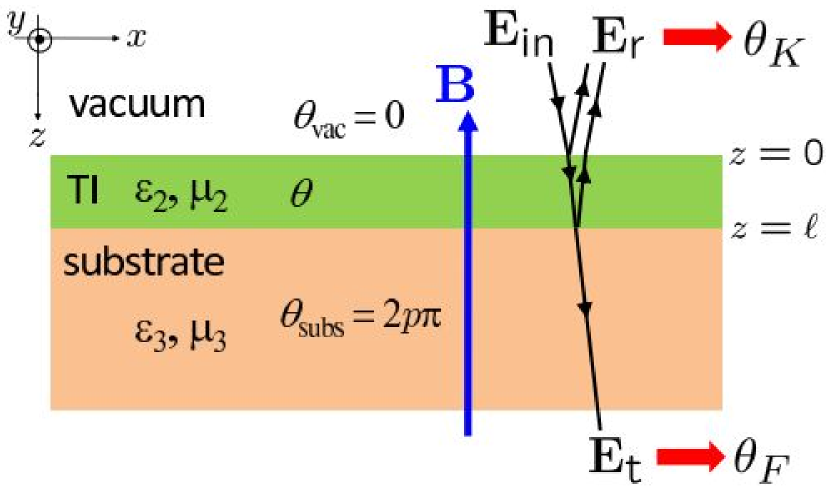
  $\mathbf{E}_{\mathrm{in}} = E_{\mathrm{in}}\hat{\mathbf{x}}$

- Reflected Light : $\mathbf{E}_{\mathrm{r}} = E_{\mathrm{r}}^{x}(-\hat{\mathbf{x}}) + E_{\mathrm{r}}^{y}\hat{\mathbf{y}}$

  - Kerr Angle : $\tan\theta_K = E_{\mathrm{r}}^{y} / E_{\mathrm{r}}^{x}$

- Transmitted Light : $\mathbf{E}_{\mathrm{t}} = E_{\mathrm{t}}^{x}\hat{\mathbf{x}} + E_{\mathrm{t}}^{y}\hat{\mathbf{y}}$

  - Faraday Angle : $\tan\theta_F = E_{\mathrm{t}}^{y} / E_{\mathrm{t}}^{x}$

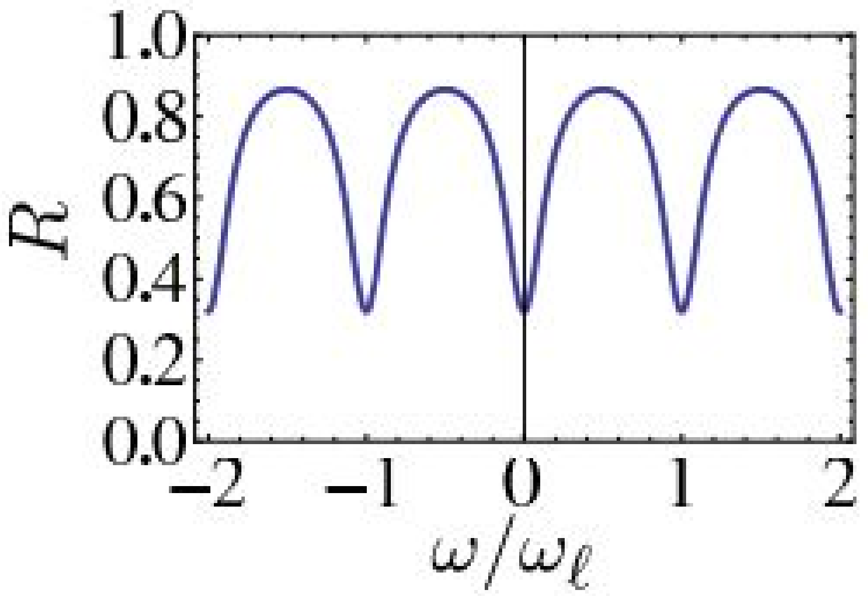


What parameters determine the rotation of the light polarization?

