Model for a Quantum Hall Effect without Landau Levels

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Background about this paper

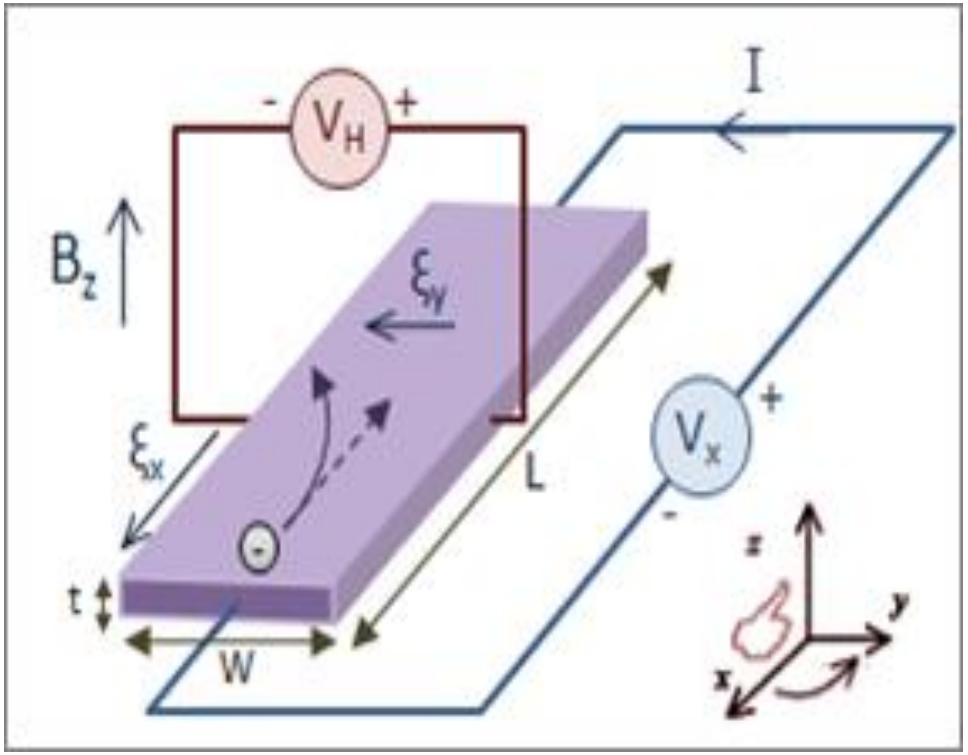
- Author: F. Duncan M. Haldane
  Eugene Higgins Professor in Princeton University
  Nobel Laureate of 2016

- This paper has been cited >1500 times
  Impact on later study of topological physics and 2D material such as graphene

Model for Classical Hall Effect

- **Hall effect:** Appearance of a voltage difference across a conductor when a transverse electric field and perpendicular magnetic field are applied.

- $\boldsymbol{V_H = \frac{I_x B_z}{nte}}$, n is charge carrier density, e is one electron charge, and t is the thickness of conductor.



Hall effect measurement setup. Apply E field in x-direction and B field in z-direction. Measure Hall voltage in y-direction

Model of Quantum Hall Effect

Quantum Hall Effect (QHE): quantization of conductance in strong magnetic fields and low temperature

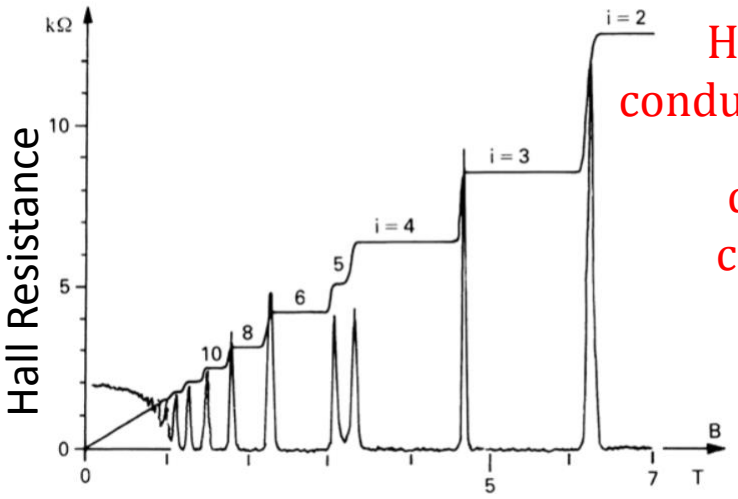
Integer Quantum Hall Effect $\sigma_{xy} = \frac{e^2\nu}{h}$ Fractional Quantum Hall Effect

