Fate of the False Vacuum

Or: How We Learned to Stop Worrying and Love Instanton Calculations

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The paper

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Fate of the false vacuum: Semiclassical theory*

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It is possible for a classical field theory to have two homogeneous stable equilibrium states with different energy densities. In the quantum version of the theory, the state of higher energy density becomes unstable through barrier penetration; it is a false vacuum. This is the first of two papers developing the qualitative and quantitative semiclassical theory of the decay of such a false vacuum for theories of a single scalar field with nonderivative interactions. In the limit of vanishing energy density between the two ground states, it is possible to obtain explicit expressions for the relevant quantities to leading order in h; in the more general case, the problem can be reduced to solving a single nonlinear ordinary differential equation.

Motivation

- We are often confronted with systems which can settle into any of a large number of local energy minima.
- Often the minimum, or vacuum, a system falls into is not the true ground state. We call this minimum the false vacuum.
- The system can reach the true ground state either via thermal or quantum fluctuations.
- In this talk, we discuss a method of approximating the time scale on which a false vacuum is stable to quantum fluctuations in a quantum field theory (QFT).

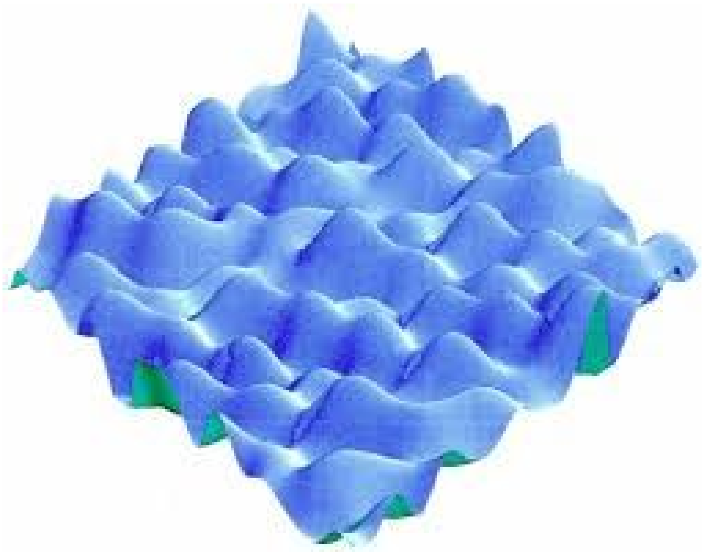
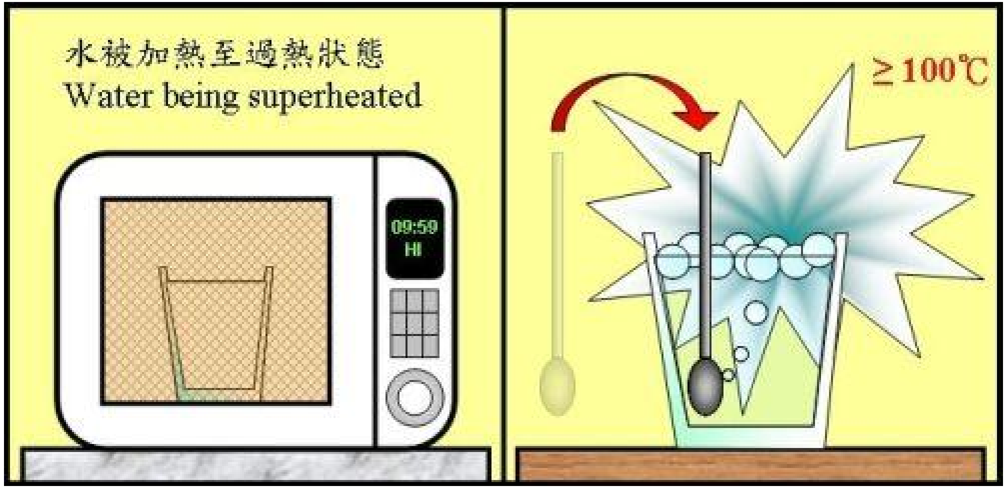


Fig: The string theory landscape

General idea: superheated liquids

- Heat water past its boiling temperature and into its “superheated liquid” phase (the “false vacuum”)
- There will be some chance a bubble of vapor (the “true vacuum”) forms. If the bubble is big enough, it will expand and drive the whole liquid into the vapor phase.

