Wave-Based Turing Machine: Time Reversal and Information Erasing

S. Perrard, E. Fort, and Y. Couder
Phys. Rev. Lett. **117**, 094502 (2016)

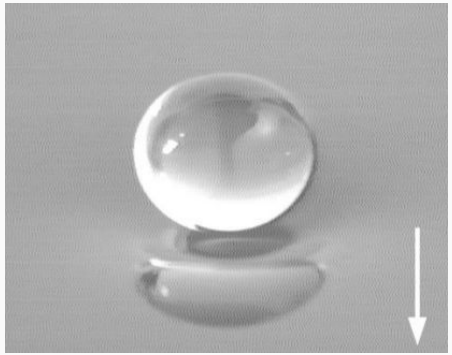
Journal Club Presentation: PHYS 596

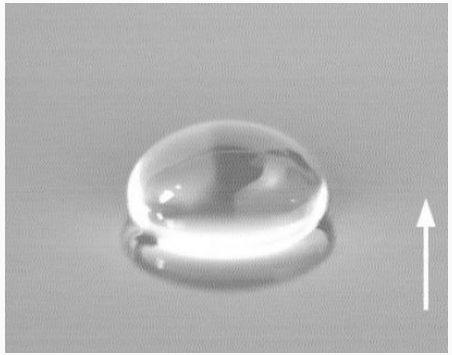
Elle Shaw, Raman Sohal,
Varsha Subramanyan, Saavanth Velury

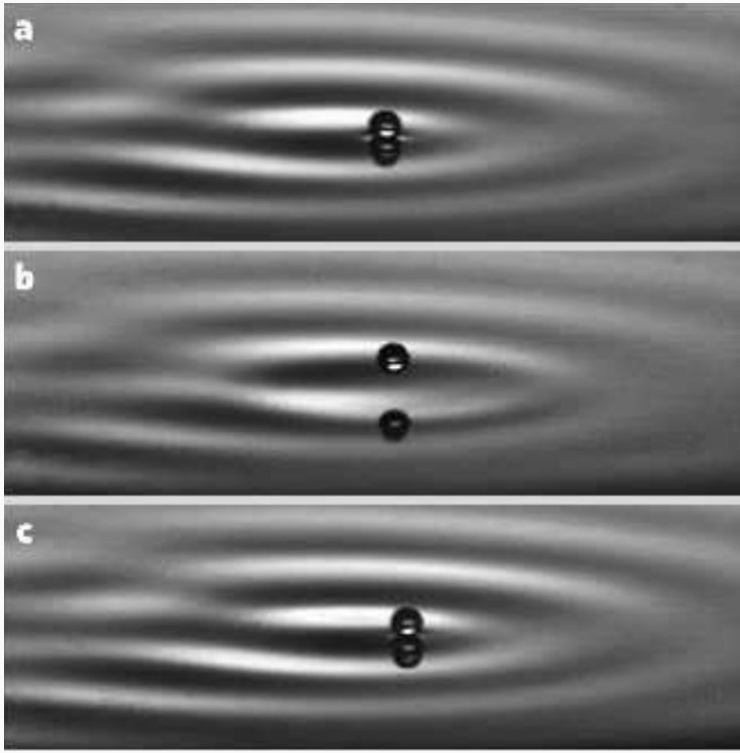
December 2, 2016

Walkers - Coupling Droplets to Waves

- Driven vertical oscillation of fluid can produce bouncing droplets
- Just below a critical value of the driving force, droplets begin to move at a constant velocity
- The coupled droplet-wave system is called a **Walker**







Y. Couder, S. Protiere, E. Fort, and A. Boudaoud, Nature (London) 437, 208 (2005)

Y. Couder, E. Fort, C.-H. Gautier, and A. Boudaoud, Phys. Rev. Lett. 94, 177801 (2005)


When the vibration amplitude is increased, the drop achieves resonance with its Faraday wave field and is propelled along the bath surface.