$\nu = 1,2,3 \ldots$

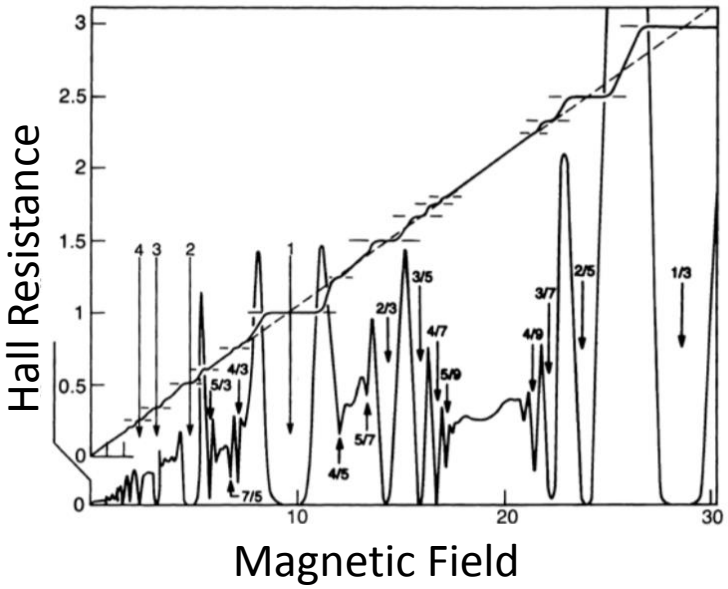
$\nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7} \cdots$

Hall conductance

quantum of conductance



(von Klitzing)



(Tsui and Störmer)

Interpreting the Integer and Fractional Quantum Hall Effects

Integer QHE: Free electrons moving in a magnetic field defined in terms of canonical momenta (momentum and vector potential)

$$\hat{H} = \hat{\Pi}_x^2 + \hat{\Pi}_y^2 \qquad\qquad \hat{\boldsymbol{\Pi}} = \hat{\boldsymbol{p}} + e\hat{\boldsymbol{A}}$$

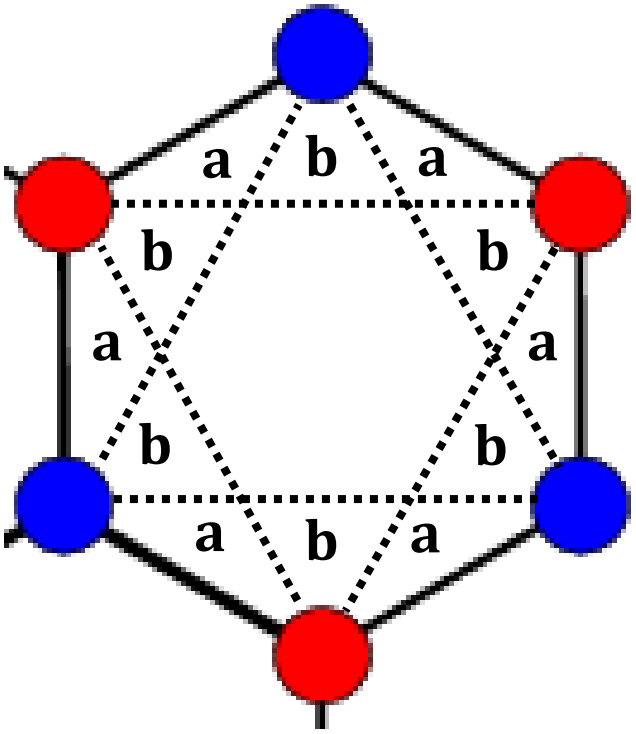
Similar mathematically to quantum harmonic oscillator

$$E = \hbar\omega_B\left(n + \frac{1}{2}\right) \qquad\qquad \omega_B = \frac{eB}{m}$$

These energies are known as Landau levels.