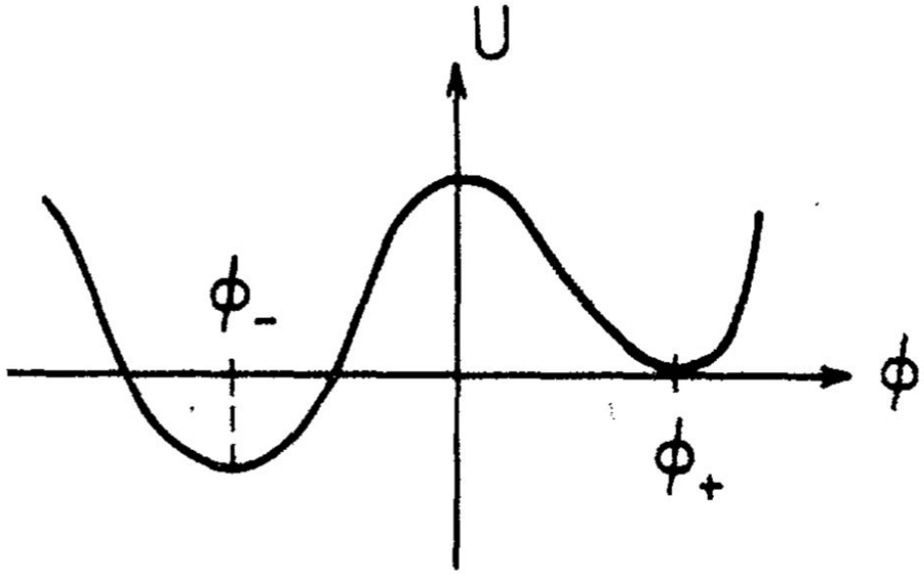


The quantum problem is not all that different from the superheated liquid

- We could easily be living in a false vacuum like the superheated liquid, but where quantum tunneling would drive our transition to the true vacuum.
- The probability of a bubble forming in the superheated liquid is set by the temperature. In the quantum case, this probability is set by $\hbar$.
- We will therefore be interested in looking for dominant tunneling processes, or instantons, in an expansion in powers of $\hbar$.

