Electrodynamics of a split-ring Josephson resonator in a microwave line

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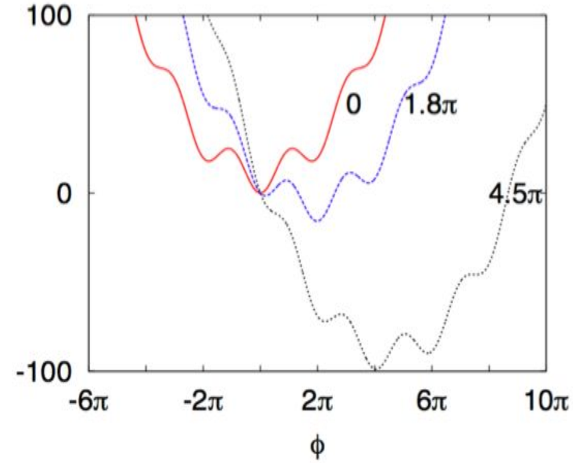

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Electrodynamics of a split-ring Josephson resonator in a microwave line

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We consider the coupling of an electromagnetic wave to a split-ring Josephson oscillator or radio-frequency superconducting quantum interference device in the hysteretic regime. This device is similar to an atomic system in that it has a number of steady states. We show that one can switch between these with a suitable short external microwave pulse. The steady states can be characterized by their resonant lines which are of the Fano type. Using a static magnetic field, we can shift these spectral lines and lift their degeneracy.

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Outline

1. This slide
2. Introduction / Background
3. Resistive Shunted Junction model
4. Numerical Analysis of State Switching
5. Critique / Citations & Impact
6. Summary

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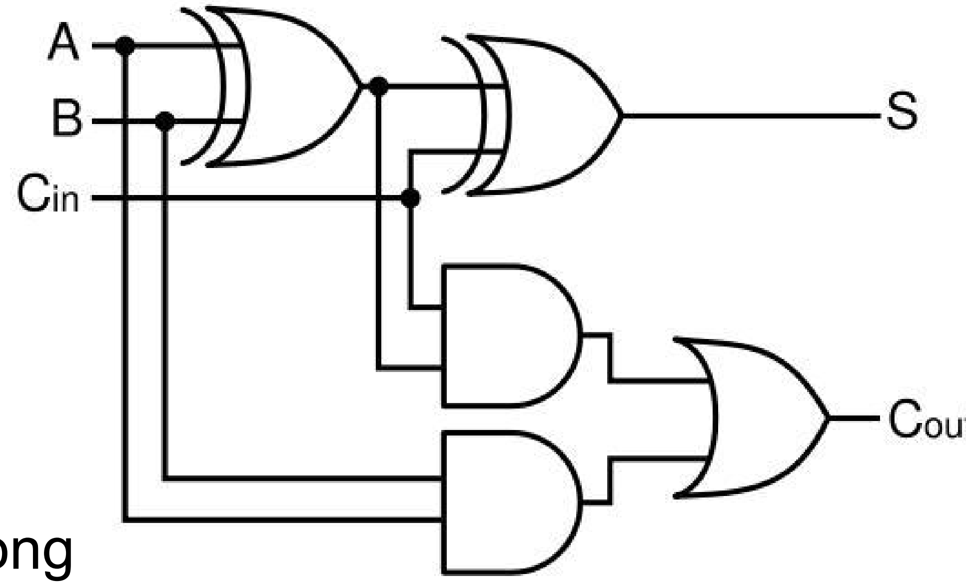
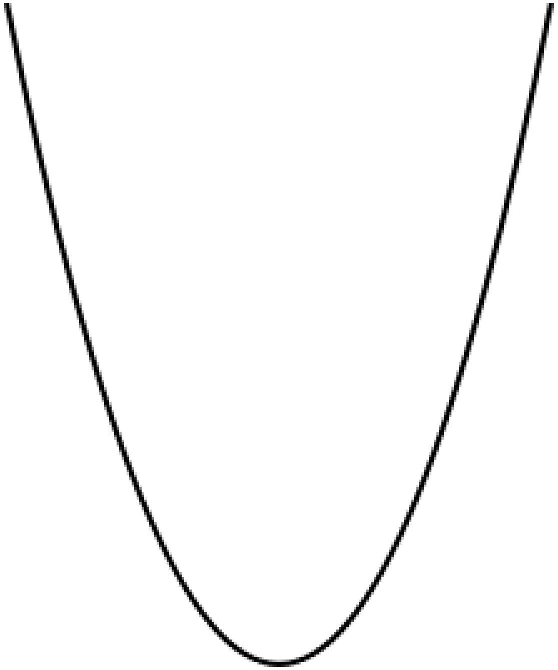
Electronic circuits



- Electrical components are connected by wires
- Voltage switches on/off along wires ⇒ signal
- Components combine signals and effect desired functionality
- **Clock signals** are very important, especially with precision electronics!