Fractional QHE involves interactions and is much more complicated

Haldane's Toy Model for Quantum Hall Effect



Pictorial representation of Haldane model

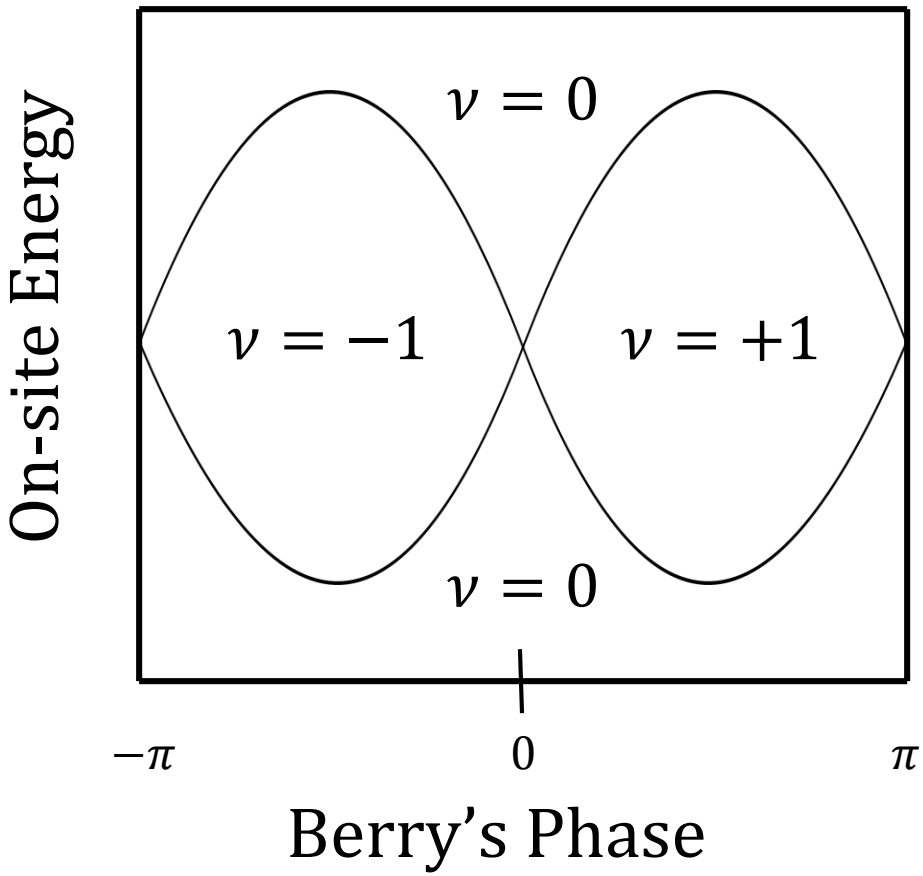
- Energy $+M$ on red sites and $-M$ on Q blue sites
- Nonuniform $\vec{B}$ in regions a $(+)$ and b $(-)$
- $\vec{B}_{tot} = 0$ so no Landau Levels
- Lines indicate hopping between nearest neighbors (solid) and next-nearest neighbors (dashed)

Haldane Model Generates Quantum Hall Effect

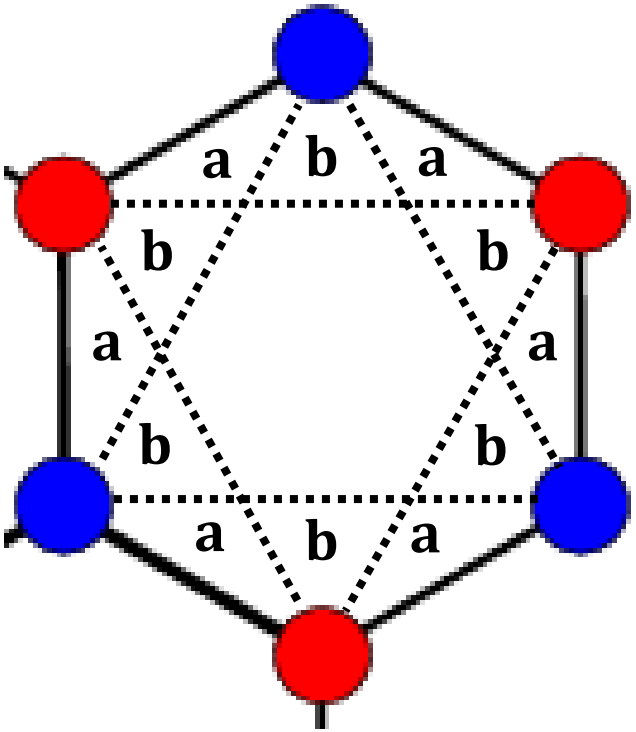
$$\sigma_{xy} = \frac{\nu e^2}{h}$$

Same result as canonical quantum Hall effect

Conductance Phase Diagram



Haldane Model and Topology



Pictorial representation of Haldane model

- $\vec{B}$ fields and hopping produce a Berry's phase
- This particular Berry's phase gives rise to a Berry's curvature
- Integrate this Berry's curvature over all space gives same quantized σ_{xy} as QHE
- Quantized in terms of topological Chern number

