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Negative Absolute Temperature for Motional Degrees of Freedom

S. Braun,[1,2] J. P. Ronzheimer,[1,2] M. Schreiber,[1,2] S. S. Hodgman,[1,2] T. Rom,[1,2] I. Bloch,[1,2] U. Schneider[1,2]*

Absolute temperature is usually bound to be positive. Under special conditions, however, negative temperatures—in which high-energy states are more occupied than low-energy states—are also possible. Such states have been demonstrated in localized systems with finite, discrete spectra. Here, we prepared a negative temperature state for motional degrees of freedom. By tailoring the Bose-Hubbard Hamiltonian, we created an attractively interacting ensemble of ultracold bosons at negative temperature that is stable against collapse for arbitrary atom numbers. The quasimomentum distribution develops sharp peaks at the upper band edge, revealing thermal equilibrium and bosonic coherence over several lattice sites. Negative temperatures imply negative pressures and open up new parameter regimes for cold atoms, enabling fundamentally new many-body states.

Absolute temperature T is one of the central concepts of statistical mechanics and is a measure of, for example, the amount of disordered motion in a classical ideal gas. Therefore, nothing can be colder than $T = 0$, where classical particles would be at rest. In a thermal state of such an ideal gas, the probability P_i for a particle to occupy a state i with kinetic energy E_i [illegible]

with energy. If we were to extend this formula to negative absolute temperatures, exponentially increasing distributions would result. Because the distribution needs to be normalizable, at positive temperatures a lower bound in energy is required, as the probabilities P_i would diverge for $E_i \rightarrow -\infty$. Negative temperatures, on the other hand, demand an upper bound in energy (1, 2). In

In Fig. 1A, we schematically show the relation between entropy S and energy E for a thermal system possessing both lower and upper energy bounds. Starting at minimum energy, where only the ground state is populated, an increase in energy leads to an occupation of a larger number of states and therefore an increase in entropy. As the temperature approaches infinity, all states become equally populated and the entropy reaches its maximum possible value S_{max}. However, the energy can be increased even further if high-energy states are more populated than low-energy ones. In [illegible] decreases with energy, [illegible] thermodynamic definition of temperature (8) ($1/T = \partial S/\partial E$), results in [illegible]. The temperature is discontinuous [illegible] entropy, jumping from positive to negative infinity. This is a consequence of the historic definition of temperature. A continuous and [illegible] increasing temperature scale would be given by $-\beta = -1/k_BT$, also emphasizing [illegible] temperature states are hotter than positive temperature states, i.e., in thermal contact, heat would flow from a negative to a positive temperature system.

Because negative temperature systems can absorb entropy while releasing energy, they give rise to several counterintuitive effects, such as

PHYS 596

Team 1

Shreya, Nina,

Nathan, Faisal

What is negative absolute temperature?

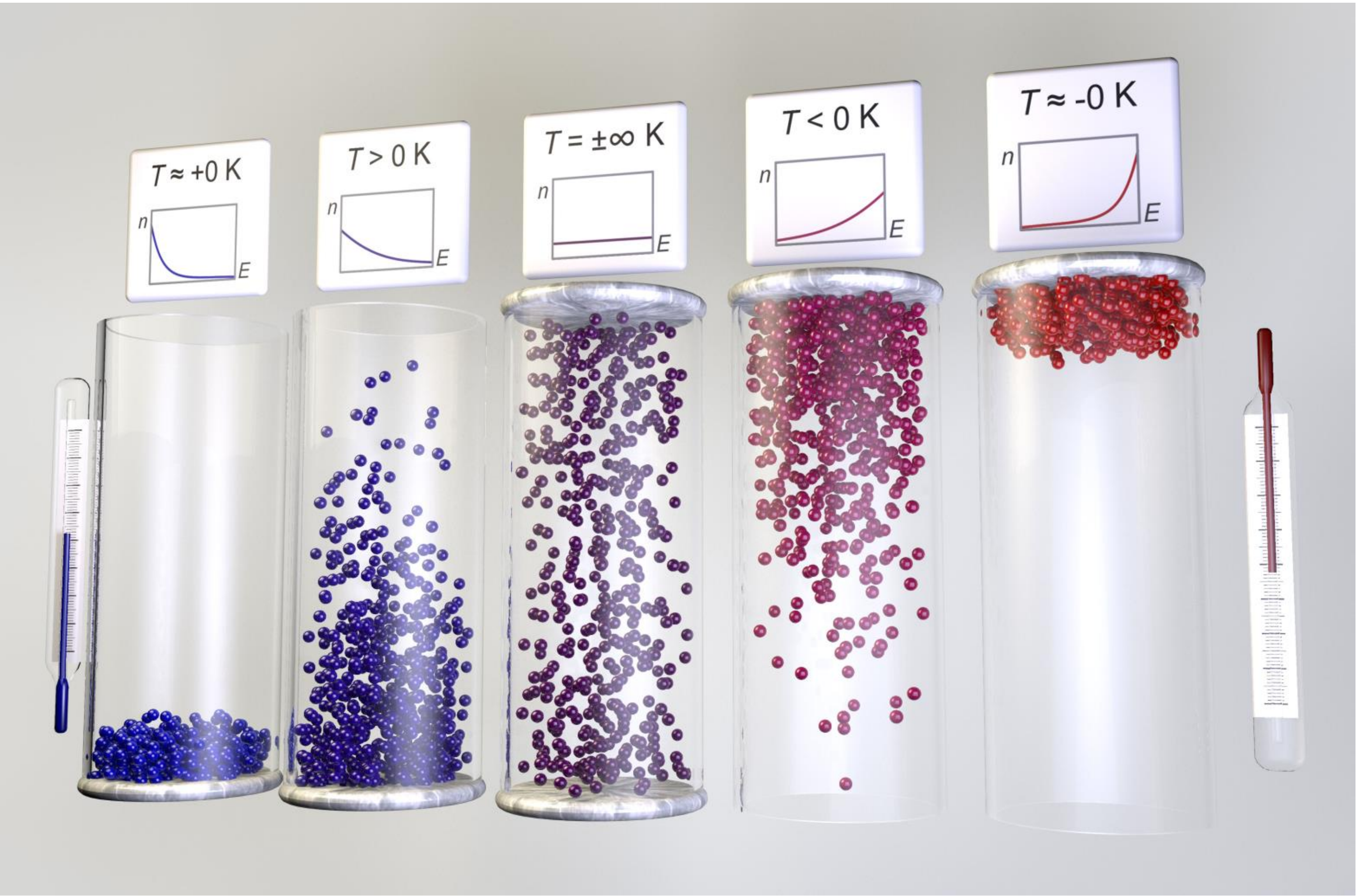
- An ensemble of particles is said to have negative absolute temperature if higher energy states are more likely to be occupied than lower energy states.