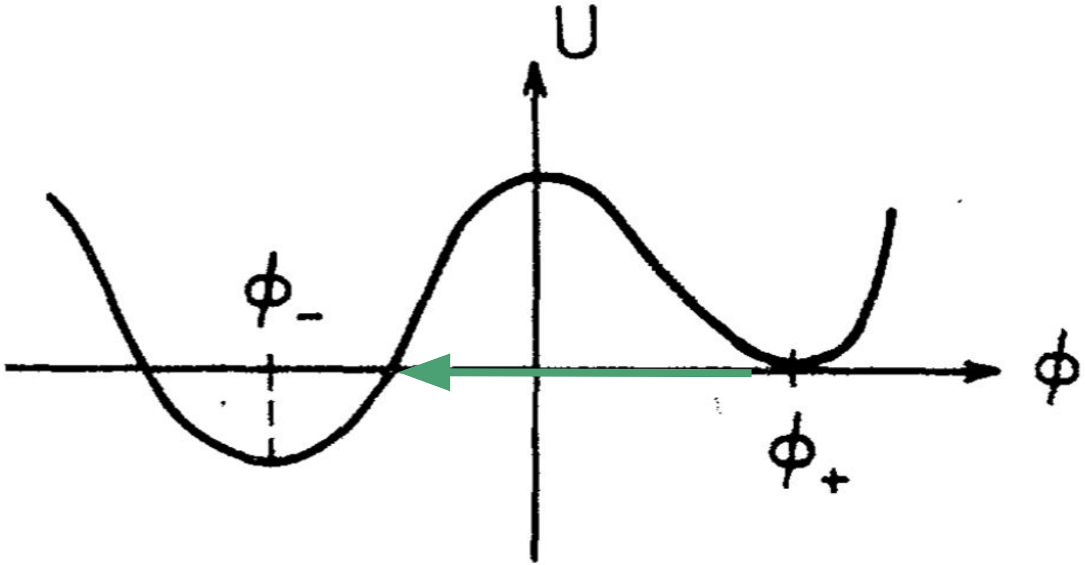
A false vacuum in scalar field theory

- Consider a scalar field ϕ. This field can describe a particle like the Higgs or the inflaton. It can describe the mean field of some magnetic system, etc.
- This field lives in the tilted potential U shown here, with a false vacuum at ϕ_+ and the true vacuum at ϕ_-.



Decay events of interest

We will be interested in quantum tunneling from the false to true vacuum. We will not consider fluctuations which hop the field over the barrier separating the false and true vacuum.



Tunneling amplitude

We are interested in calculating the probability amplitude for tunneling from the false to true vacuum per unit time. We first note that the full transition amplitude is

$$\langle \phi_-(t) | \phi_+(0) \rangle = \langle \phi_- | e^{iHt/\hbar} | \phi_+ \rangle = \int \mathcal{D}\phi e^{iS/\hbar}$$

Where the integral is over field configurations which satisfy the right boundary conditions.

Notice that the oscillatory behavior of the propagator makes this problem seem intractable.

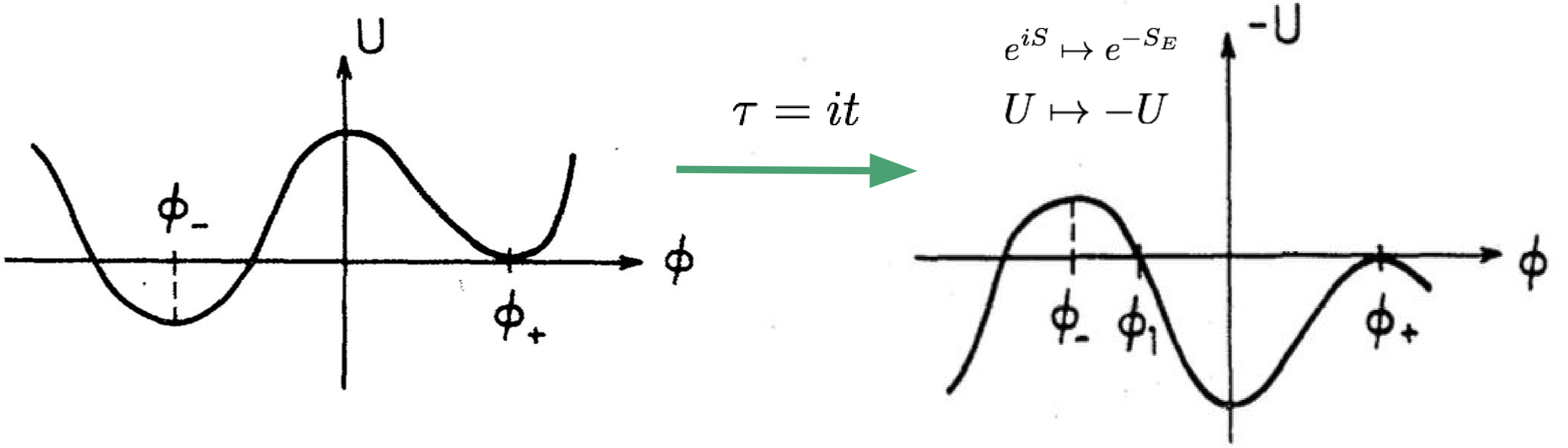
However, we have the major tool of the field theorist at our disposal…

....Wait for it...

Analytic Continuation!