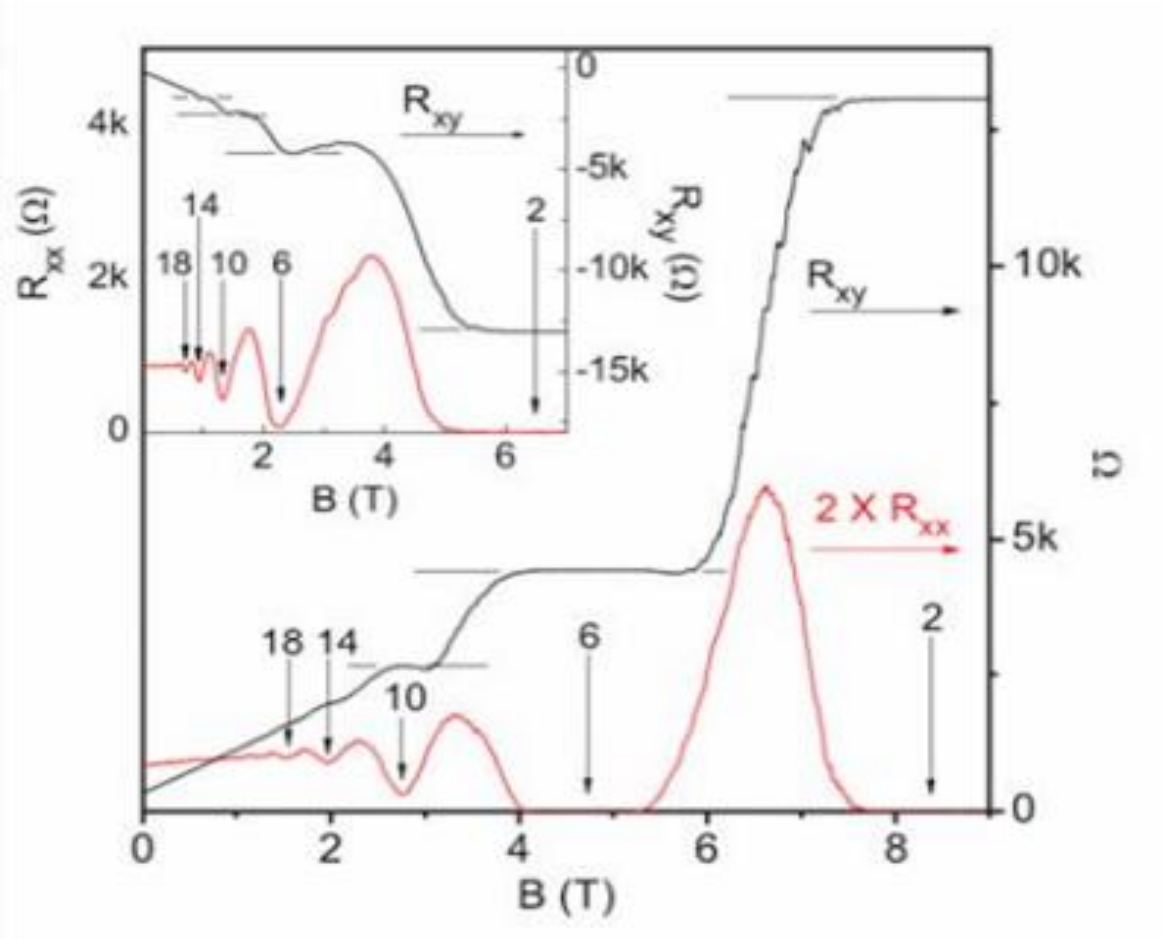
Critique

- Clear
- New perspective on QHE
- Parity anomaly not well motivated
- Hard to implement experimentally