How to Deal with Timing Using Electronics



Hint: What has a natural frequency of oscillation?

How to Deal with Timing Using Electronics

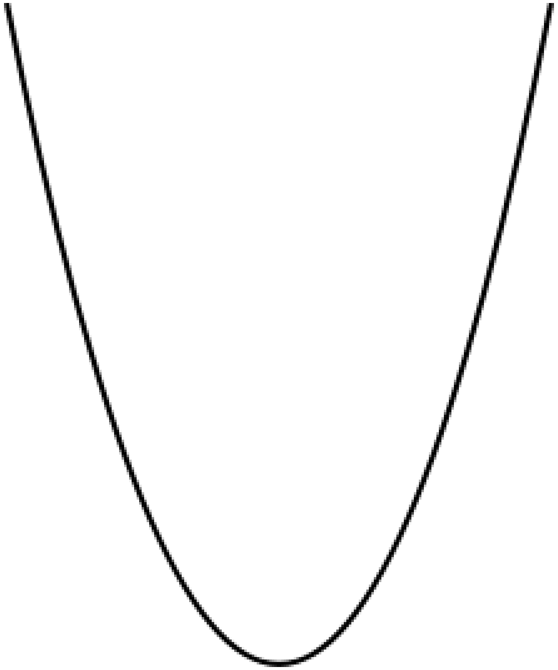
Build a harmonic oscillator in current!

$$\ddot{I} + \omega_0^2 I = 0$$

From Kirchhoff's laws, the above describes an inductor-capacitor (LC) circuit.

Natural frequency given by:

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



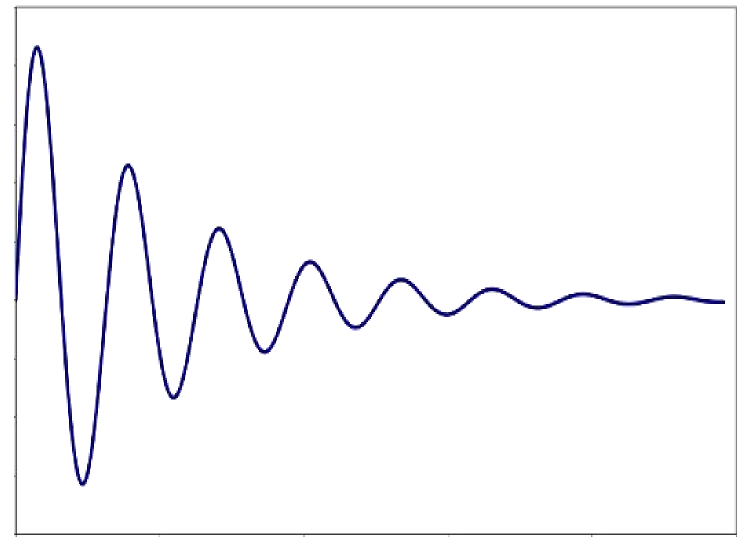
Hint: What has a natural frequency of oscillation?

Problems with the "Real" World

Modern electronics are built out of normal materials.

Circuit components have non-zero resistances!

L-C → ***R***-L-C → ***damping***



Characteristic damping timescale

$$\alpha = \frac{1}{RC}$$

Dimensionless "Quality factor"

$$Q \equiv \frac{\alpha}{\omega_0} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

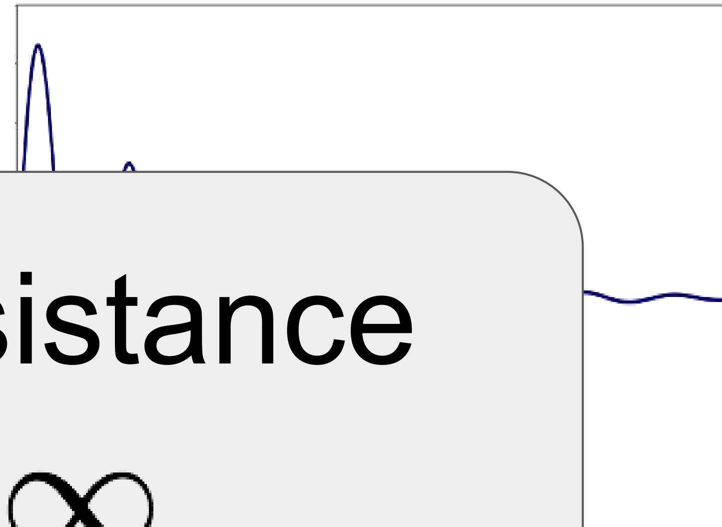
Problems with the “Real” World

Modern electronics are built out of normal materials.

