Measurement of the Instantaneous Velocity of a Brownian Particle

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Team 8

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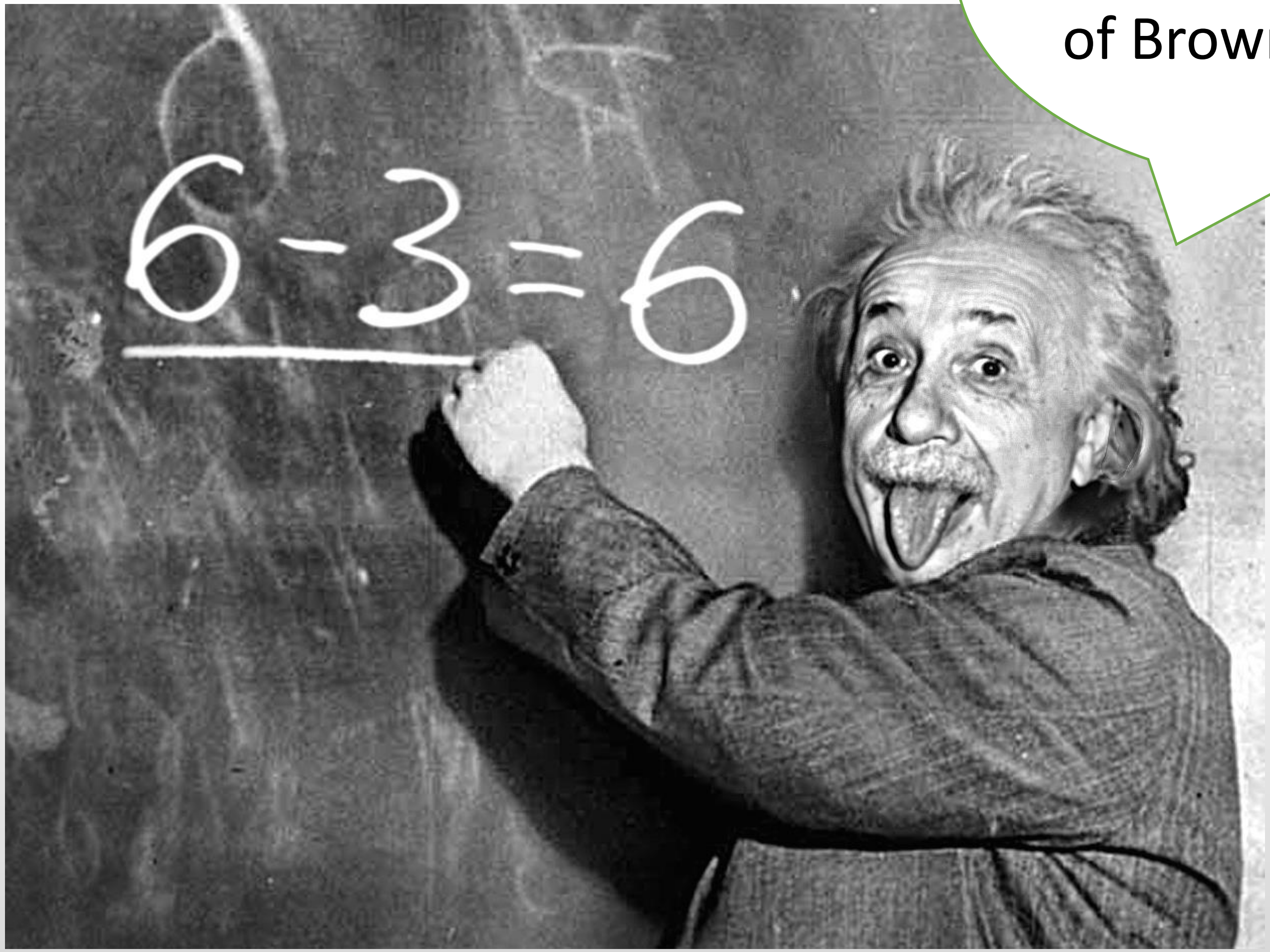
Li, T, et al. "Measurement of the Instantaneous Velocity of a Brownian Particle." Science 328.5986 (2010):1673-1675. Web.