Current Related Research

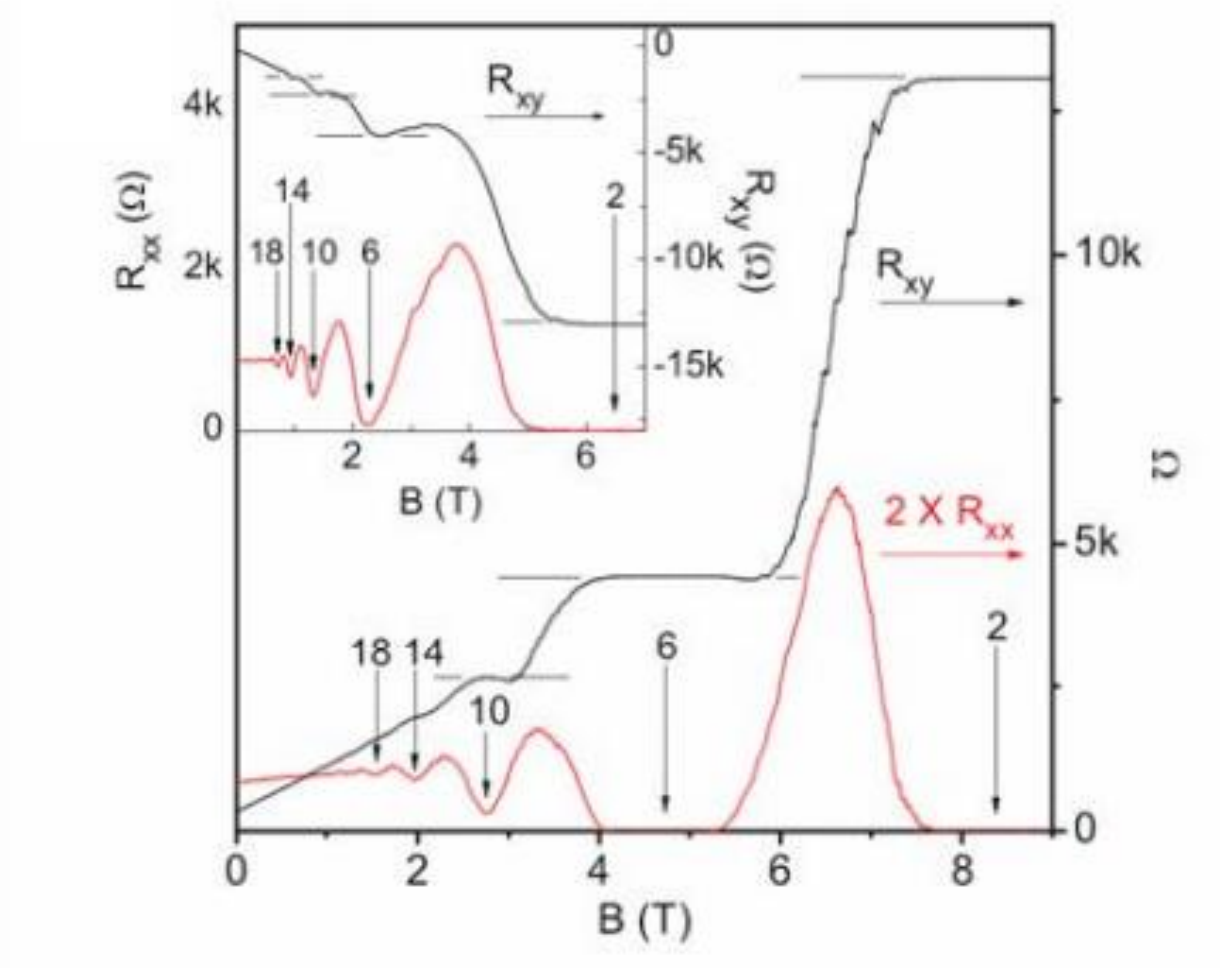
- Experimental Observation of Quantum Hall Effect and Berry's Phase in Graphene (By Yuanbo Zhang 2005)
- Experimental Observation of the Quantum Anomalous Hall Effect in a Magnetic Topological Insulator (By Qi-Kun Xue, Shou-Cheng Zhang 2013)
- Attempts to extend Haldane model to explain fractional Quantum Hall Effect
- Quantum Spin Hall Effect

QHE in Graphene Result (2005)



Quantized Magnetoresistance (Red) and
Hall Resistance(black) of a graphene device at T = 30 mK and Vg = 15 V

QHE in Graphene Result (2005)