Circu
resis

L-C

Chara



Get rid of resistance

$$Q \rightarrow \infty$$

$$\alpha = \frac{1}{RC}$$

Dimensionless “Quality factor”

$$Q \equiv \frac{\alpha}{\omega_0} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Josephson Junctions

Basic idea:

Embed insulator between two superconductors

Cooper pairs tunnel between superconductors

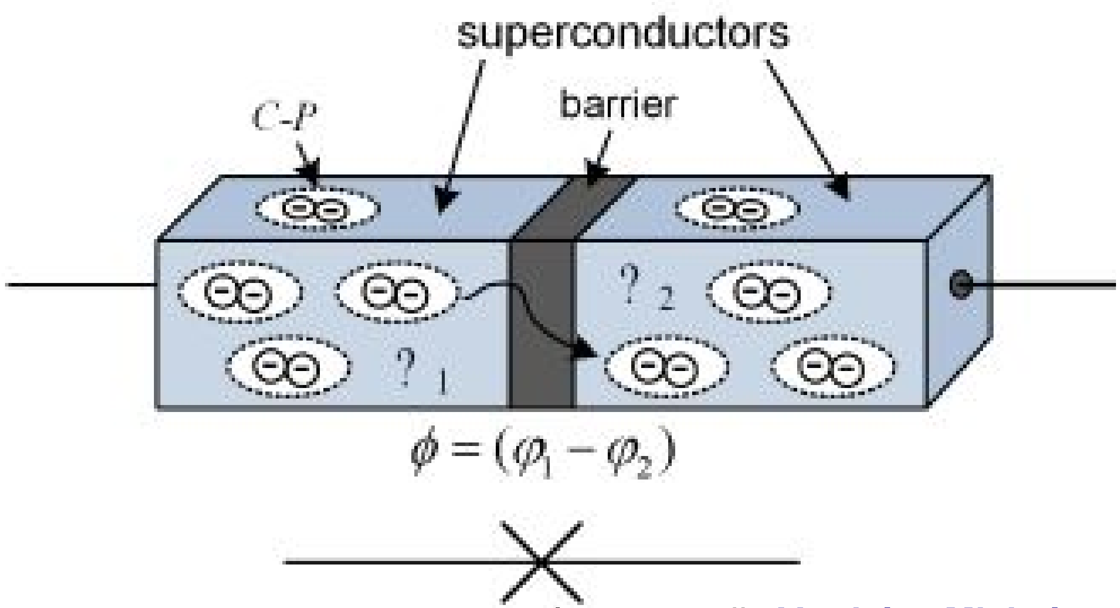


Image credit: **Vratislav Michal**

Josephson Junctions

Basic idea:

Embed insulator between two superconductors

Build a resonator using Josephson Junction.

"Resistive Shunted Junction" (RSJ) model

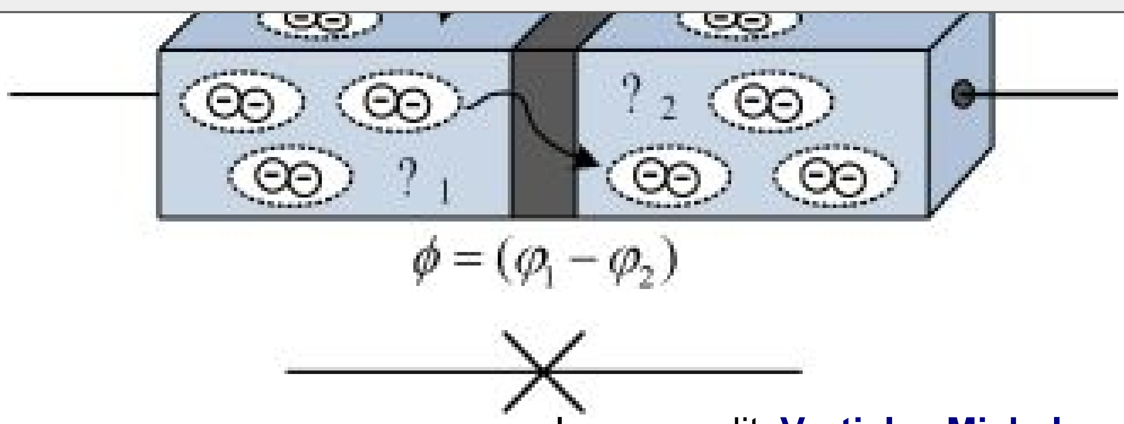


Image credit: **Vratislav Michal**

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Resistive Shunted Junction model

$$\nabla \cdot \boldsymbol{D} = 0 \qquad \nabla\, \boldsymbol{x}\, \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla\, \boldsymbol{x}\, \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \qquad \nabla \cdot \boldsymbol{B} = 0$$

The choice of λ ≈100 cm which corresponds to microwaves with ω ~ GHz. In this regime λ >> the diameter of the ring (ζ) ≈ 10 um