Motivation



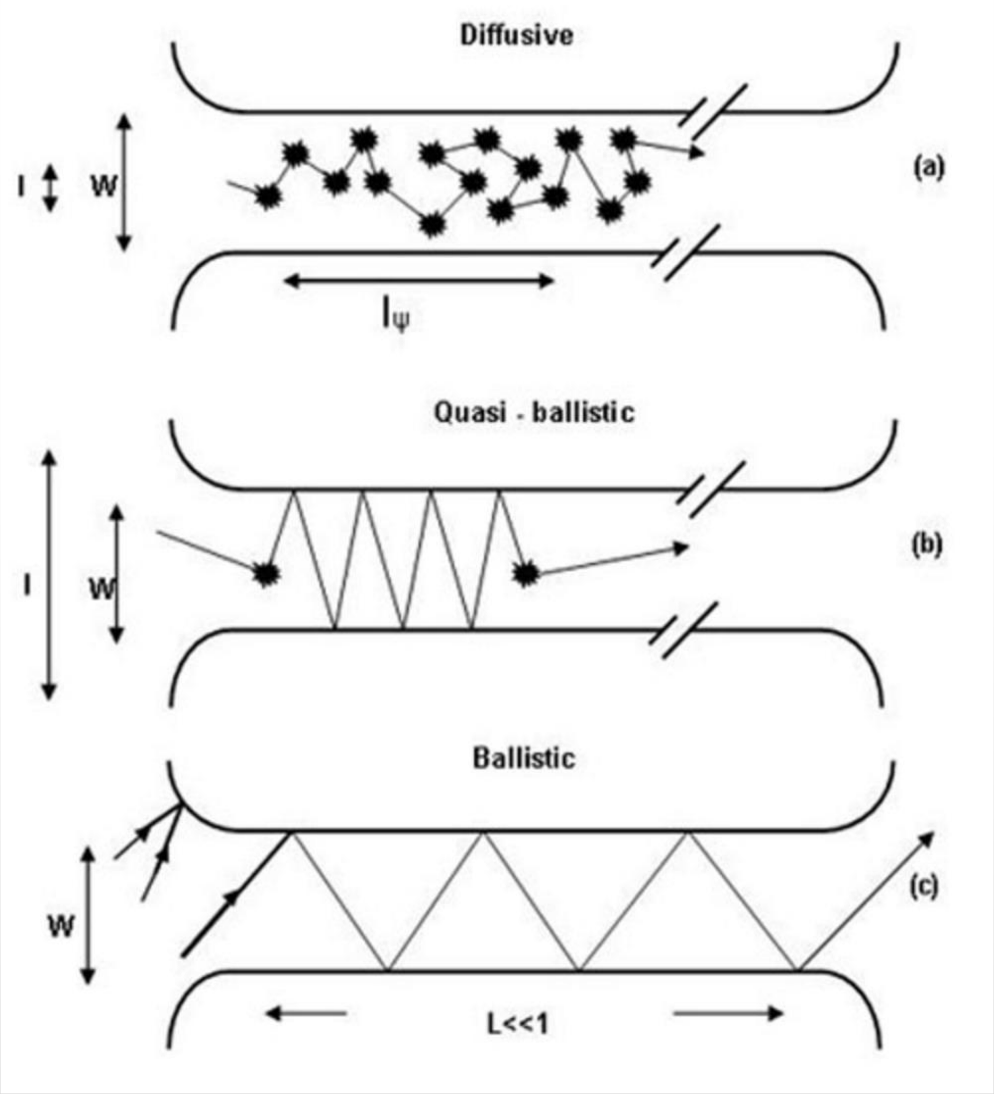
We can't measure the instantaneous velocity of Brownian motion!

- Apply trapping techniques to macroscopic particles
- Study Brownian motion in a gas

Brownian Motion Overview

- Brownian motion applies to many fields
- Different types of transport with various timescales:

Diffusive Motion	Ballistic Motion
Over long time scales compared to relaxation times	Over short time scales compared to relaxation times
Random motion	Dominated by inertia



Comparison of Brownian Motion Models

Model	Mean Square Displacement of Brownian Particle	Conditions
Einstein	$\langle[\Delta x(t)]^2\rangle = 2\frac{k_B T}{\gamma}t$	$t \gg \frac{m}{\gamma}$, free
Modified Einstein	$\langle[\Delta x(t)]^2\rangle = \frac{2mk_B T}{\gamma^2}\left[\frac{\gamma}{m}t - 1 + e^{-\gamma t/m}\right]\left(= \frac{k_B T}{m}t^2\right)$	$t > \frac{m}{\gamma}$, free
Langevin	$\langle[\Delta x(t)]^2\rangle = \frac{2k_B T}{m\omega^2}\left[1 - e^{\frac{-t\gamma}{2m}}\left((\cos(\omega_1 t) + \frac{\gamma\sin(\omega_1 t)}{2m\omega_1}\right)\right]$	$\forall t$, SHO