$$P_i \propto e^{-E_i/kT}$$

- If high energy states are more populated than low energy states, entropy decreases with energy.

$$\frac{1}{T} \equiv \frac{\partial S}{\partial E}$$

Negative absolute temperature



Previous work on negative temperature

- The first experiment to measure negative temperature was performed at Harvard by Purcell and Pound in 1951.
- By quickly reversing the magnetic field acting on a nuclear spin crystal, they produced a sample where the higher energy states were more occupied.
- Since then negative temperature ensembles in spin systems have been produced in other ways. Oja and Lounasmaa (1997) gives a comprehensive review.

Negative temperature for motional degrees of freedom

- For the probability distribution of a negative temperature ensemble to be normalizable, we need an upper bound in energy.
- Since localized spin systems have a finite number of energy states there is a natural upper bound in energy.
- This is hard to achieve in systems with motional degrees of freedom since kinetic energy is usually not bounded from above.
- Braun et al (2013) achieves exactly this with bosonic cold atoms in an optical lattice.

What is the point?

- At thermal equilibrium, negative temperature implies negative pressure.

- This is relevant to models of dark energy and cosmology based on Bose-Einstein condensation.

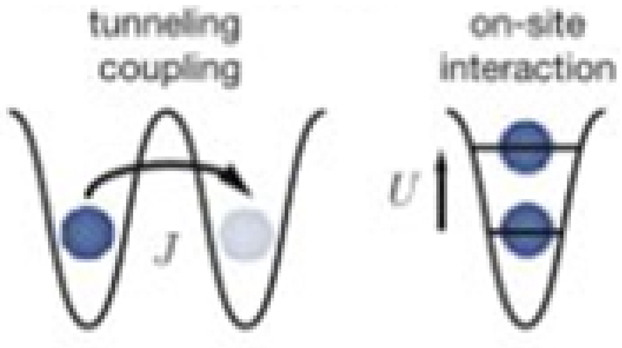
- Negative temperatures are also relevant for quantum simulations of many body systems that are not symmetric with respect to inversion of kinetic energy, for example, Kagome lattices.

Introducing the Bose-Hubbard Hamiltonian

- Spinless bosons on a lattice are described by the Bose-Hubbard model:

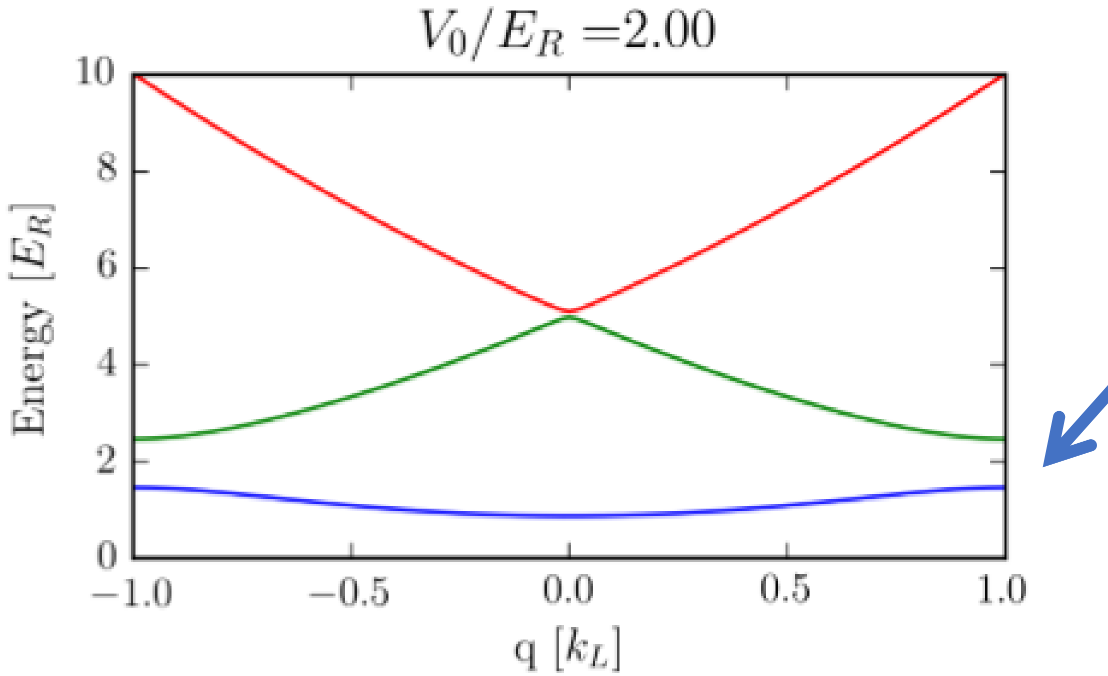
$$H = -J \sum_{\langle i,j \rangle} \hat{b}_i^\dagger \hat{b}_j + \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1) + V \sum_i \mathrm{r}_i^2 \hat{n}_i$$