The supercurrent is defined as

$$I = I_c \sin \frac{\Phi}{\Phi_0}$$

where Φ_0 (h/2e) is the reduced flux quantum and I_c is the critical current

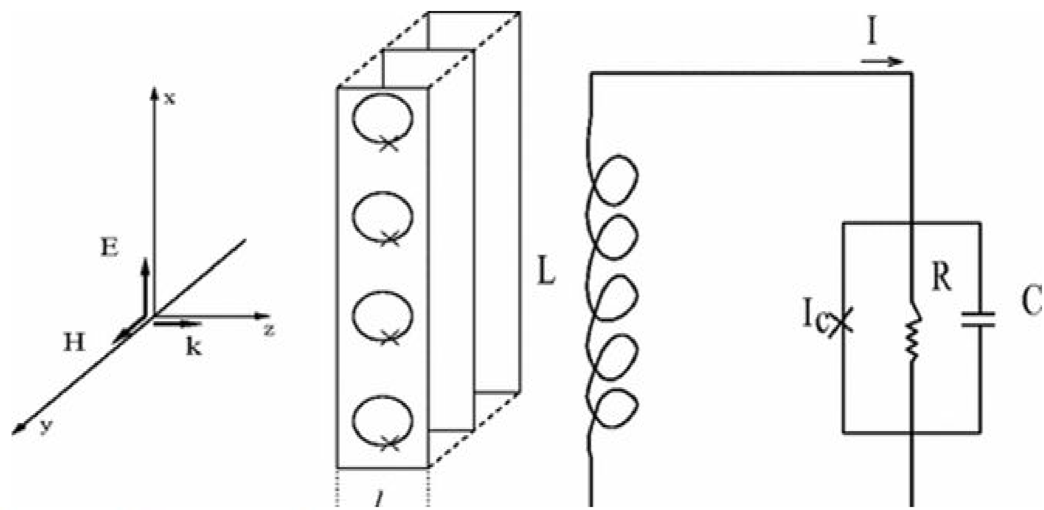


Fig 1. Left panel shows the incident electromagnetic wave on the split ring Josephson junction resonator in the (x, z) plane. The right panel shows the equivalent circuit of the model.

Resistive Shunted Junction model

After applying Ohm's law and Kirchhoff's law for the equivalent circuit and performing many lines of math with defining the flux as

$$\Phi(t) = SH(t),$$

A second order differential equation is revealed that describes the evolution of $\varphi(\mathrm{t}) = \frac{\Phi(t)}{\Phi_0} = \frac{SH(t)}{\Phi_0}$

$$\therefore \frac{\partial^2 \varphi}{\partial t^2} + \frac{1}{RC}\frac{\partial \varphi}{\partial t} + \frac{I_c}{C\Phi_0}\sin\varphi = -\frac{1}{LC}\frac{SH(0,t)}{\Phi_0}.$$

Resistive Shunted Junction model

The natural units of time, flux, and space are

$$\omega_T = \frac{1}{\sqrt{LC}}, \quad \Phi_0, z_0 = \frac{c}{\omega_T\sqrt{\varepsilon}},$$

where ω_T is the Thompson frequency and z_0 is the inverse of the Thompson wave number.

With these parameters dimensionless variables can be introduced

$$h = \frac{SH}{\Phi_0}, \qquad \tau = \omega_T t, \qquad \xi = z/z_0$$

Resistive Shunted Junction model

In terms of these new variables the differential equation is expressed as

Damping term

Forcing term

$$\varphi_{,\tau\tau} + \alpha\varphi_{,\tau} + \varphi + \beta \sin\varphi = -h,$$

Josephson Junction

where the parameters $\alpha, \beta, \text{and } \kappa$ are

$$\alpha^* = \frac{1}{R}\sqrt{\frac{L}{C}}, \quad \beta^* = \frac{LI_c}{\Phi_0}, \quad \kappa = \frac{n_r \zeta S \omega_T}{L\sqrt{\varepsilon}}$$

*Most Important parameters of the paper

Characterize the system in terms of its fixed points

Maxwell's equations and the physical constraints of the system produce this system of coupled differential equations in the phase:

Josephson Junction

Forcing term

$$
\begin{aligned}
\varphi_{,\tau} &\equiv \psi \\
\psi_{,\tau} &= -\alpha\psi - \beta\sin(\varphi) - \varphi + h
\end{aligned}
$$

Damping term

Math tells us the fixed points are given by phase as

$$(0,0) \quad (\varphi^*, 0) \qquad\qquad -\beta\sin(\varphi^*) - \varphi^* + h = 0$$