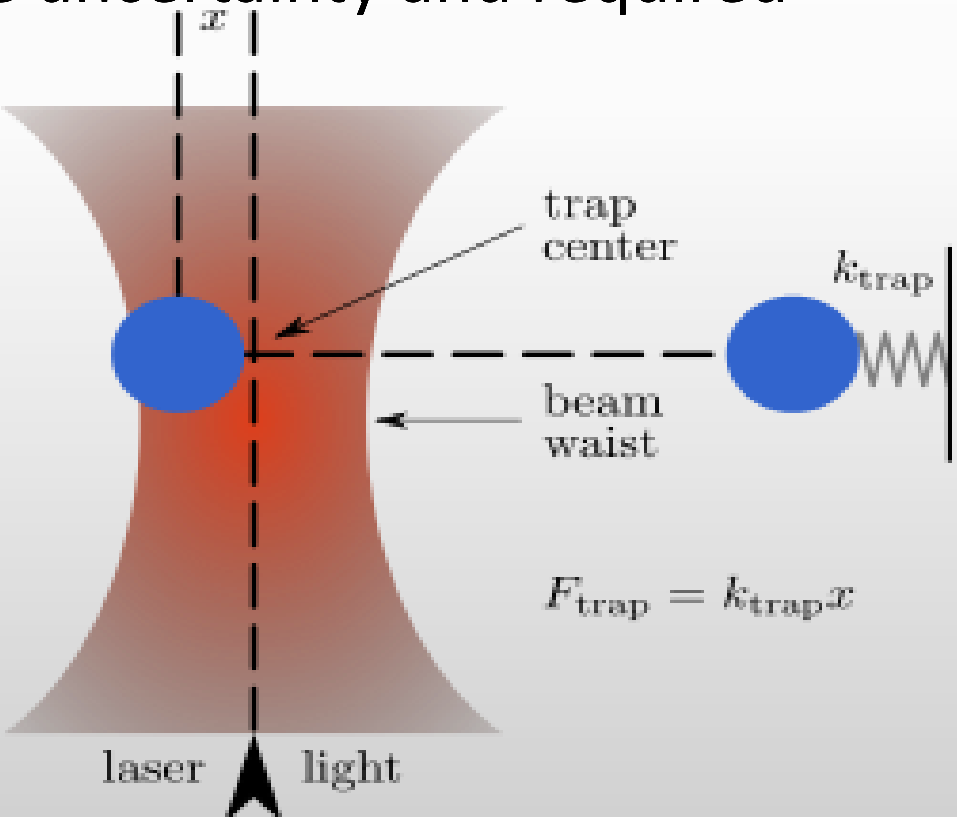
Experimental Approach

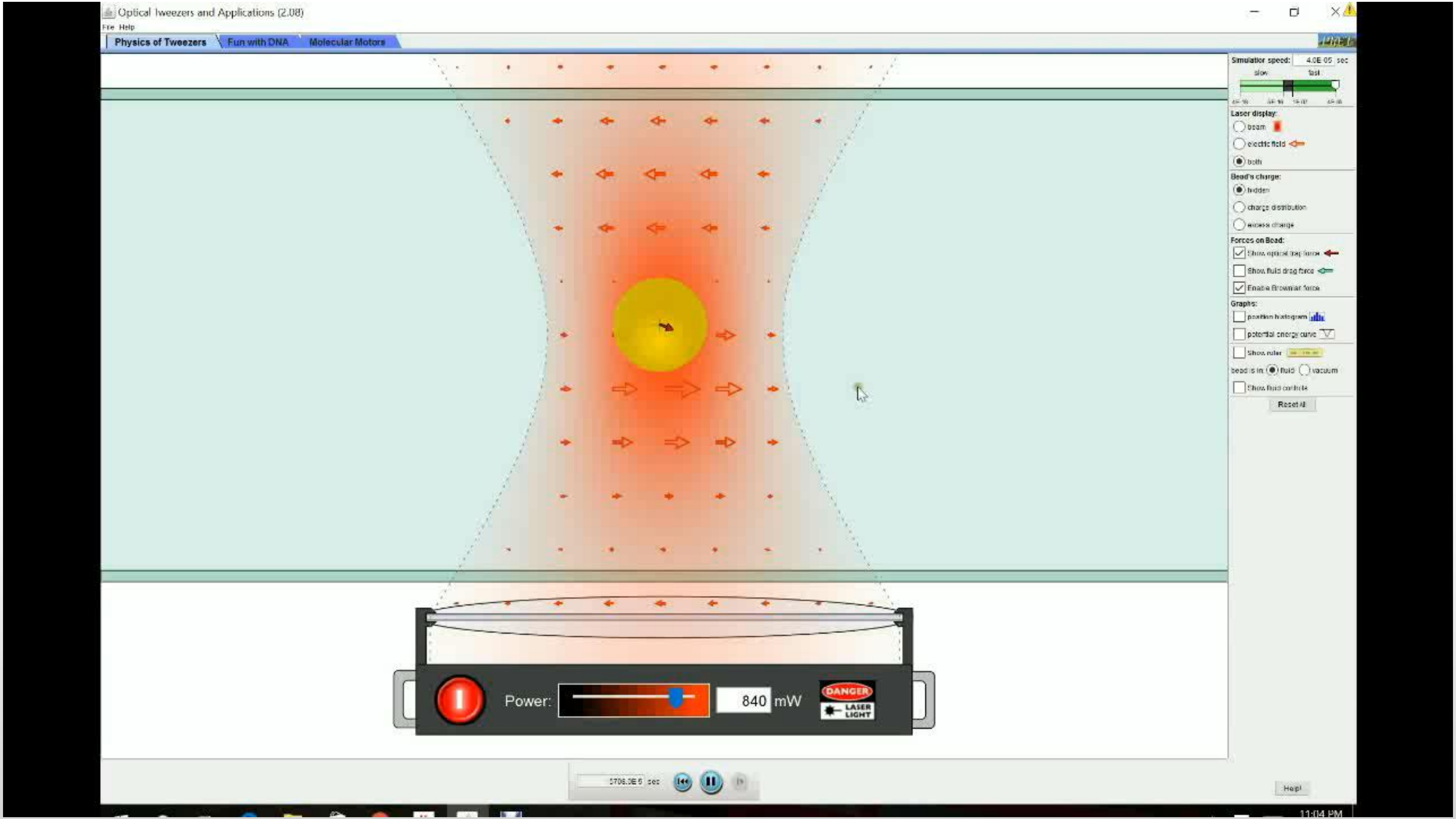
Use optical tweezers to make a 3D trap

- Allows us to suspend particle in gas
- Previous measurements in fluid had large uncertainty and required greater experimental resolution