The imaginary time path integral in pictures

By analytic continuation, we mean the rotation to imaginary time, as shown below. This eliminates the oscillatory behavior of the integrand, implying that it is dominated by the minima of the classical action with flipped potential S_E, i.e. solutions which “bounce” between the classical turning points, ϕ_+ and ϕ_1.



The uses of instantons: calculating the decay rate

Approximating the path integral by the integrand of minimum action amounts to an expansion in powers of $\hbar$, i.e. the decay rate Γ is

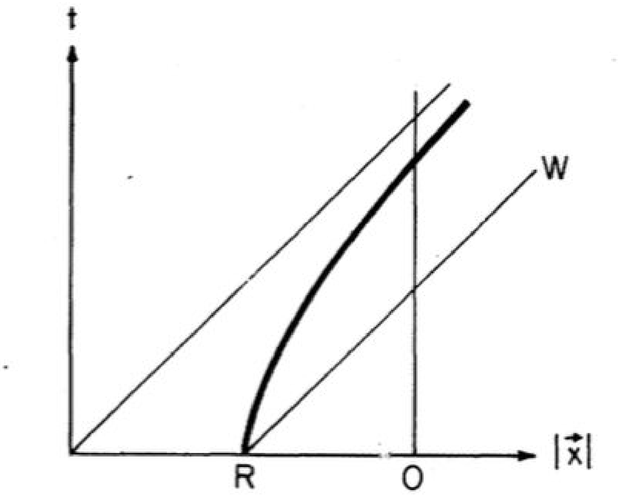
$$\Gamma = Ae^{-B/\hbar}[1 + O(\hbar)]$$

where $B = S_E[\phi_{cl}]$.

The field configurations ϕ_{cl} which minimize the action are known as instantons. As said in the previous slide, they can be thought to “bounce” between the classical turning points ϕ_+ and ϕ_1 .

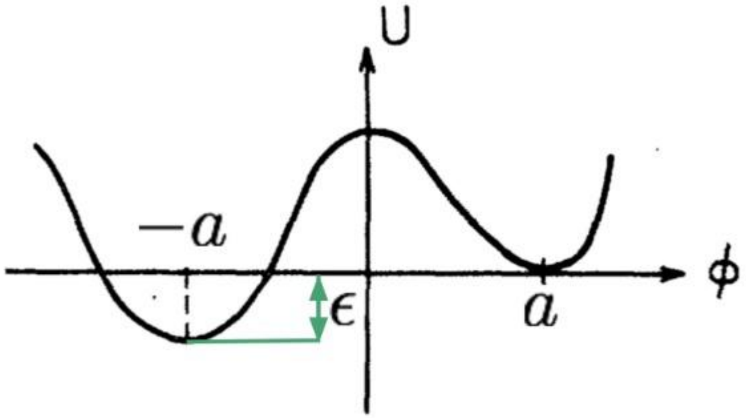
Geometric structure of the instantons: HyperBubbles!

- These instanton configurations turn out to be spherical bubbles (in the Euclidean spacetime) which expand at the speed of light.
- The region inside of the bubbles corresponds to the true vacuum, while at spatial infinity the false vacuum persists. This is just like the superheated liquid!
- The radius R of the bubble as a function of time is shown (bold curve). Notice that the speed of this expansion is asymptotically the speed of light (the slope of the line from the origin).



Perturbative calculation

In practice, actually calculating the path integral over the instanton configurations is still nigh impossible without appealing to additional perturbative methods.



Consider a symmetric, degenerate potential with minima at $\pm a$, e.g.

$$U_+ = \frac{\lambda}{8}\left(\phi^2 - \frac{\mu^2}{\lambda}\right)^2$$

where $a = \sqrt{\mu^2/\lambda}$. We then take an infinitesimal tilt

$$U = U_+ + \frac{\epsilon}{2a}(\phi - a)$$

where ϵ is the small-energy difference between the true and false vacua

Results: decay rate of the false vacuum

When the dust settles, we obtain the imaginary time instanton action

$$B = S_E = 27\pi^2 S_1^4 / 2\epsilon^3$$

where $S_1 = \int_{-a}^{a} d\phi \sqrt{2U(\phi)} = \frac{\mu^3}{3\lambda}$ is the unperturbed action of the instanton.