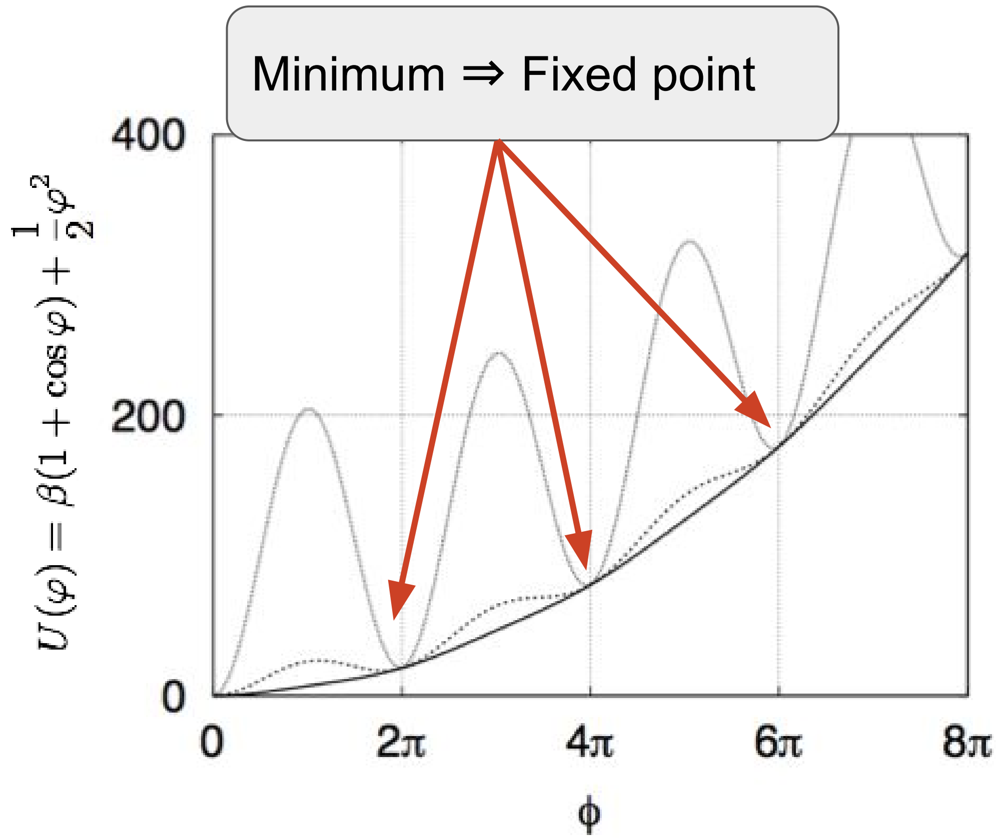
Visualizing the stable phases of the system

Plotting the "potential energy" of the system versus phase in the limit of **no damping** and **no incident flux**

Damping $\alpha \to 0$

Incident Flux $h \to 0$

Remember $\beta = \frac{LI_c}{\Phi_0}$



Effective "potential energy", U, of the system as a function of phase for different $\beta = 1, 9.76, 100$

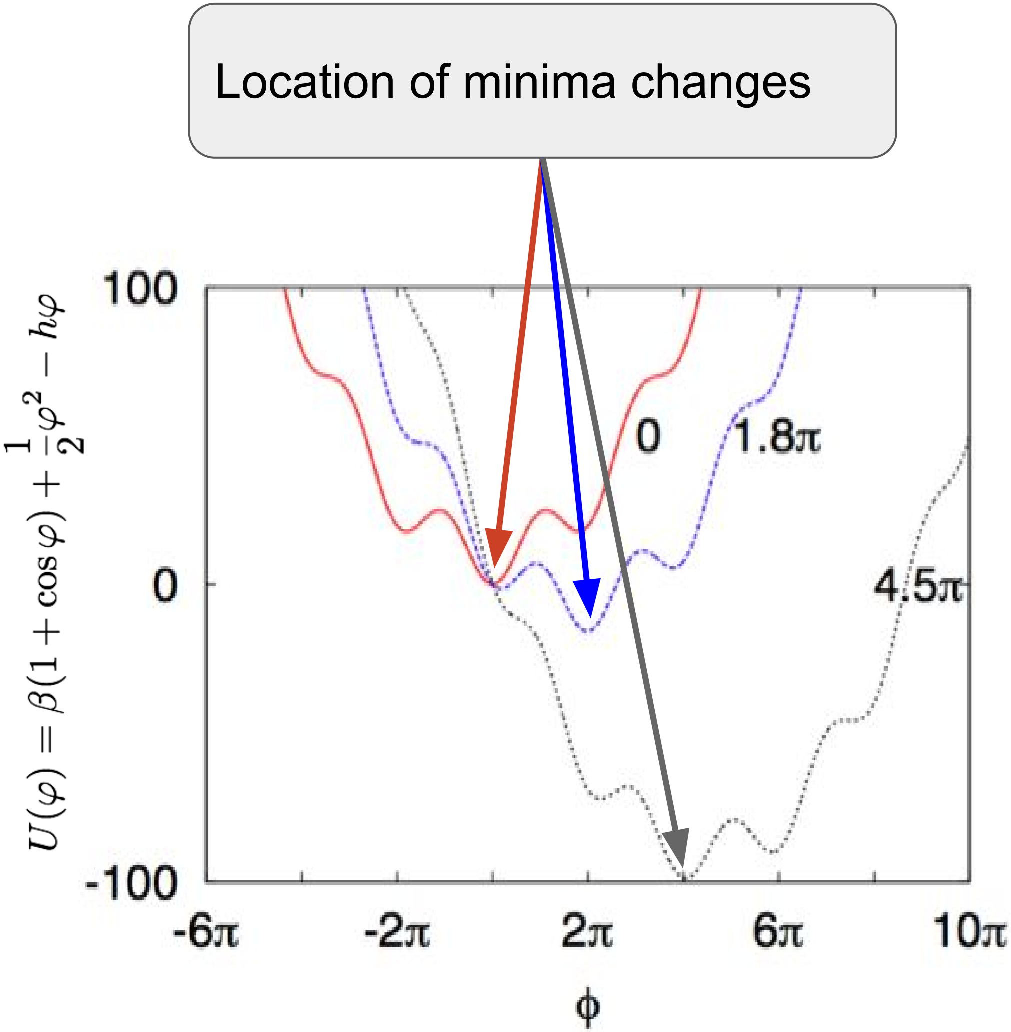
How does an applied magnetic field affect the potential?