Track motion of bead

- Determined by deflection of beam

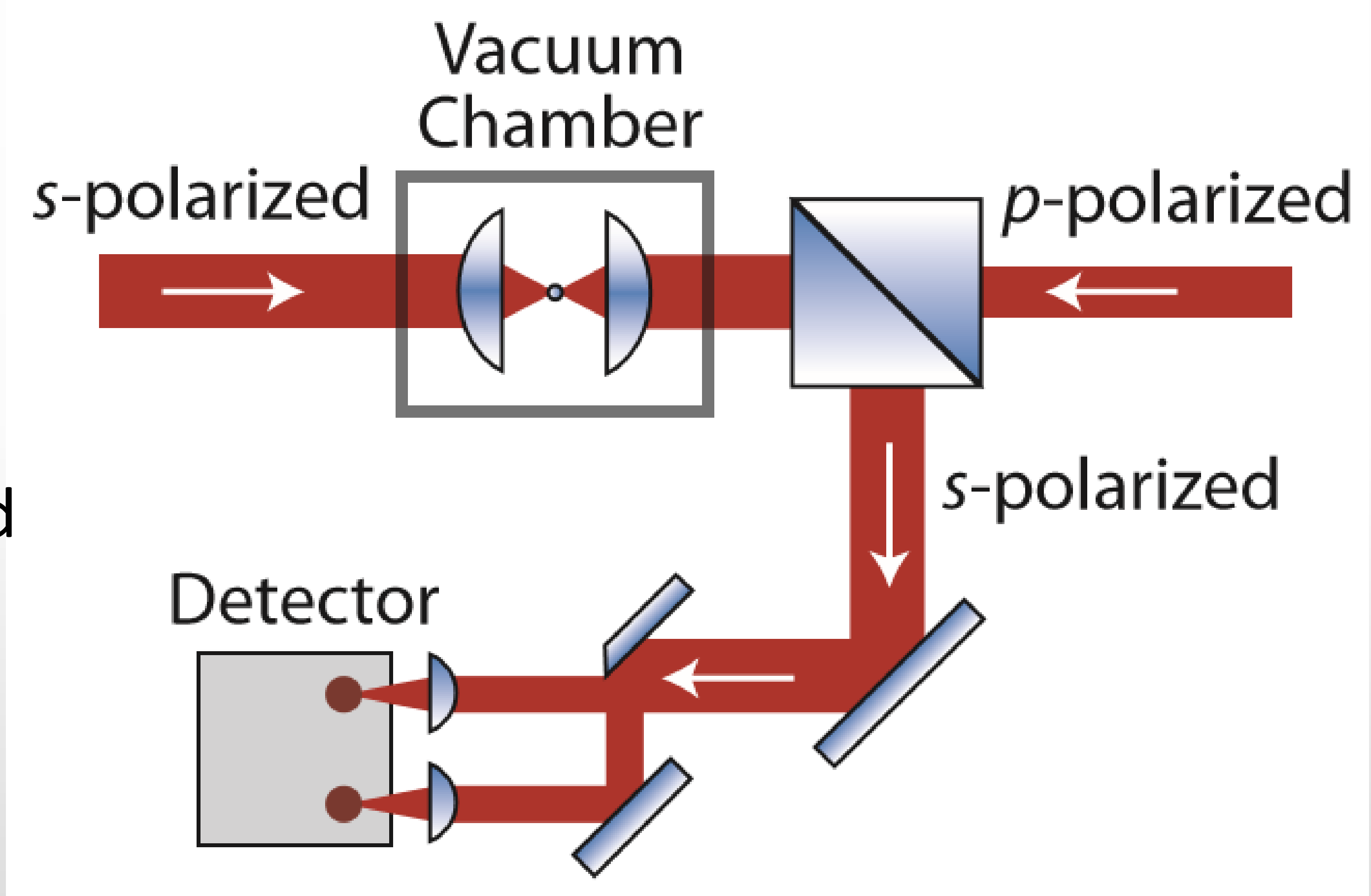




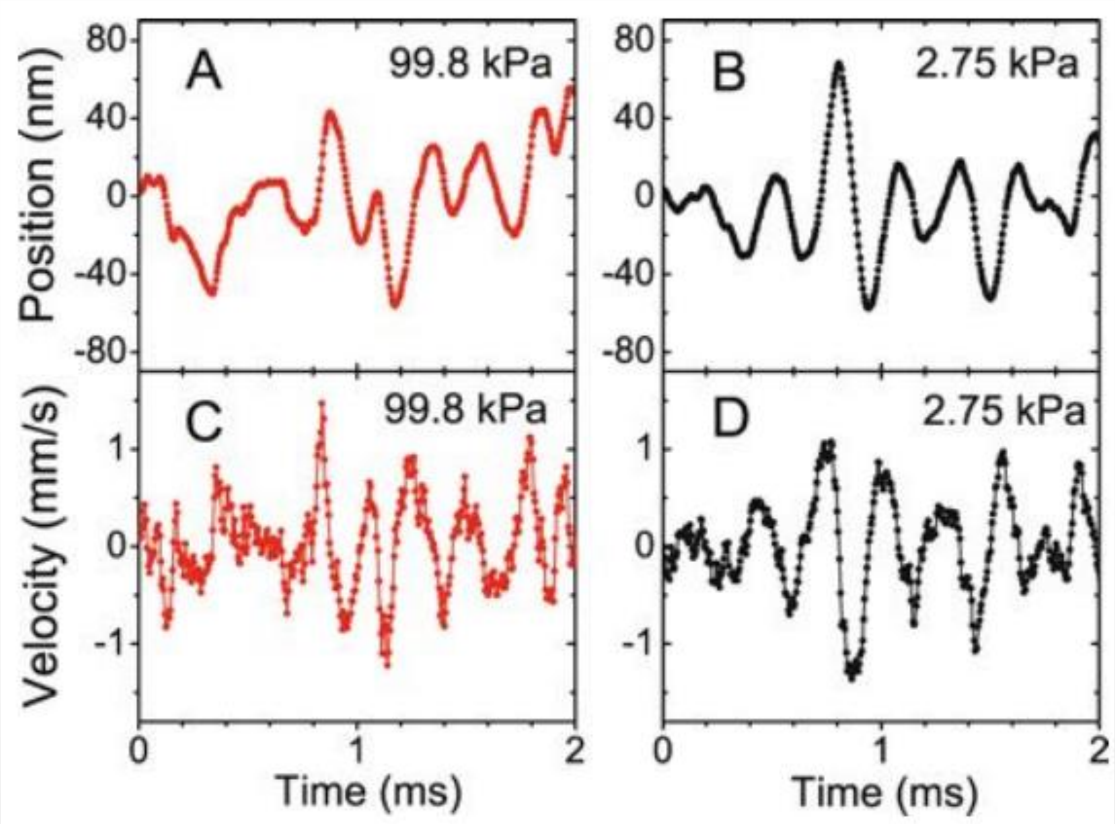
Optical Tweezers and Applications (2.08)
File Help
Physics of Tweezers
Fun with DNA
Molecular Motors
Simulation speed: 4.0E-05 sec
slow
fast
Laser display:
beam
electric field
both
Bead's charge:
hidden
charge distribution
Forces on Bead:
Show fluid drag force
Graphs:
potential energy curve
Show ruler
bead is in: fluid vacuum
Reset All
Power:
840 mW
DANGER
LASER LIGHT
sec
Help!
11:04 PM

Optical Tweezer Experimental Set-Up

- Orthogonally polarized, counter propagating beams
  - z trapping from momentum exchange
  - x and y harmonic potential from oscillating electric field
- 3 µm SiO_2 (disordered lattice) spherical bead
- 1 Å spatial resolution