While seemingly ugly, the above expression comes from the fact that the radius of an instanton bubble here is proportional to S_1/ϵ.

To get the decay rate, the above result can be plugged into

$$\Gamma \sim \exp[-B]$$

Selected cameo appearances

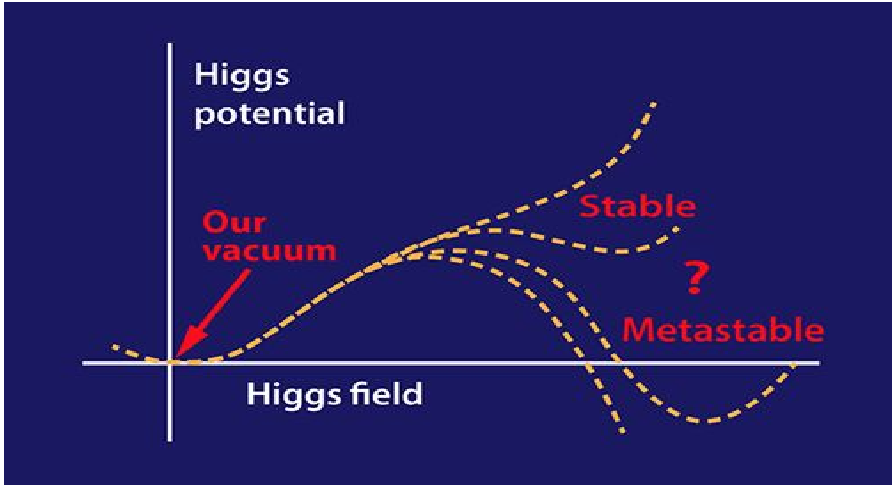
- Particle physics: stability of the electroweak vacuum
- String theory/cosmology: the KKLT construction
- Condensed matter physics: spin glasses

Particle physics: stability of the electroweak vacuum

- The Standard Model predicts gauge bosons get their mass from the Higgs field. Its potential of this field is given by

$$V(\phi) = -\frac{1}{2}\mu^2(\phi^\dagger\phi) + \lambda(\phi^\dagger\phi)^2$$

- LHC result for the Higgs and top quark masses imply $\lambda = -0.14 \pm 0.006$ at high energies, where the quartic term dominates [4]. Without new physics, this leads to a new, true vacuum at large field values. This is cause to fear for our lives.
- Lucky for us, we can safely believe that the lifetime of this false vacuum is larger than the age of the universe



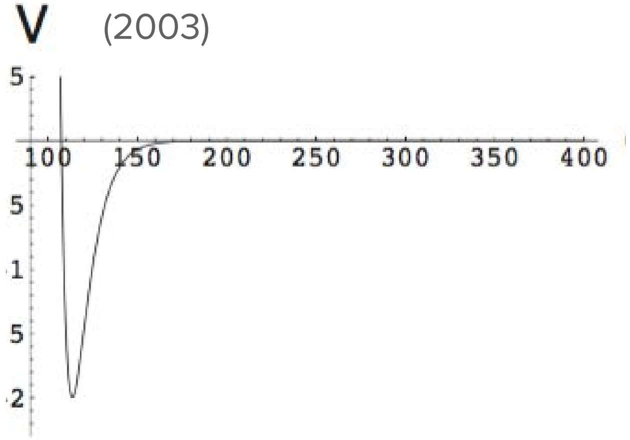
[3] A. Kusenko, APS Physics 8, 108 (2015).