Allowing non-zero flux changes potential as

$$U \to U' = U - h\varphi$$

Like a ball hanging from a spring in a gravitational field

Careful choice of incident flux can align minima of different order!



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Switching between equilibrium states

$$\varphi_{,\tau\tau} + \alpha\varphi_{,\tau} + \varphi + \beta \sin\varphi = -h, \tag{4}$$

Damping term

Josephson Junction

Forcing term (External field)

Multiply Eq. (4) by $\varphi_{,\tau}$ and integrate over time to get the energy difference,

$$E(\tau_2) - E(\tau_1) \equiv \left[\frac{1}{2}{\varphi_{,\tau}}^2 + \beta(1-\cos\varphi) + \frac{1}{2}\varphi^2\right]_{\tau_1}^{\tau_2}$$

$$= \int_{\tau_1}^{\tau_2} d\tau\, h\varphi_\tau - \alpha\int_{\tau_1}^{\tau_2} d\tau\, \varphi_\tau^2. \tag{11}$$

External short pulse

$$h(\tau) = a\delta(\tau),$$ short pulse.

$$\varphi_{,\tau\tau} + \alpha\varphi_{,\tau} + \varphi + \beta\sin\varphi = a\delta(\tau). \quad (4)$$

Damping term

Josephson Junction

Forcing term (Short pulse)

Integrate Eq. (4) over a small interval around 0.

φ is a continuous function. Take the limit $\epsilon \to 0$,

$$[\varphi_\tau]_{-\epsilon}^{\epsilon} + \alpha[\varphi]_{-\epsilon}^{\epsilon} + \int_{-\epsilon}^{\epsilon} d\tau(\beta\sin\varphi + \varphi) = a. \qquad (14)$$

External short pulse continued

φ_τ cannot be continuous at 0.

Assuming $\varphi_\tau(0_-) = 0$ we get $\varphi_\tau(0_+) = a$

A short pulse gives momentum to the system.

$$\varphi_{,\tau\tau} + \alpha\varphi_{,\tau} + \varphi + \beta \sin\varphi = a\delta(\tau). \quad (4)$$