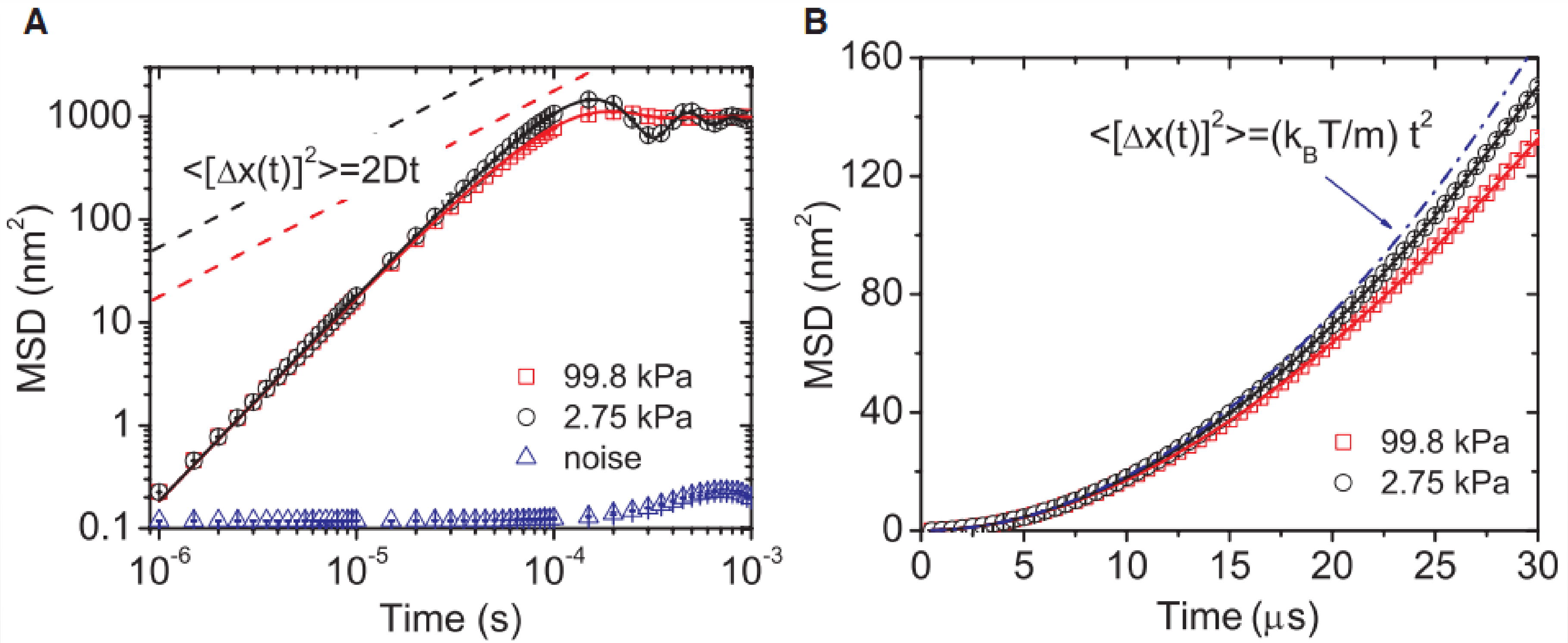


Measured Positions and Velocities of the Bead



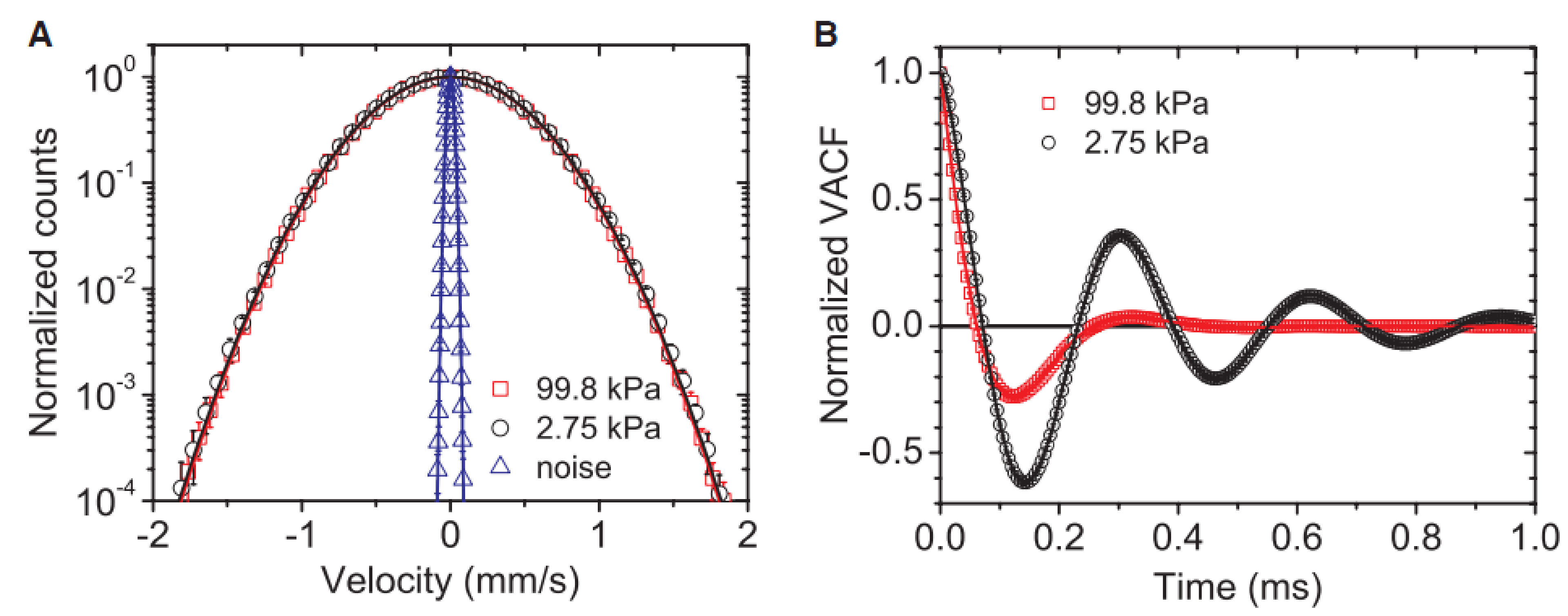
1D motion of 3-µm-diameter silica in different pressure

Measured Mean Squared Displacement of the Bead



"Mean Squared Displacement" $\langle[\Delta x(t)]^2\rangle$		
dashed line	long-time prediction by Einstein Eq.	don't agree
dash-dotted line	short-time prediction by Langevin Eq.	fit well
dotted line	experiment data	

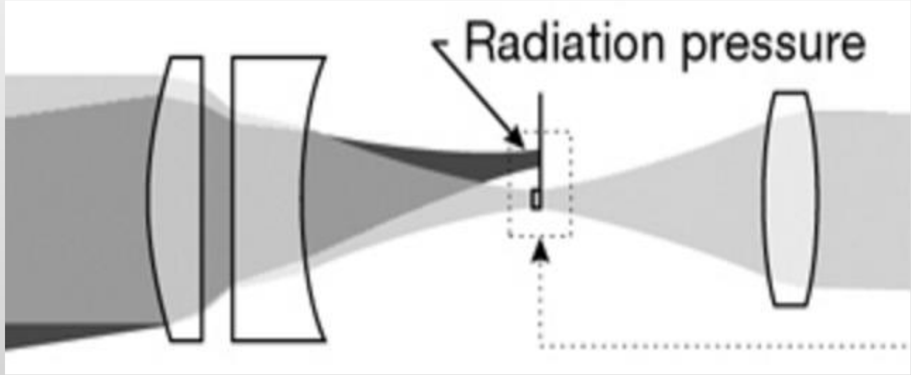
Measured Velocities Fit to Maxwell-Boltzmann