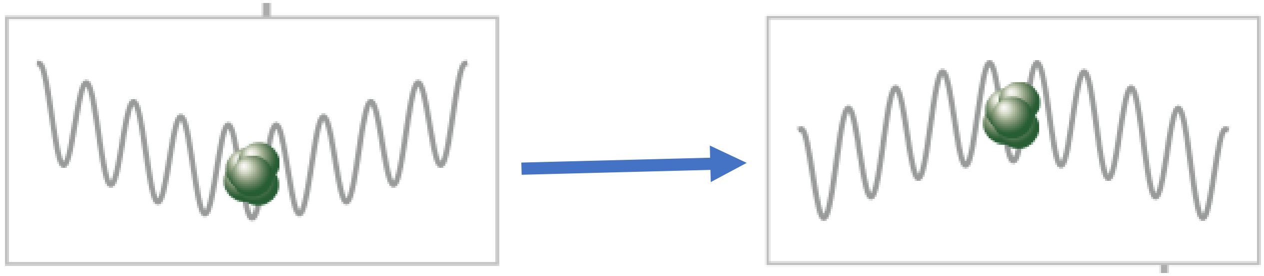
Kinetic/tunneling On-site interactions Trapping



- In specific regimes of J, U and V, the energies of this Hamiltonian are bounded. This means negative temperature states are possible.

Achieving Upper Bounds in Energy

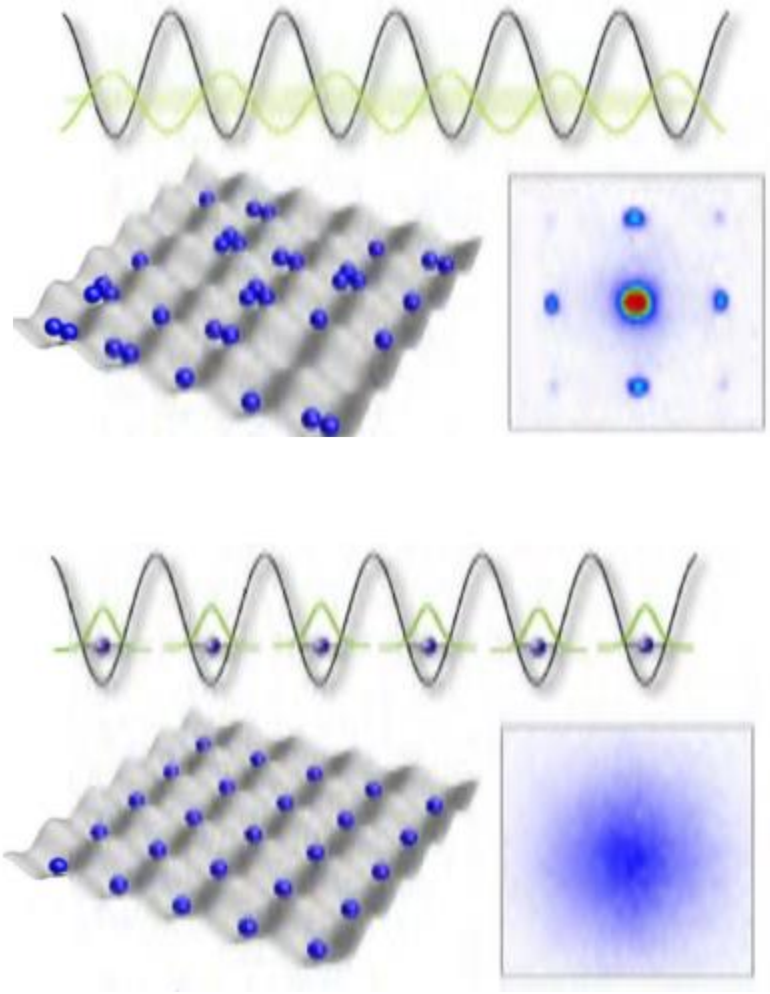




Kinetic Energy is bounded

Interaction and Potential Energy?