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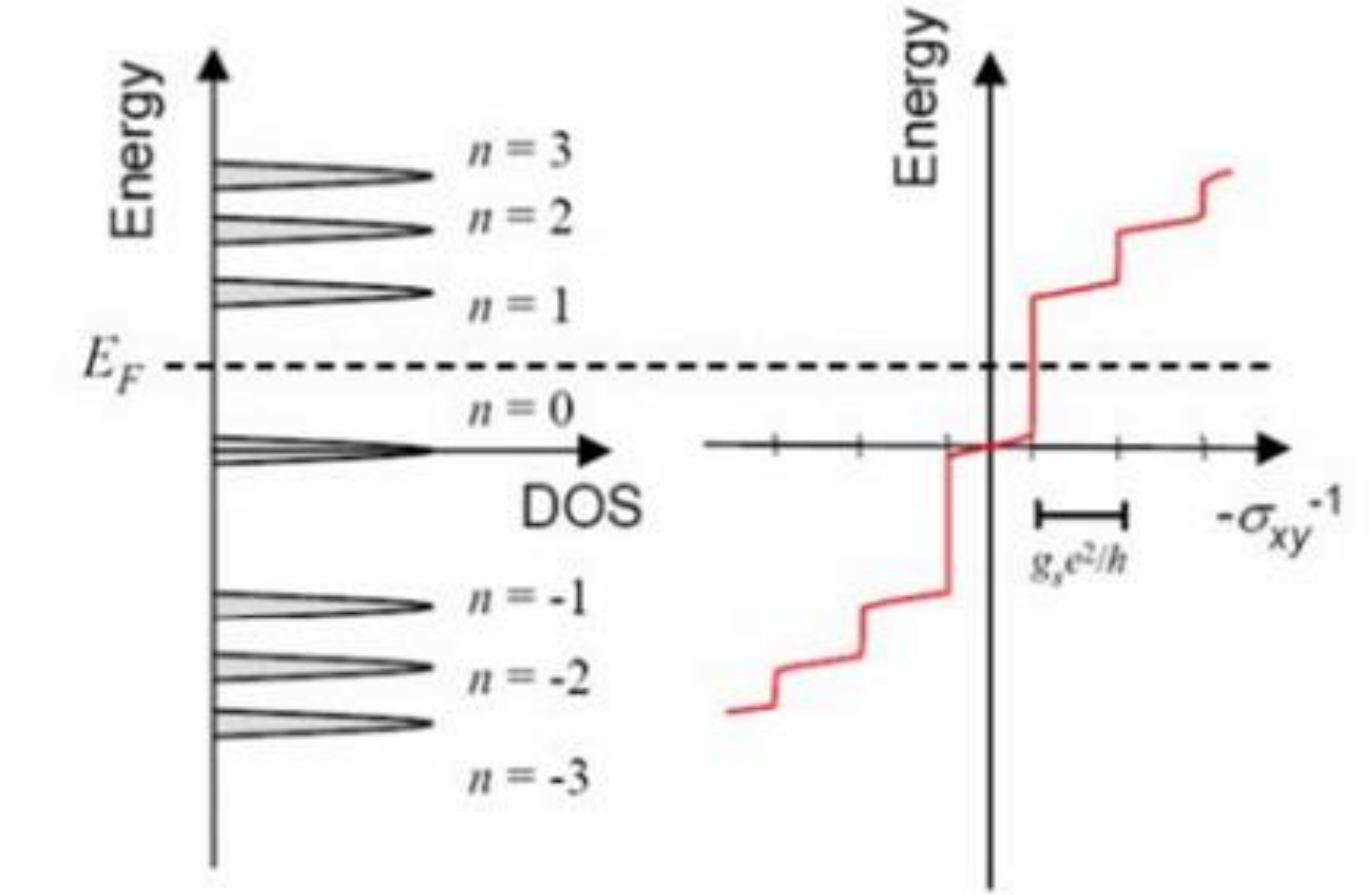
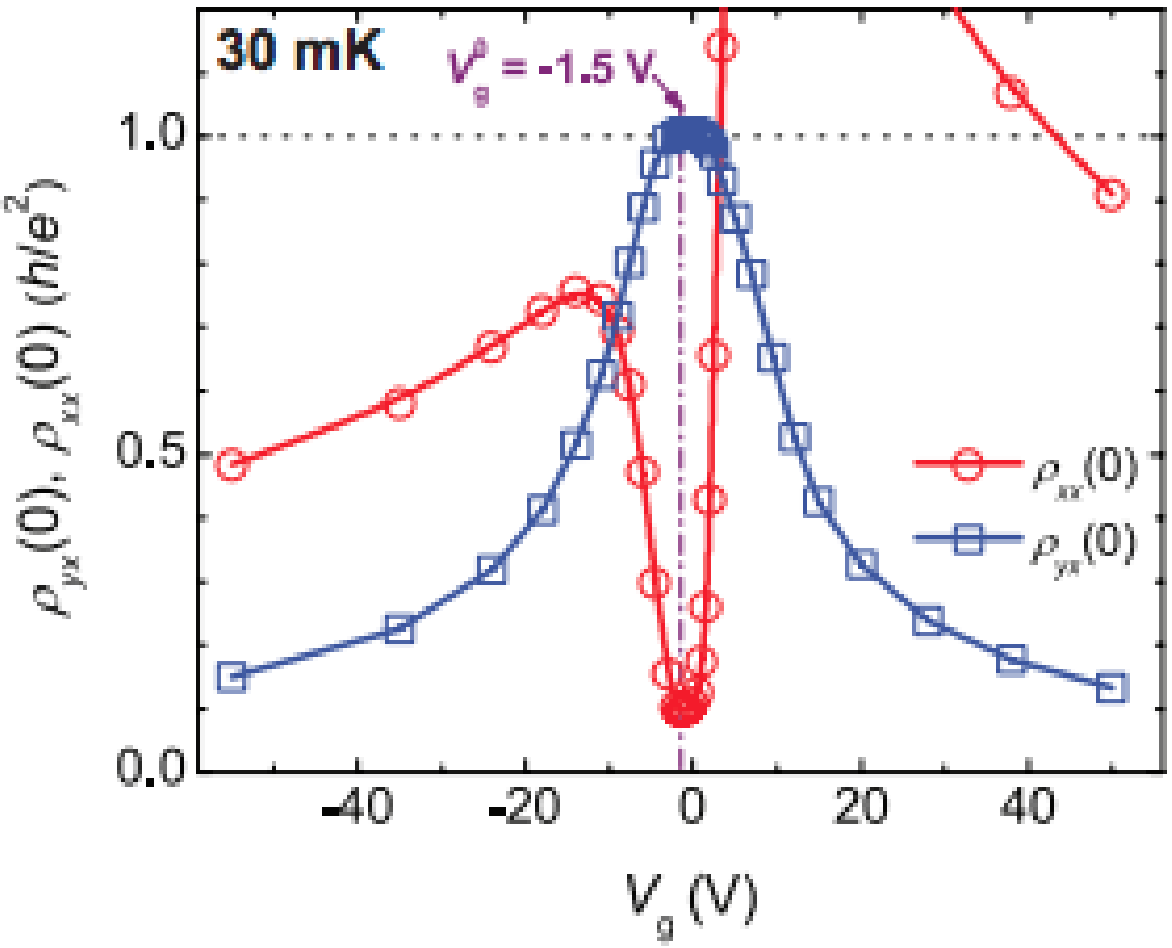