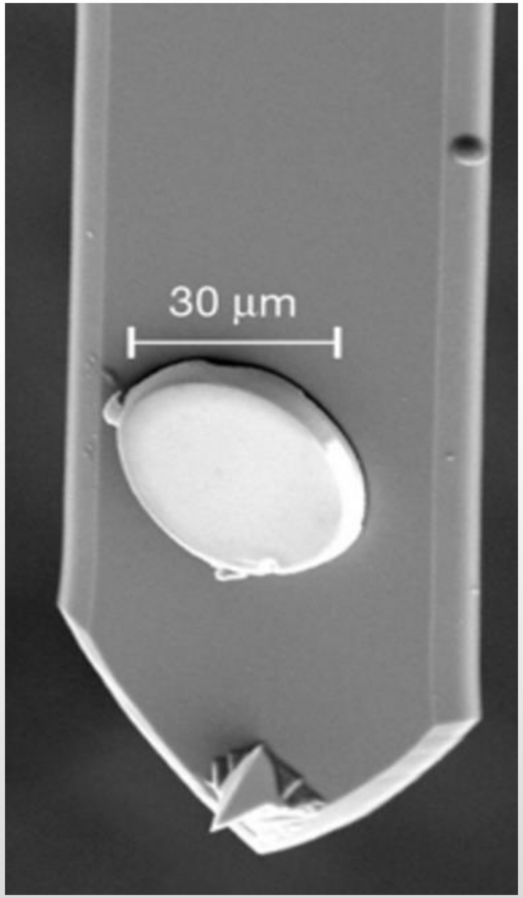


- Dotted line --- Experimental distribution
- Solid line --- Maxwell-Boltzmann distribution

Possible Next Steps Using This Laser Cooling Method

- Feedback controlled laser cooling of several macroscopic objects
- Measuring entropy production from the paths taken of the particle
- Quantum effects in mechanical systems