Ground States of the Bose – Hubbard Model

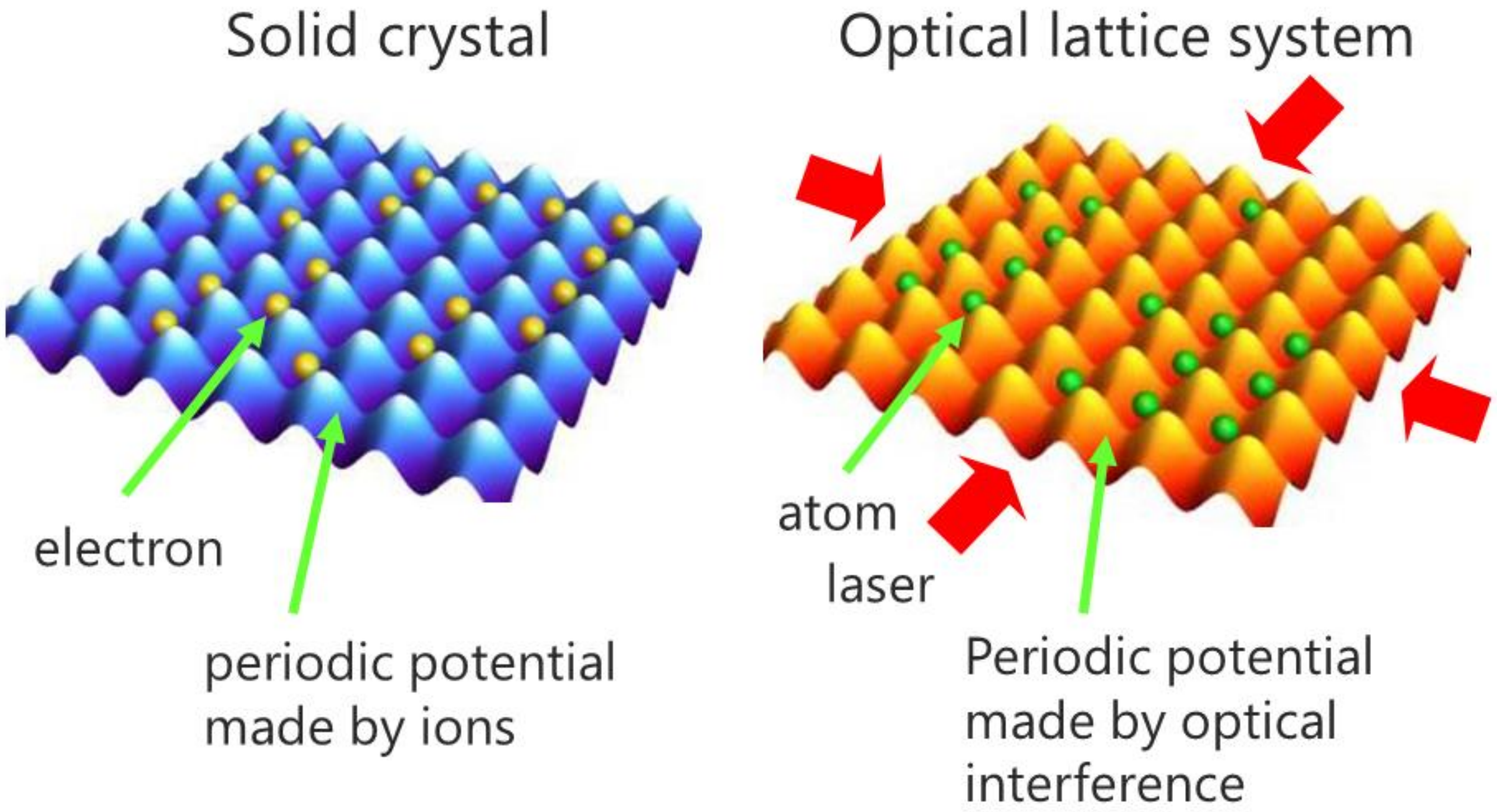


- Two ground states – Superfluid and the Mott Insulator
- Superfluid : all particles exist in the same state (q = 0), lowest energy
- Mott Insulating Phase
- Can go from one to another by changing U/J

Cold atoms in optical lattices

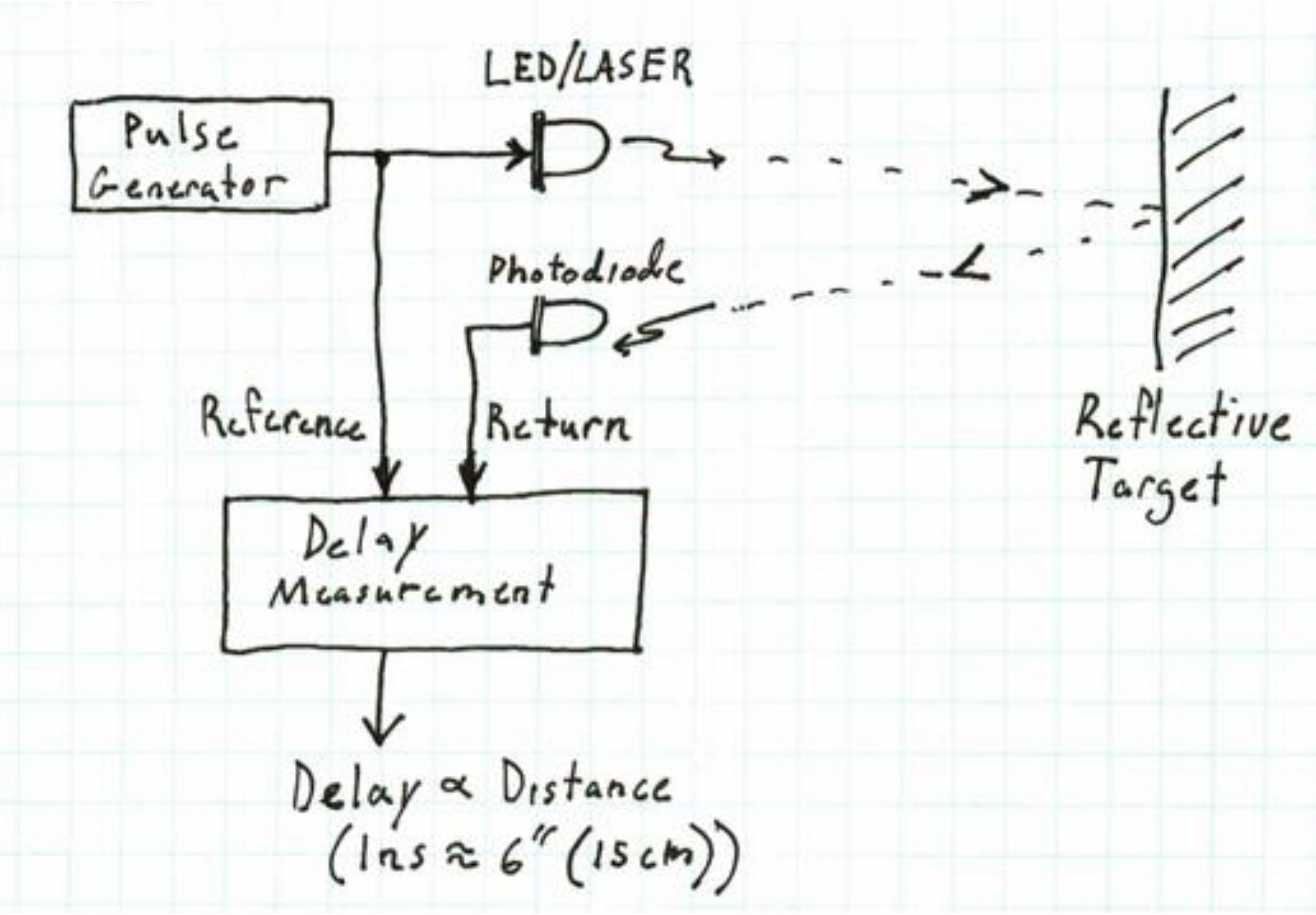
- Need a system that faithfully simulates the Bose – Hubbard Model
- System --> Ultra cold atoms (bosonic or fermionic)
- (cold) atoms move in a potential set up by light (optical lattice) such that the whole system is isolated from the environment (don't want interaction with other positive temperature systems)
- Highly tunable with respect to almost all parameters!