Wave field stores memory of droplet trajectory

- Integro-Differential equation of motion describes coupling of oil droplet position (x_P) to the wave field it produces (described by Bessel functions):

$$m\ddot{\boldsymbol{x}}_p + D\dot{\boldsymbol{x}}_p = \frac{F}{T_F}\int_{-\infty}^{t} \frac{\mathrm{J}_1(k_F|\boldsymbol{x}_p(t)-\boldsymbol{x}_p(s)|)}{|\boldsymbol{x}_p(t)-\boldsymbol{x}_p(s)|}(\boldsymbol{x}_p(t)-\boldsymbol{x}_p(s))\mathrm{e}^{-(t-s)/(T_F M_e)}\,\mathrm{d}s$$

- **Memory** quantified as:

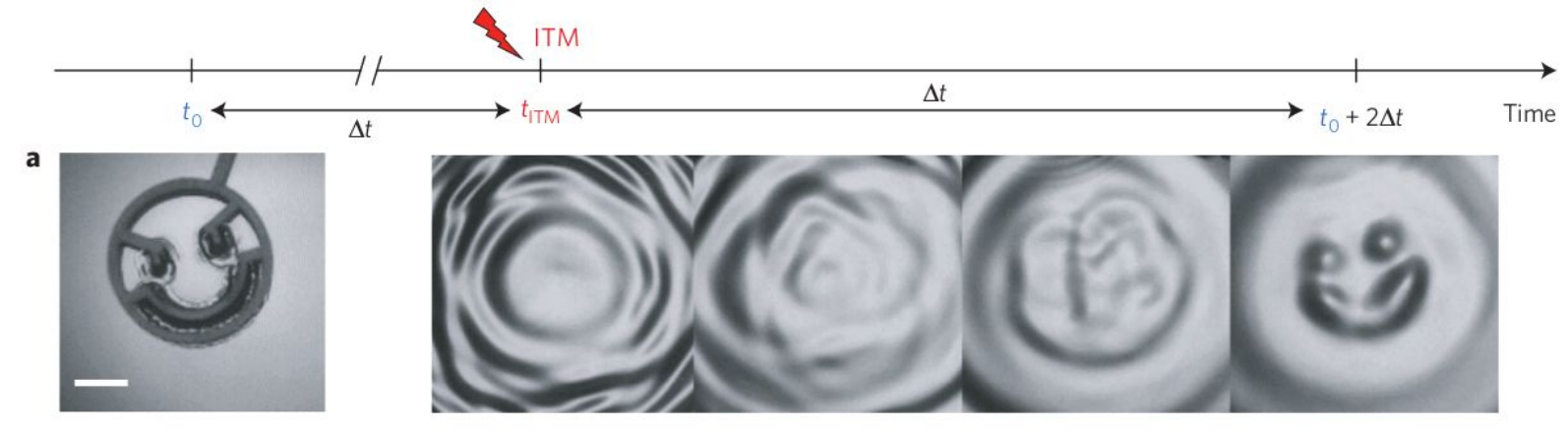
$$M_e = M_e(\gamma) \equiv \frac{T_d}{T_F\,(1-\gamma/\gamma_F)}$$

- Stability analysis: chaotic dynamics possible in confined geometry

A. U. Oza, R. R. Rosales, and J. W. M. Bush, J. Fluid Mech. 737, 552 (2013)

Waves exhibit time-reversal symmetry in chaotic regime

- Chaos – sensitivity to initial conditions
- Particles and waves behave differently
  - Waves respect time reversal symmetry in the chaotic regime while particles do not.
- What happens to a **particle coupled to a wave field**?