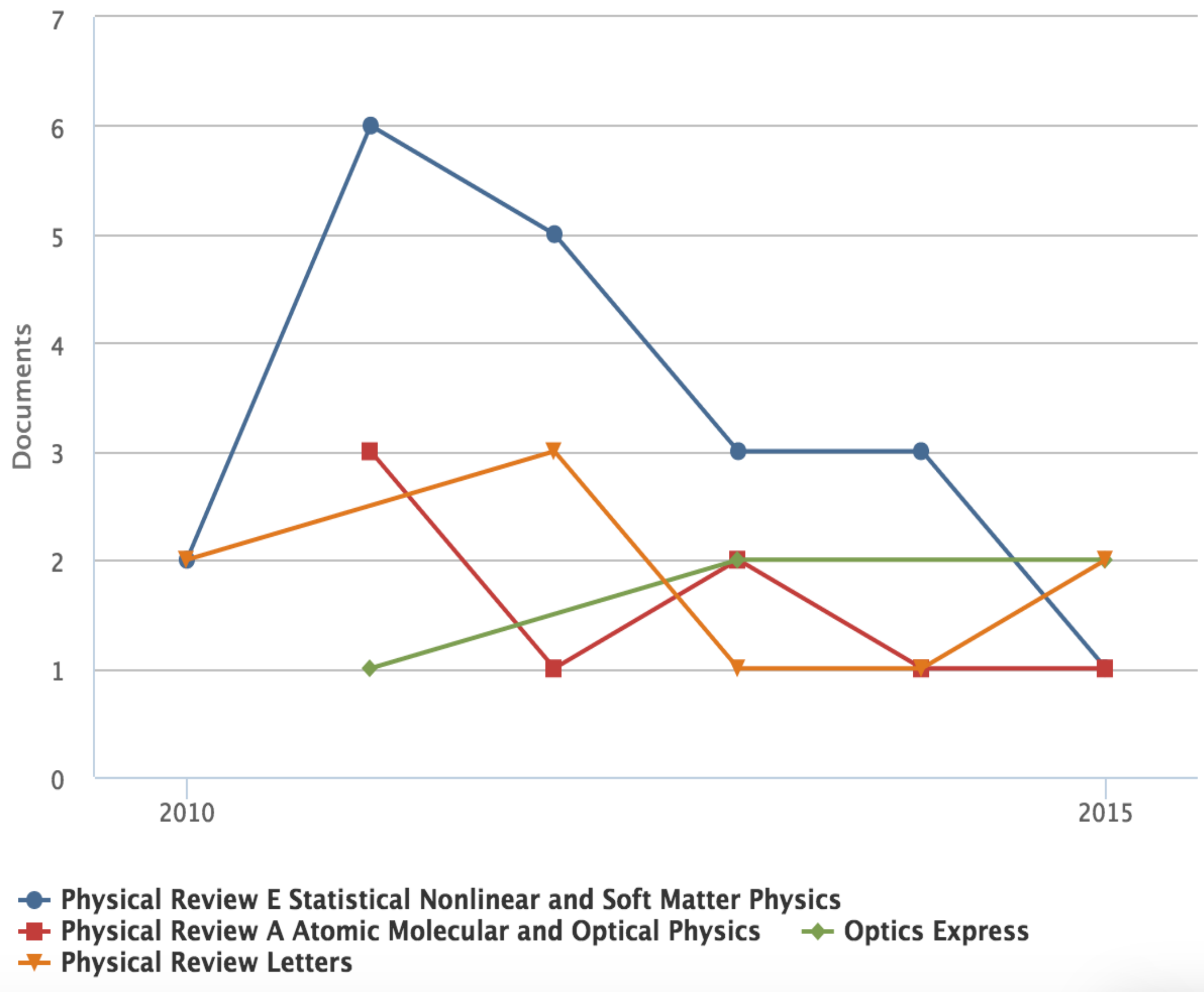




Citations

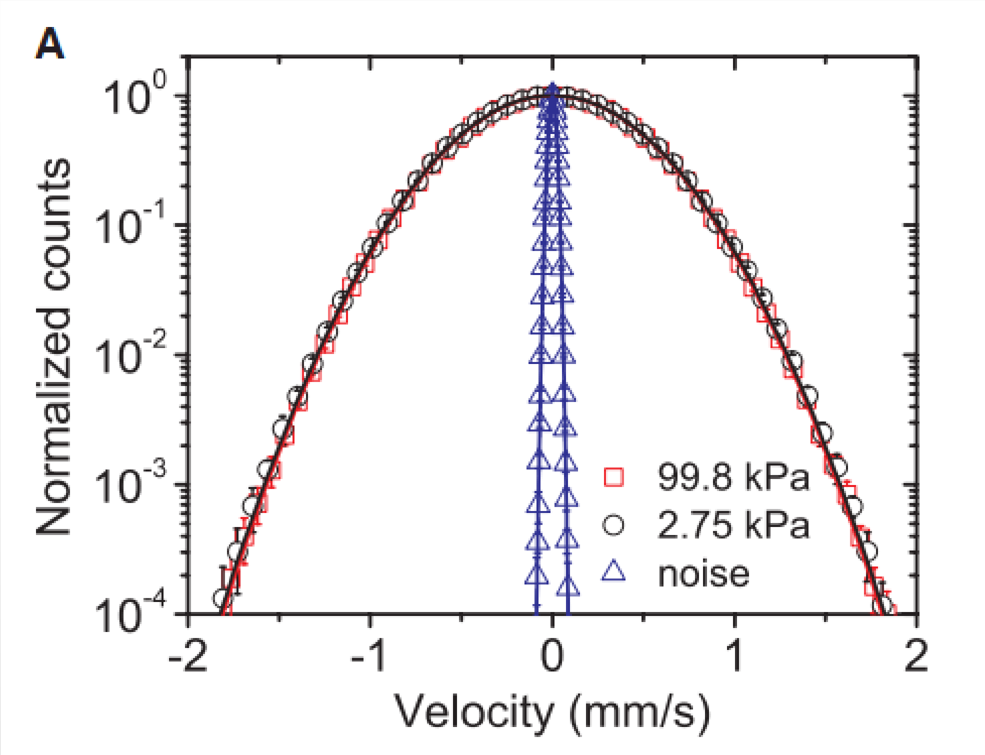
- Published in Science, 2010
- Google Scholar Citations: 202
- Scopus Citations: 144
- Citations by outside groups: 124

Year	Documents
2015	18
2014	24
2013	35
2012	33
2011	29
2010	5



Critiques

- Authors claim heat absorption from laser is negligible
- Comparing apples to oranges: SHO vs free space
- Error bars not discussed
- Normalized counts not defined



Summary

- Assumptions about inability to measure these velocities is wrong!
- Measurements align very well with the Maxwell-Boltzmann distribution
- Novel measurement opens a new sphere of applications

Acknowledgments

All the groups that sacrificed themselves by presenting before us RIP

Einstein Model of Diffusive Motion

The diffusion equation:

$$\frac{\partial P}{\partial t} = D \frac{\partial^2 P}{\partial x^2}$$

For free particle, near STP, $t \gg \frac{m}{\gamma}$, $D = \frac{k_B T}{\gamma}$

Yields