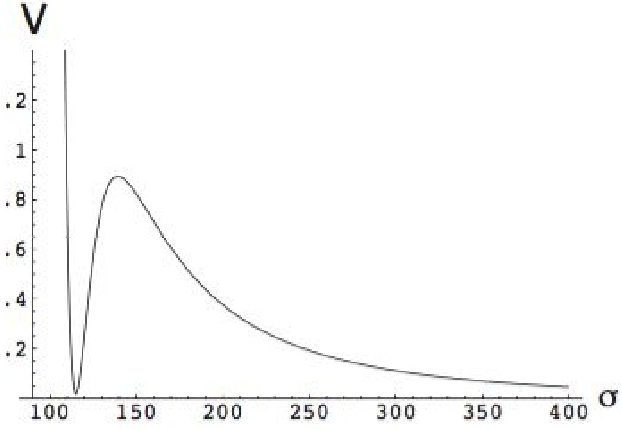
[4] G. Degrassi *et al.*, JHEP 1208 (2012) 098.

String theory: KKLT

- In string theory, vacua corresponding to universes of negative curvature are ubiquitous (top).
- It is really hard to construct vacua in string theory giving a universe undergoing accelerated expansion like our own (de Sitter, bottom). Such universes have positive curvature.
- The KKLT construction [5] was the first realization of a de Sitter vacuum in string theory. But *this vacuum is only metastable*.
- The authors actually use a gravitational version of the formalism we discussed (also developed by Coleman) to deduce the lifetime of their de Sitter vacua, which they find to be longer than the age of the universe (~14 billion years).

[5] S. Kachru, R. Kallosh, A. Linde, S. Trivedi, PRD 68, 046005 (2003)





Condensed matter: spin glasses

- Magnet stuck with a metastable, disordered spin configuration

- J_{ij} -- random variable which determines nearest-neighbor interactions between spins, generates a rugged free-energy landscape

- As we reduce the temperature, system becomes "trapped" in a local, disordered local energy minimum that is not the true ground state

$$H = -\sum_{\langle ij \rangle} J_{ij} S_i S_j$$

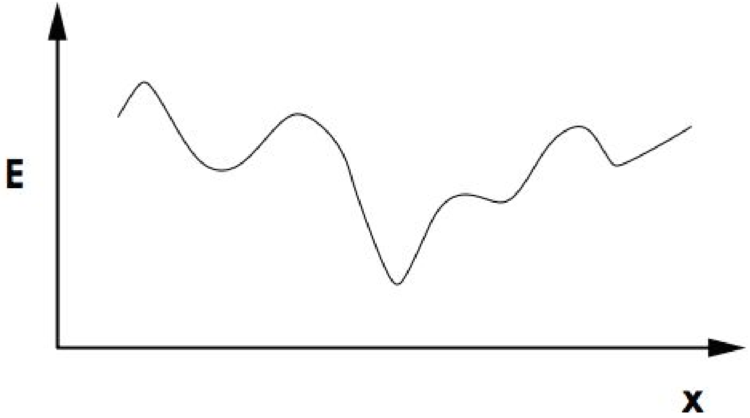


Fig: rough example of a spin glass energy landscape.

Connection to Coleman's analysis

- Replace quantum fluctuations with thermal ones, although quantum spin glasses are also very interesting.
- Provides platform to study stability of glass to thermal fluctuations
- Applications beyond materials: neural networks and the brain [7]

[6] M. Advani, S. Lahiri, and S. Ganguli, J. Stat. Mech. 03, P03014 (2013).