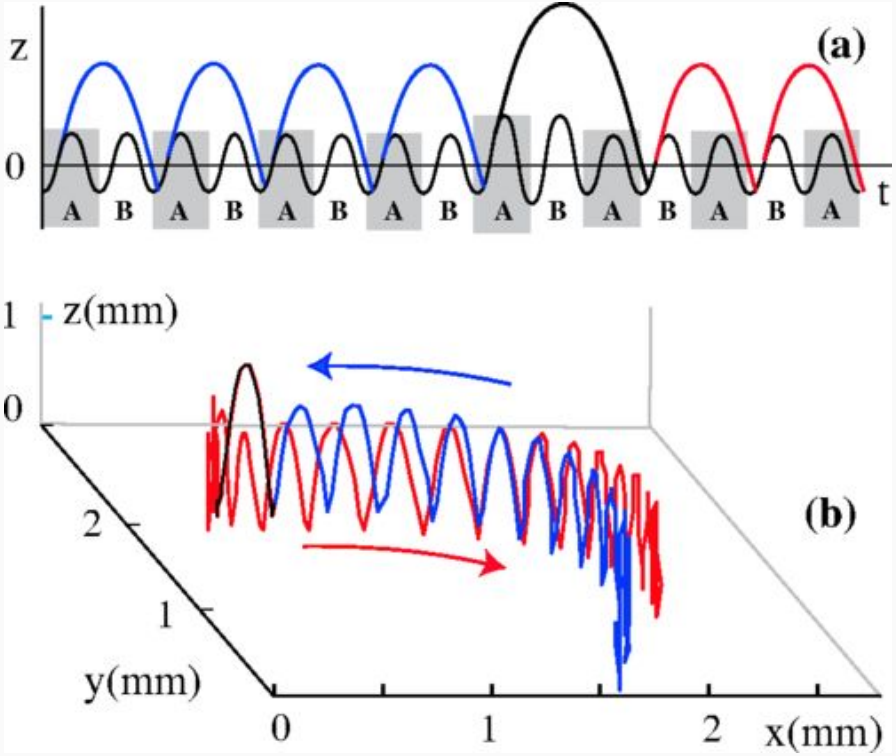


Water waves are prepared in a given shape and allowed to propagate. At ‘ITM’ the waves are reversed and propagate ‘back in time’, reforming the initial shape.

V. Bacot, M. Labousse, A. Eddi, M. Fink, and E. Fort, Nat. Phys. (2016)

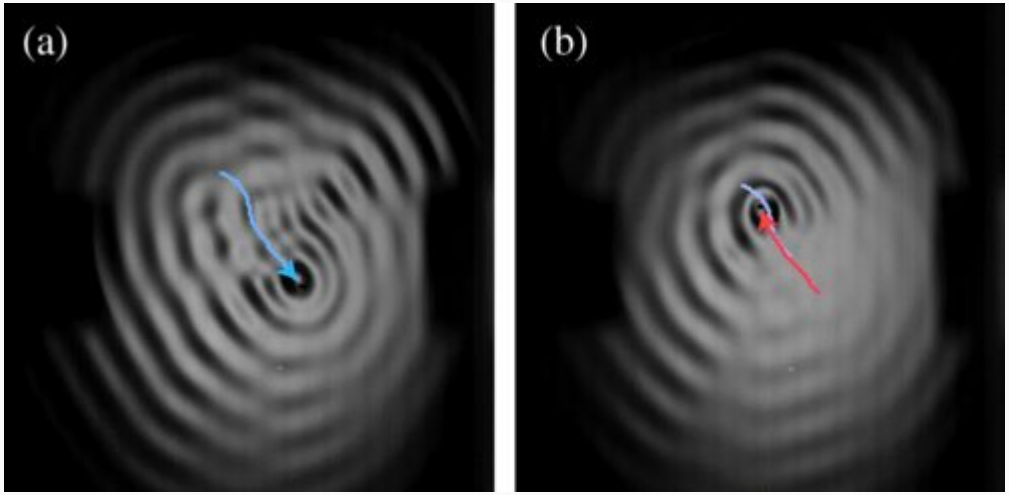
How is it done?! (Experimental implementation of time-reversibility)

- Bath of silicon oil with a certain viscosity is oscillated vertically
- The drop is propelled by the interaction with the waves
- A π shift is induced by a brief and controlled disturbance of the forcing oscillation - the drop receives a kick of larger amplitude
- The phase shift of π corresponds to reversal of time



Waves play the role of a global information repository

- Walkers respect time reversal symmetry (just like waves)
- Temporal reversibility emerges from an erasing process of the pre-generated wave field
- The dynamics of the walker can be understood in terms of writing, storing, reading and erasing processes



Visualization of forward (blue) and backward (red) trajectories. Smaller amplitude waves can be observed in reverse trajectory in Fig. (b)