- Generally the Kerr and Faraday angles will depend on many parameters (dielectric constants of materials, length, frequency of light, multiple reflections, θ , θ_{subs})
- It seems dubious that we could extract the exact quantization of the TME from just the Kerr/Faraday angles
- But the paper suggests a trick that simplifies the expressions - Measure the angles at reflectivity minima and maxima (simplifies the expressions significantly)

Reflectivity minima

- Find specific frequency so that we have minimum reflectivity $R \equiv |\mathbf{E}_{\mathrm{r}}|^2/|\mathbf{E}_{\mathrm{in}}|^2$



- Equation between Faraday and Kerr angles:

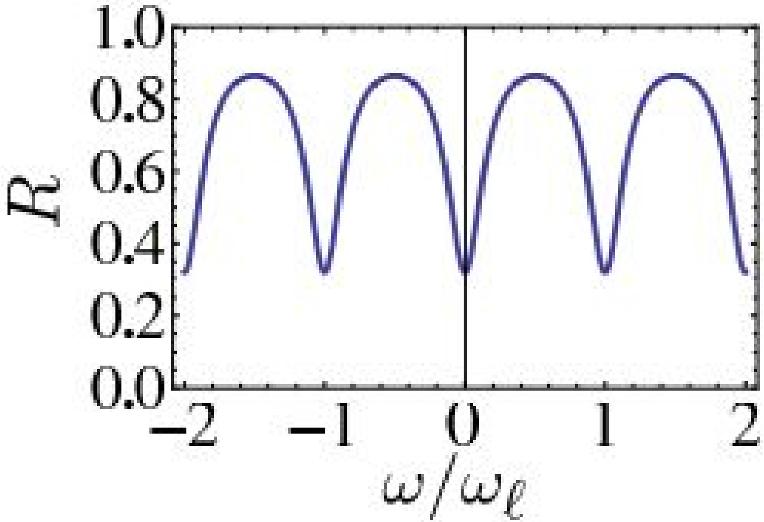
$$\frac{\cot\theta'_F + \cot\theta'_K}{1+\cot^2\theta'_F} = \alpha p, \qquad p \in \mathbb{Z}.$$

$$\alpha = e^2/\hbar c.$$

(Independent on other properties of the materials)

Reflectivity maxima

- ω is now tuned to a reflectivity maxima
- Kerr and Faraday angles - more compicated formulas



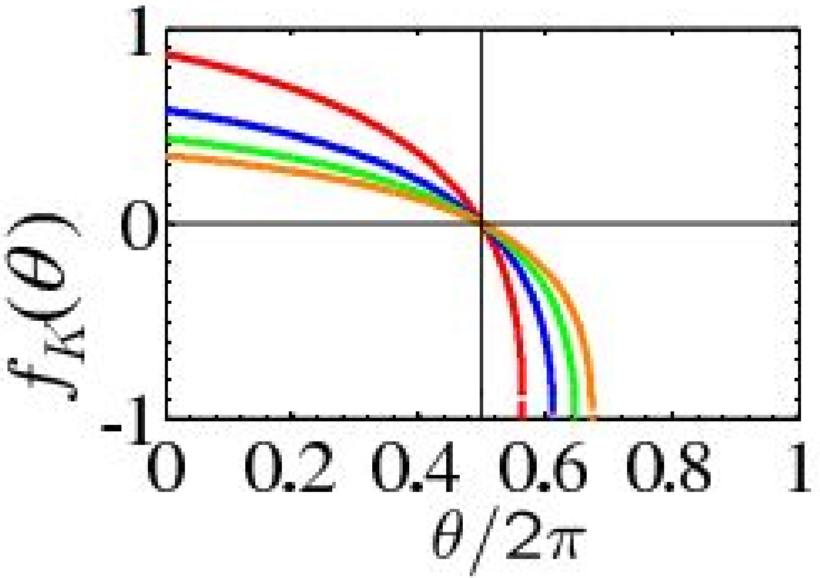
$$\tan\theta''_F = \frac{2\alpha\left(p - \frac{\theta}{2\pi} + Y_3\frac{\theta}{2\pi}\right)}{Y_3 + Y_2^2 - 4\alpha^2\frac{\theta}{2\pi}\left(p - \frac{\theta}{2\pi}\right)}, \quad \tan\theta''_K = \frac{4\alpha\left[Y_2^2\left(p - \frac{\theta}{2\pi}\right) - \tilde{Y}_3^2\frac{\theta}{2\pi}\right]}{\tilde{Y}_3^2 - Y_2^4 + 4\alpha^2\left[2Y_2^2\frac{\theta}{2\pi}\left(p - \frac{\theta}{2\pi}\right) - \tilde{Y}_3^2\left(\frac{\theta}{2\pi}\right)^2\right]},$$