How did they measure a negative temperature state?



- Bose-Einstein condensate in dipole trap
- Uses counter propagating laser beams to create a spatially periodic polarization pattern which can trap neutral atoms

How an Optical Time of Flight Probe Measures Negative Temperature

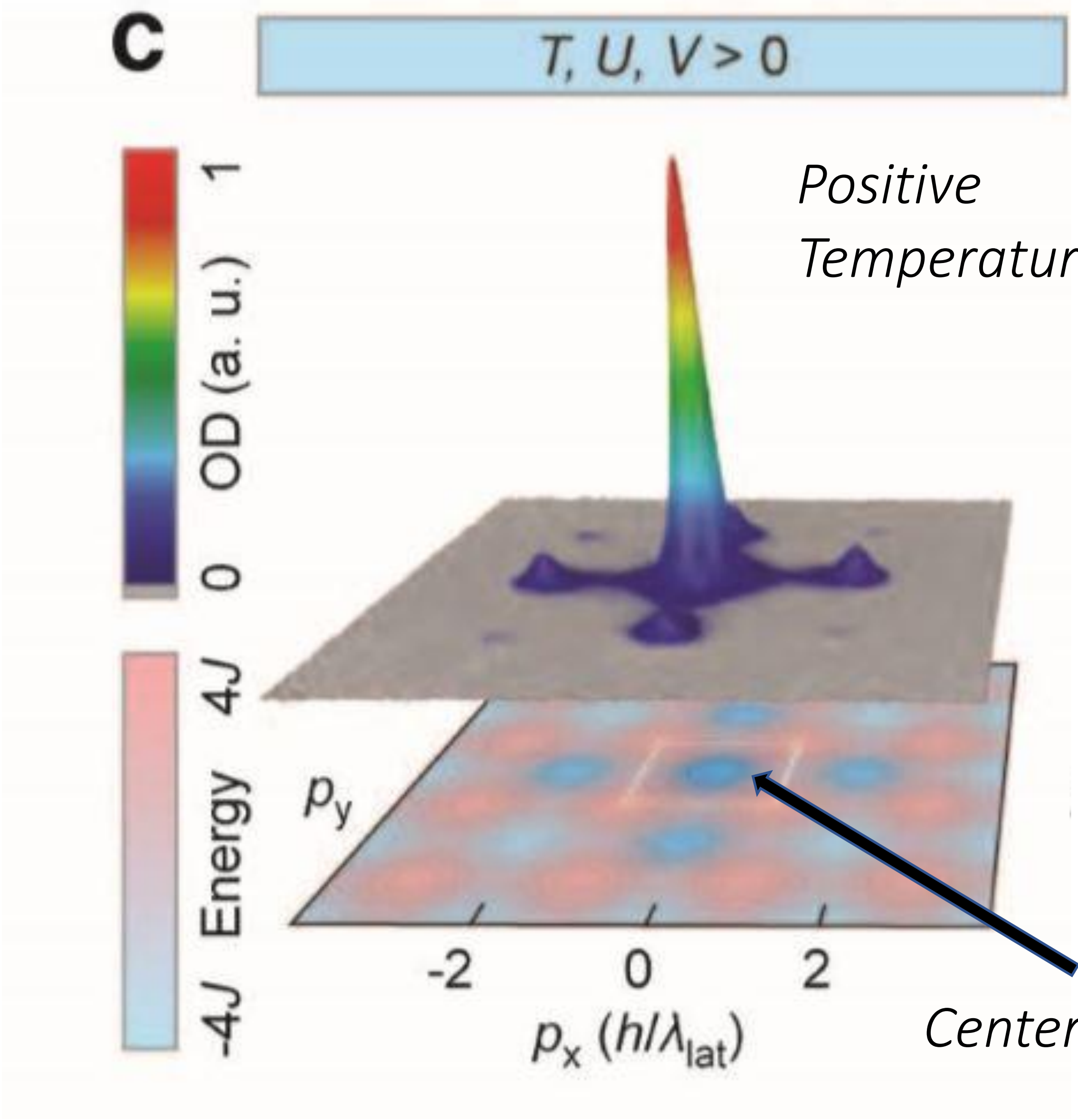


- Equation relating momentum and time:

$$pc = \frac{L(m_0c^2)}{\sqrt{t^2c^2 - L^2}}$$

- Possibly recording final intensity of the laser to measure optical density/absorbance

Momentum Distribution for Positive and Negative Temperature States

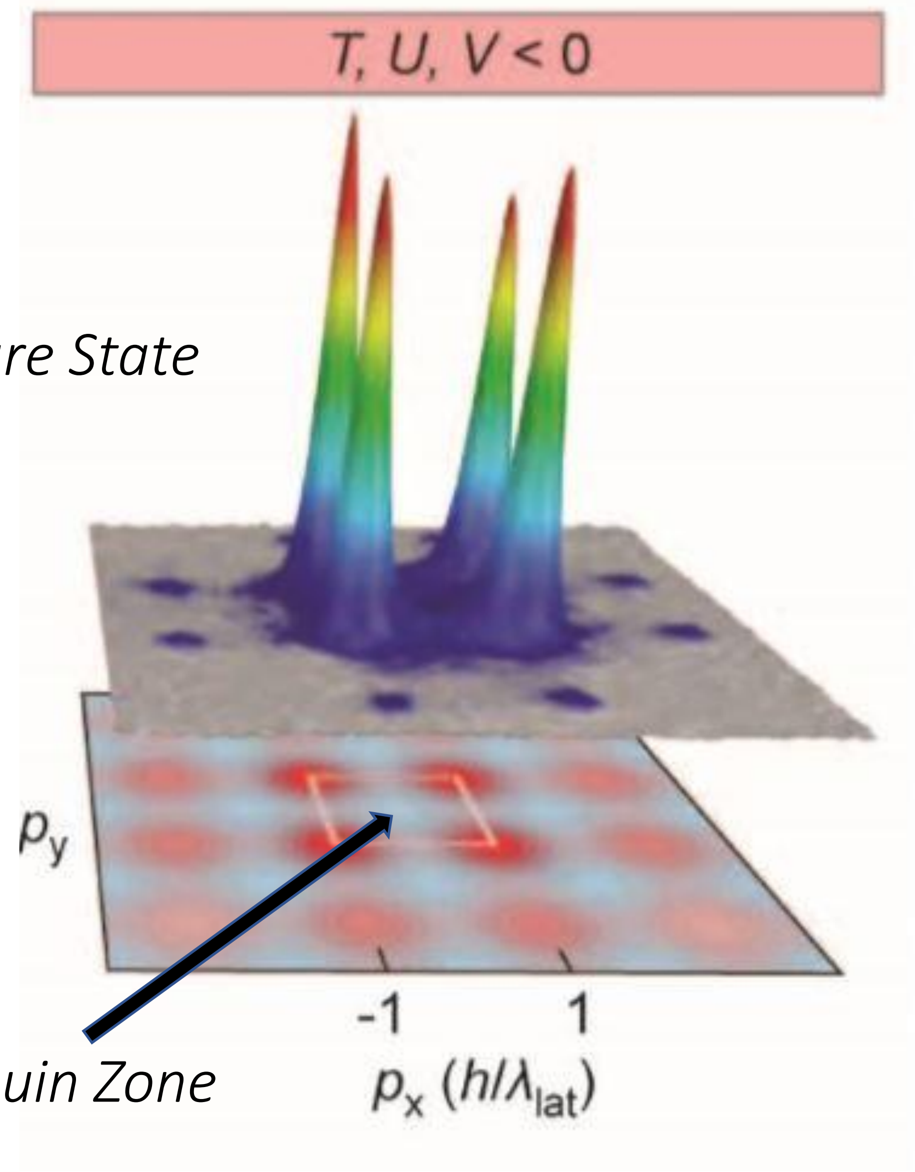


Positive Temperature State

Center of Brillouin Zone

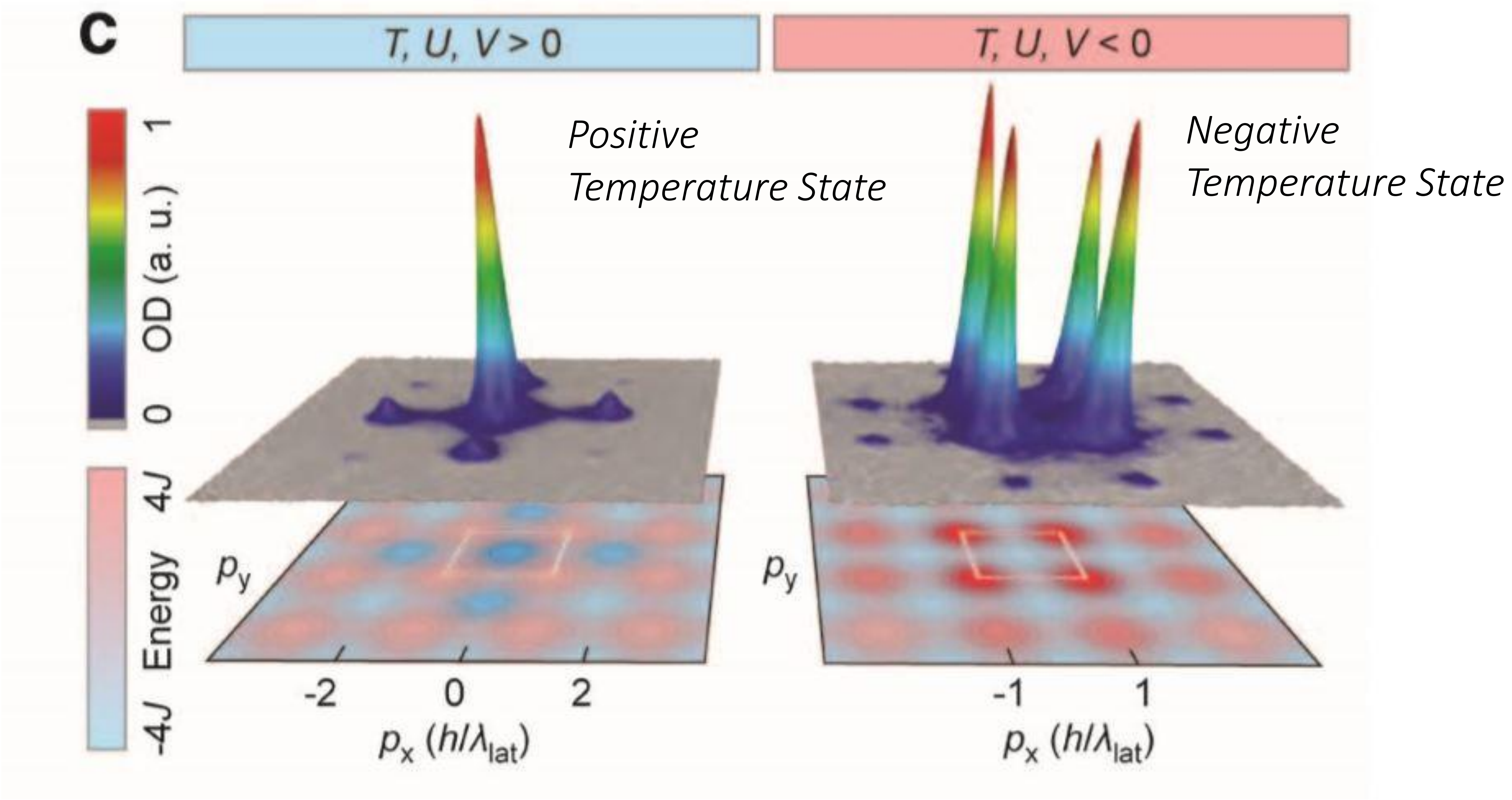
Momentum Distribution for Positive and Negative Temperature States