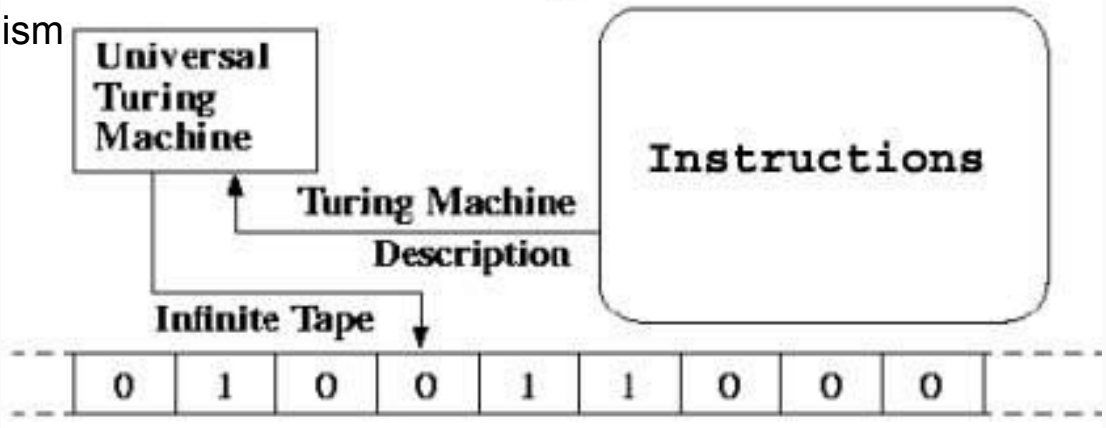
Goal: Creating a Wave-Based Turing Machine Analogy

What is a Turing machine?

It's a hypothetical computing machine that can simulate any computer algorithm using a predefined set of rules or operations to determine a result from a set of input values.

Basic Operations

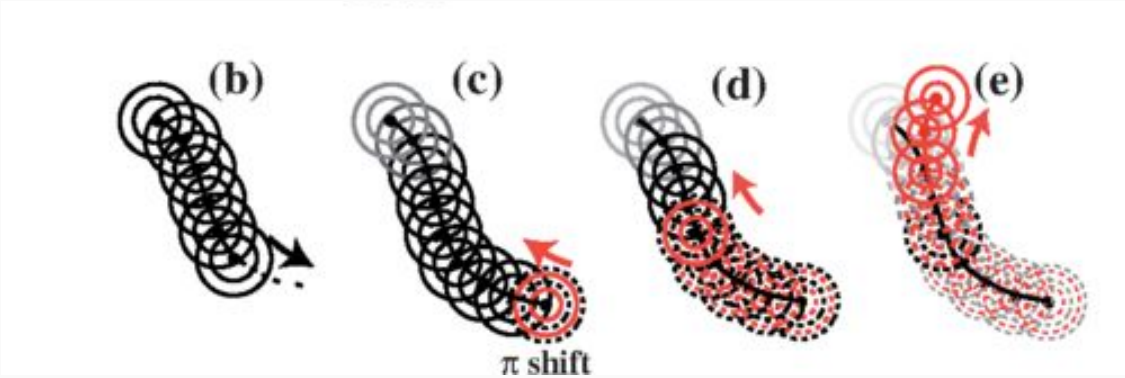
- Write - 1s and 0s
- Store - Tape is storage mechanism
- Read - Machine accepts input
- Erase - Can write over data
- Move Tape (Reversibility)



Goal: Creating a Wave-Based Turing Machine Analogy

Wave-Based Turing Machine

- “Write” - Bounce encodes positional information by generation of a standing Bessel wave
- “Store” - Global wave field stores history of the trajectory for some given time
- “Read” - Local slope of the interface gives horizontal ‘kick’ to the drops next jump
- **“Erase” - pi shift → emitted waves have phase opposite the initial waves and bounce by bounce the drop erases the previous wave field.**



Bessel waves created by each bounce (black), then the subsequent annihilation of the waves as the path is retraced following the π phase shift (red)

Critique and Analysis

- As the authors themselves claim, implementation of the Turing Machine in this way is difficult and impractical.
- It might not be feasible to store useful, digital data (as we know it today) despite the similarity to Turing Machines
- Possible future work
  - Further exploration of the information theory connection
  - Ways to quantify the information being stored or erased
  - Information theoretic ways of quantifying “memory” (like Von Neumann entropy) can be established through the thermodynamics of the system

Outlook

- Hot off the press! (No Citations, yet!)
- Relatively small field of study
- Could have larger impacts as the relationship between walkers and quantum mechanics is explored, as walkers behave analogous to several quantum systems (Hydrodynamic Quantum Analogs)
  - Exhibit particle diffraction, quantum tunneling, quantized orbits, the Zeeman effect
- Might also have pedagogical applications
  - Visualization of wave functions
  - Time evolution of wave functions
  - Wave-particle duality