$$\tilde{Y}_3^2 = Y_3^2 + 4\alpha^2\left(p - \frac{\theta}{2\pi}\right)^2$$

$$Y_i \equiv \sqrt{\varepsilon_i/\mu_i}$$

<u>Universal Function</u> f

- Use previous formulas to find an equation of the form: $f(\theta'_K, \theta'_F, \theta''_K, \theta''_F; p, \theta) = 0$
- f does not depend explicitly on any material parameter ε_i, μ_i
- We can find the zero crossing $f(\theta) = 0$
- Hence find the parameter θ.
- Demonstrate the quantization of the TME effect in the TI bulk by verifying it is always 0 or π !

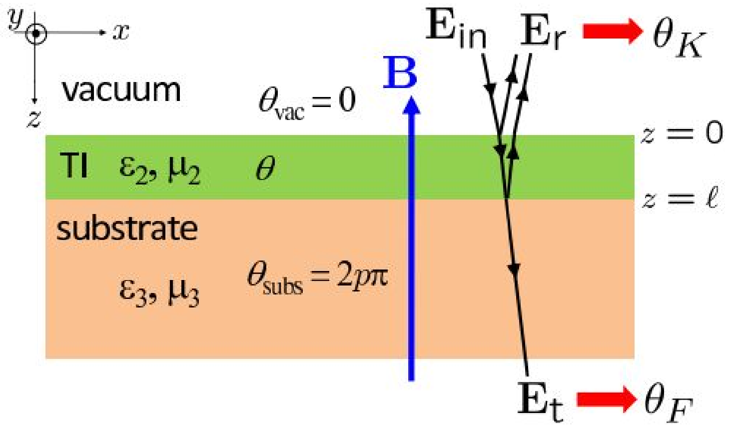


Summary

- Authors propose an optical experiment to directly measure the topological quantization phenomenon.
- By considering both the Kerr and Faraday effects, the authors derive the universal function f, which is independent of material parameters and can be used to determine θ.

Critique

- Is the experiment feasible?
  - Theory only applies when $w << E_g/\hbar$.
  - To achieve maximum or minimum reflectivity ω or the thickness of the TI film must be changed continuously.
  - Samples do not yet have sufficient quality to yield strongly developed Quantum Hall effects.
  - This paper neglects reflection from the substrate-vacuum boundary.



Impact of the paper

- This paper has been cited 88 times.
- The TME effect has yet to be observed.
- Yuanpei Lan, Shaolong Wan,and Shou-Cheng Zhang. “Generalized quantization condition for topological insulators.” Phys. Rev. B 83, 205109 (2011).
    - Considers light at oblique incidence
- Wang-Kong Tse and A. H. MacDonald. “Magneto-optical Faraday and Kerr effects in topological insulator films and in other layered quantized Hall systems.” Phys. Rev. B 84, 205327 (2011)
    - Considers both thin and thick films
    - Examines possible experimental difficulties

Thank you!