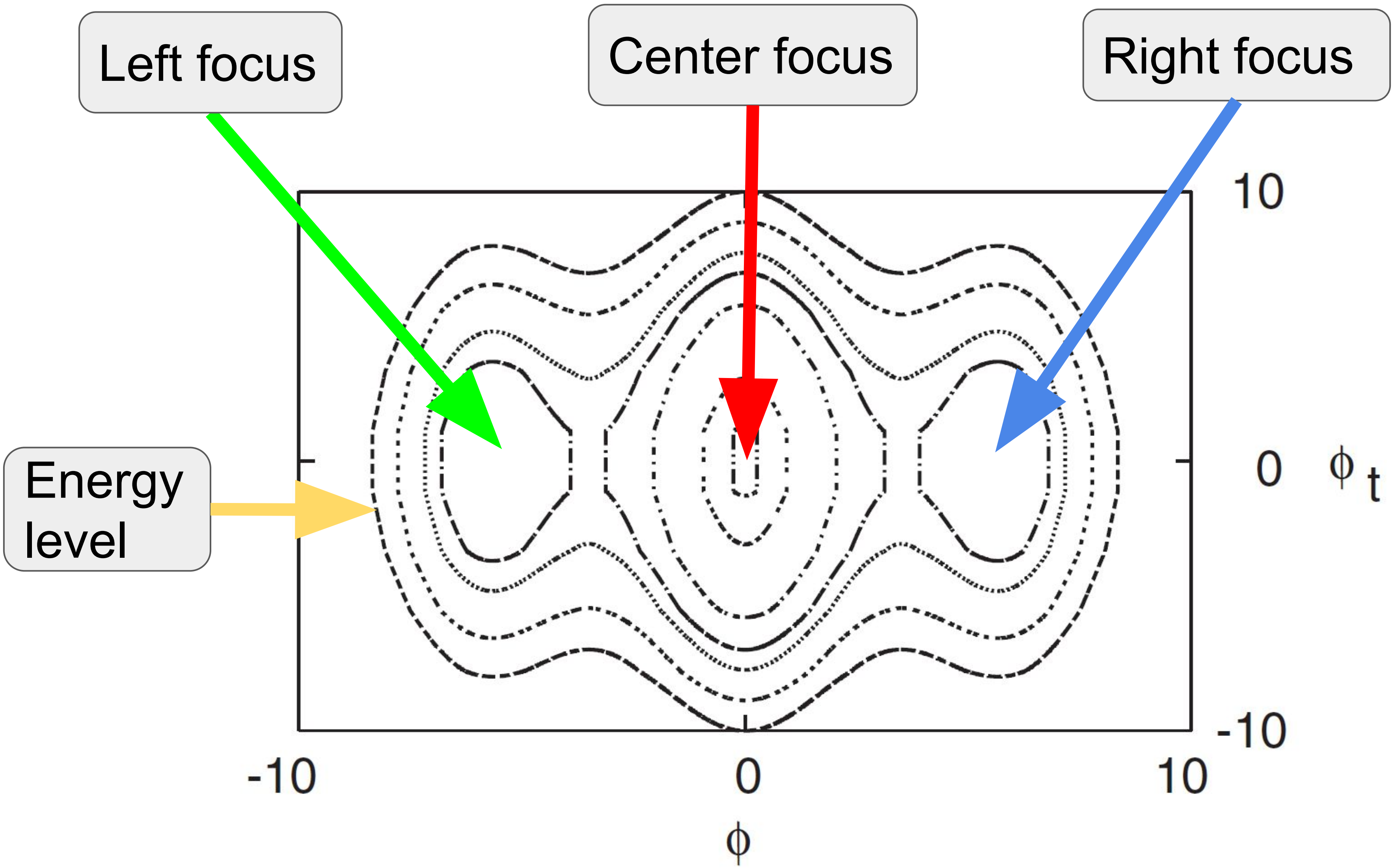
Damping term

Josephson Junction

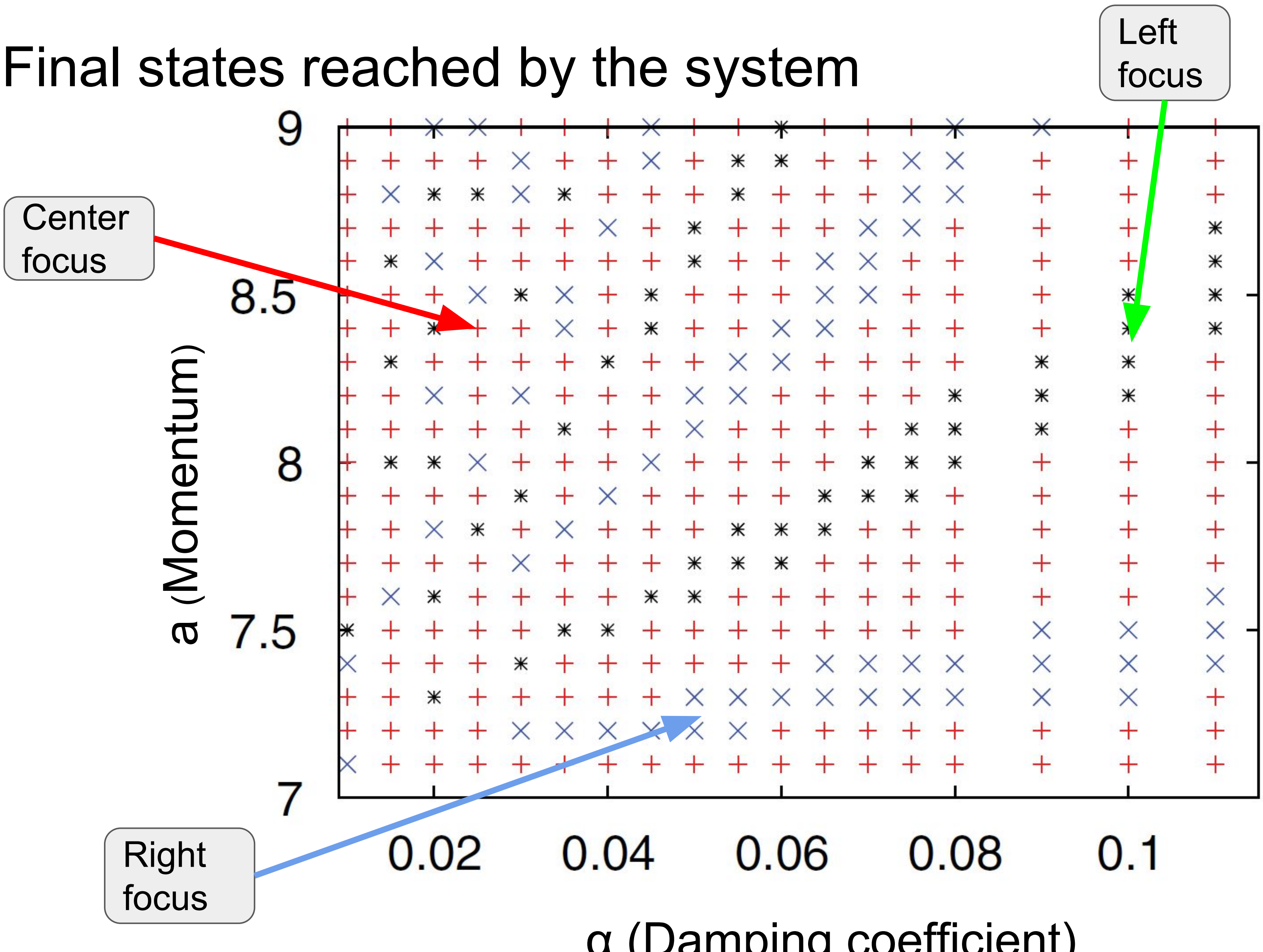
Forcing term (Short pulse)