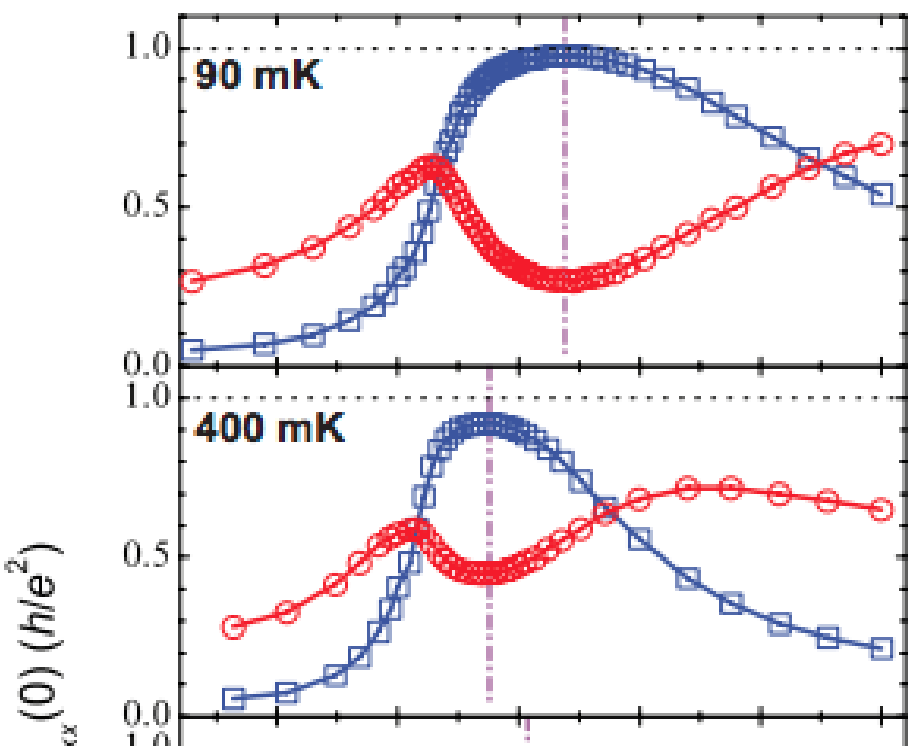
A schematic diagram of the Landau level density of states (DOS) and corresponding quantum Hall conductance (σ_xy) as a function of energy.