$$\langle [\Delta x(t)]^2 \rangle = 2Dt$$

$$\bar{v} = \sqrt{\frac{2D}{t}}$$

Modified Einstein Model

By modifying Einstein's diffusion equation

$$\frac{\partial P}{\partial t} = -\frac{1}{\gamma}\frac{\partial}{\partial x}\big(K(x)P(x,t)\big) + D\,\frac{\partial^2 P}{\partial x^2}$$

For general particle, near STP, $t > \frac{m}{\gamma}$

Yields

$$\langle[\Delta x(t)]^2\rangle = \frac{2mk_BT}{\gamma^2}\left[\frac{\gamma}{m}t - 1 + e^{-\gamma t/m}\right]$$

for $t \gg \frac{m}{\gamma}$ this is just Einstein again.

Langevin Model of Brownian Motion

Langevin Equation

$$\frac{d^2y}{dt^2} + \beta\frac{dy}{dt} + {\omega_0}^2 y = F(t)$$

For particle in SHO potential, $\forall t$, where $\beta = \frac{\gamma}{m}$

Yields

$$\langle[\Delta x(t)]^2\rangle = \frac{2k_BT}{m\omega^2}\left[1 - e^{\frac{-t\gamma}{2m}}\left((\cos(\omega_1 t) + \frac{\gamma\sin(\omega_1 t)}{2m\omega_1}\right)\right]$$

For bound particle, all t, $\omega_1 = \sqrt{\omega^2 - \frac{1}{4}\left(\frac{\gamma}{m}\right)^2}$