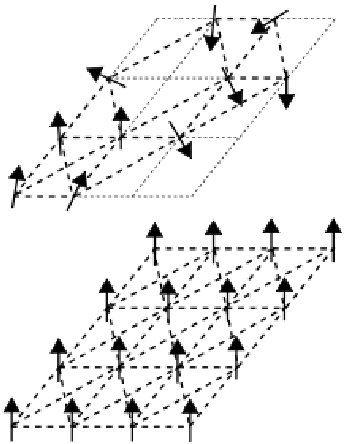
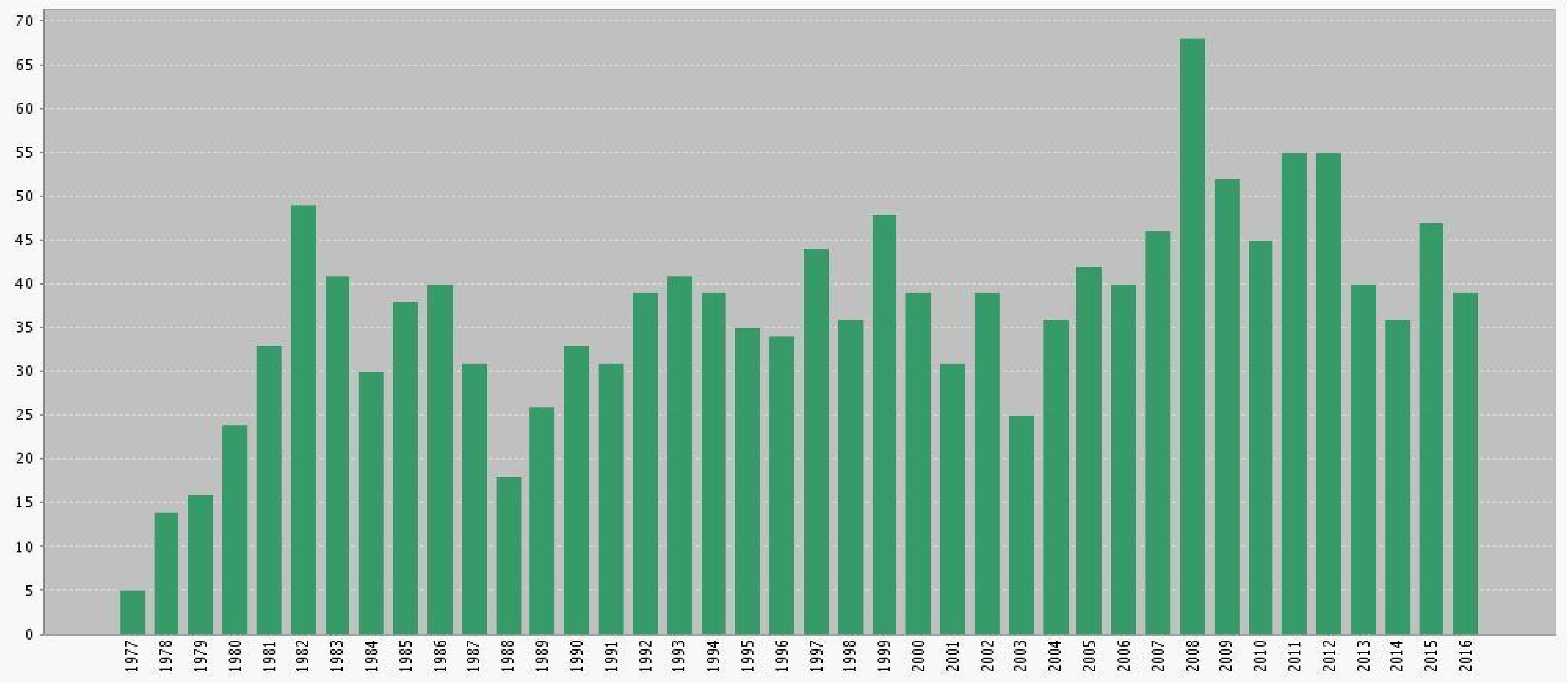


Fig: (top) example of a spin glass vacuum configuration. (bottom) ferromagnetically ordered configuration.

Concluding remarks

- This work presents a very useful semiclassical technique that not only allowed estimation of the lifetime of a false vacuum, but also conceptually paved the way for similar analyses of metastable systems in many other areas of theoretical physics.
- The technique reflects the deep relationship between QFT and statistical physics.

Number of Citing Papers (total=1480):



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3. A. Kusenko, APS Physics 8, 108 (2015).
4. G. Degrassi *et al.*, JHEP 1208 (2012) 098.
5. S. Kachru, R. Kallosh, A. Linde, S. Trivedi, PRD 68, 046005 (2003)
6. M. Advani, S. Lahiri, and S. Ganguli, J. Stat. Mech. 03, P03014 (2013).