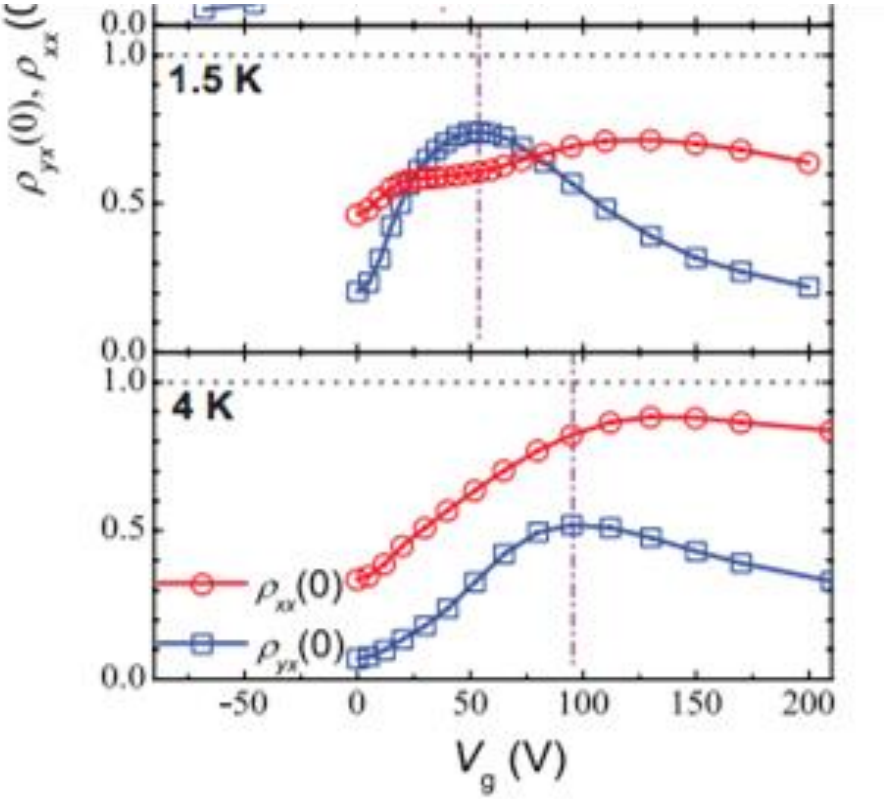
Room-temperature quantum Hall effect in graphene（2007）

Quantum Anomalous Hall Effect Observation (2013)



Quantum Anomalous Hall Effect for different Temperature





Fractional Quantum Hall Effect

- Nonabelions in the fractional quantum Hall effect(Moore/Read 1991)
- Observation of the fractional quantum Hall effect in graphene (2009)

Quantum Spin Hall Effect

- Quantum spin Hall effect (BA Bernevig, SC Zhang 2005)
- Quantum spin Hall effect in graphene(CL Kane, EJ Mele, 2005)
- Quantum spin Hall effect and topological phase transition in HgTe quantum wells(BA Bernevig, TL Hughes, SC Zhang ,2006)

Summary

- Integer QHE can occur in the absence of $\vec{B}$ fields
- Simple model allows for computation of Hall conductance
- Quantization due to accumulation of phase hints at underlying topological structure