Solve Eq. (4) numerically using Runge-Kutta algorithm, starting from (0, a) in phase space.

Phase portrait of the Hamiltonian system



Final states reached by the system



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Critiques

- Some assumptions not made explicit or justified clearly
- Could have made connection to (possible) experimental work more taught
- Plots could be more enlightening
- Occasional typos

Citation history

- Posted Jan. 5, 2012
- Published in PRB May 25, 2012
- Four citations
  - One 2015 self-citation, also cited four times
  - Other three: two separate groups, 10-30 citations

Work by other citations

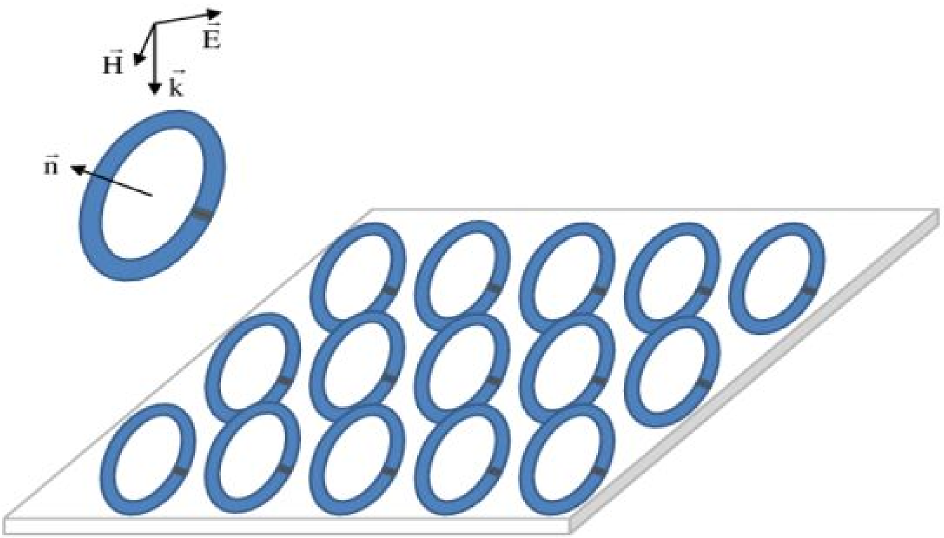
- “Multistability and switching in a superconducting metamaterial”
  - “It should be noted that here, we deal with a purely dynamic phenomenon that is not related to the multistability known from hysteretic SQUIDs”
- “Progress in superconducting metamaterials”
  - Review paper, 30 citations (17 in 2016)

Work by other citations

- “Multistability and self-organization in disordered SQUID metamaterials”
    - Studied magnetic responses of 2D rf SQUIDS (theoretical/numerical)

Subsequent work by the authors

- "Polarization rotation by an rf-SQUID metasurface"
  - PRB, March 24, 2015
  - "We study the transmission and reflection of a plane electromagnetic wave through a two-dimensional array of rf-SQUIDs."
  - Very similar model to 2012 paper



Summary

- Normal RLC circuits exhibit damping
- Resistive Shunted Junction (RSJ) model might allow for R=0 timing circuits
- Careful treatment of physical assumptions produces easy-to-analyze simplified “model” for RSJ system