Negative Temperature State

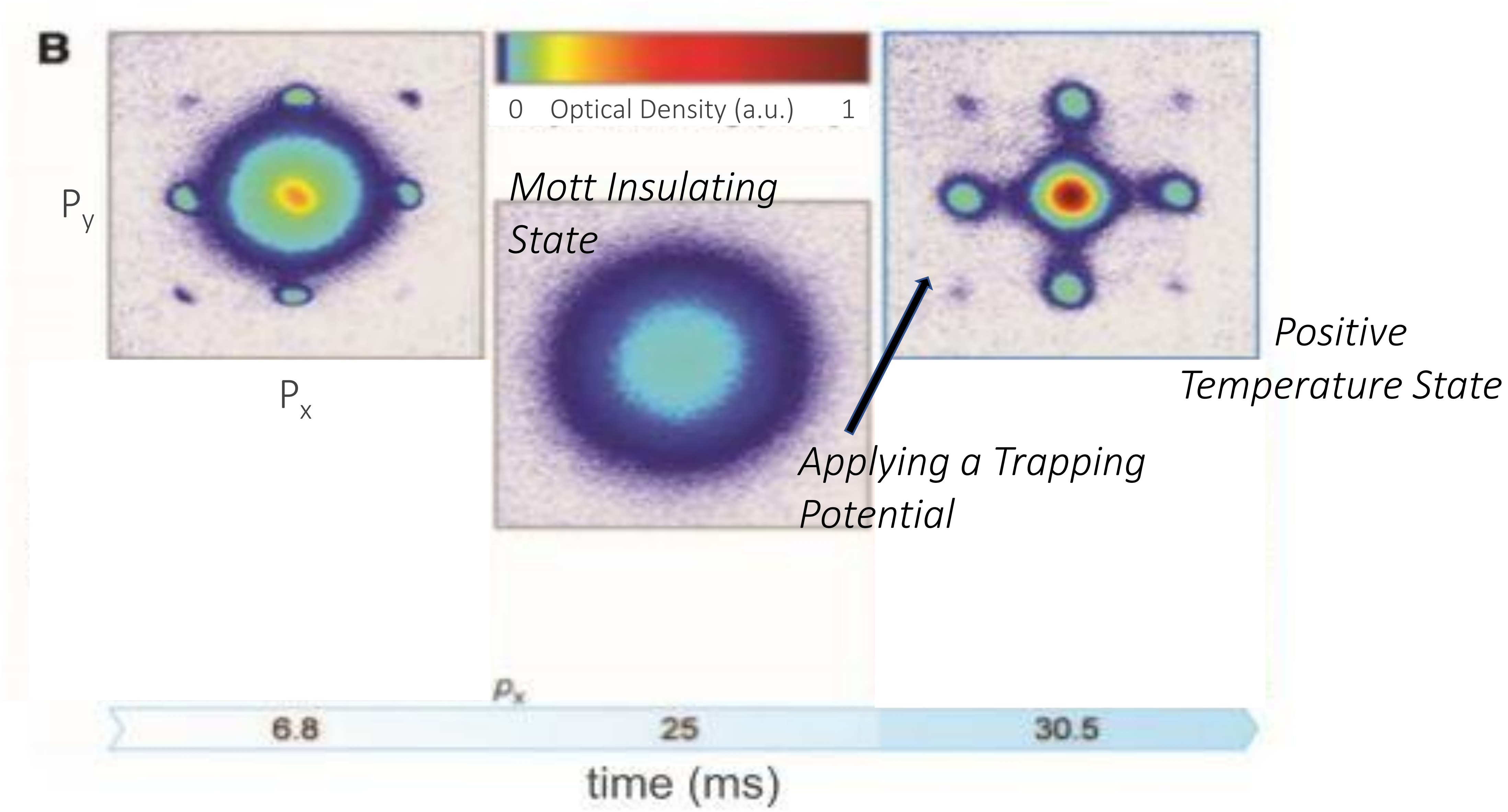


Center of Brillouin Zone

Momentum Distribution for Positive and Negative Temperature States



Change in the Momentum Distribution as a Function of Time



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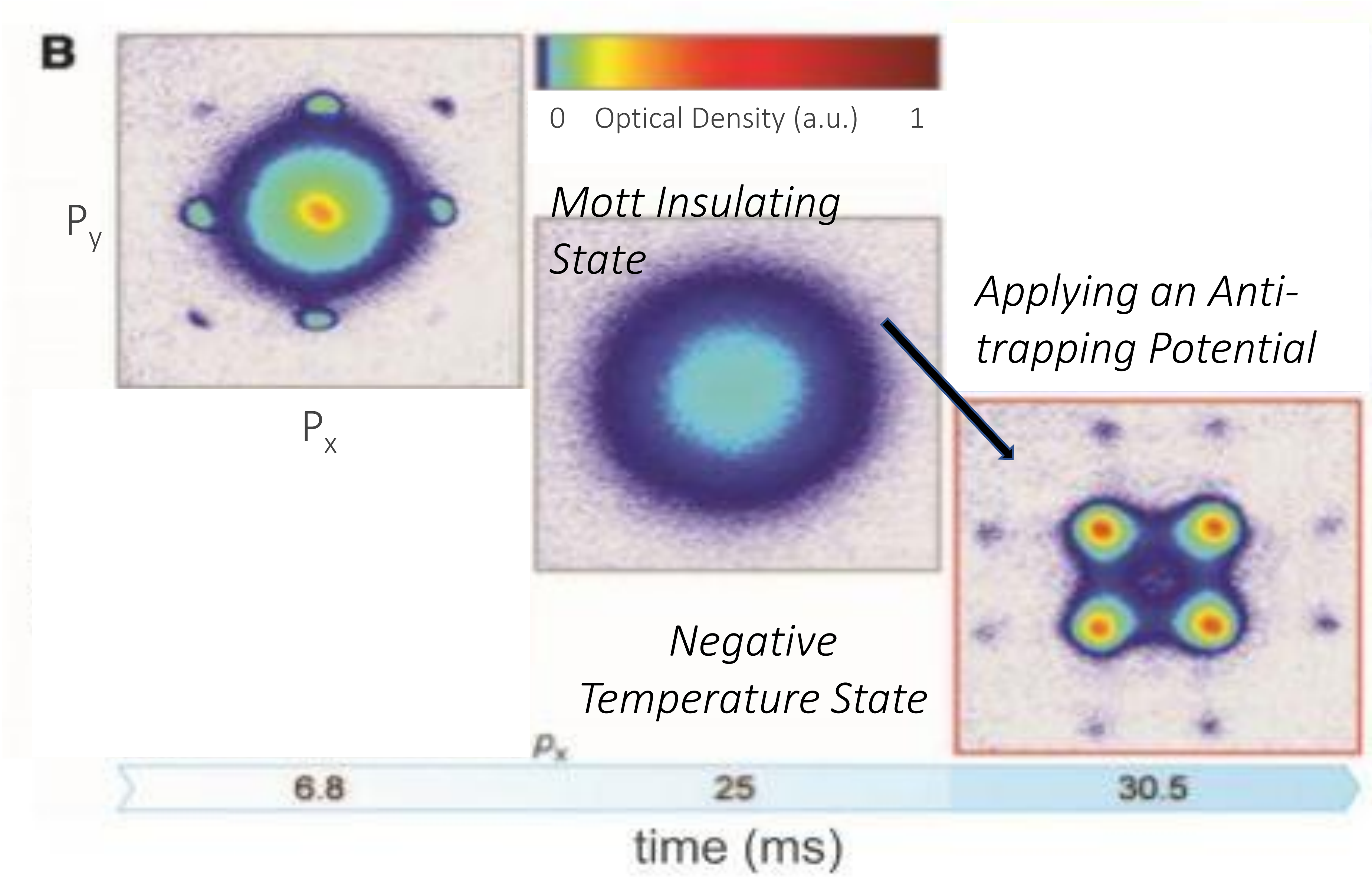

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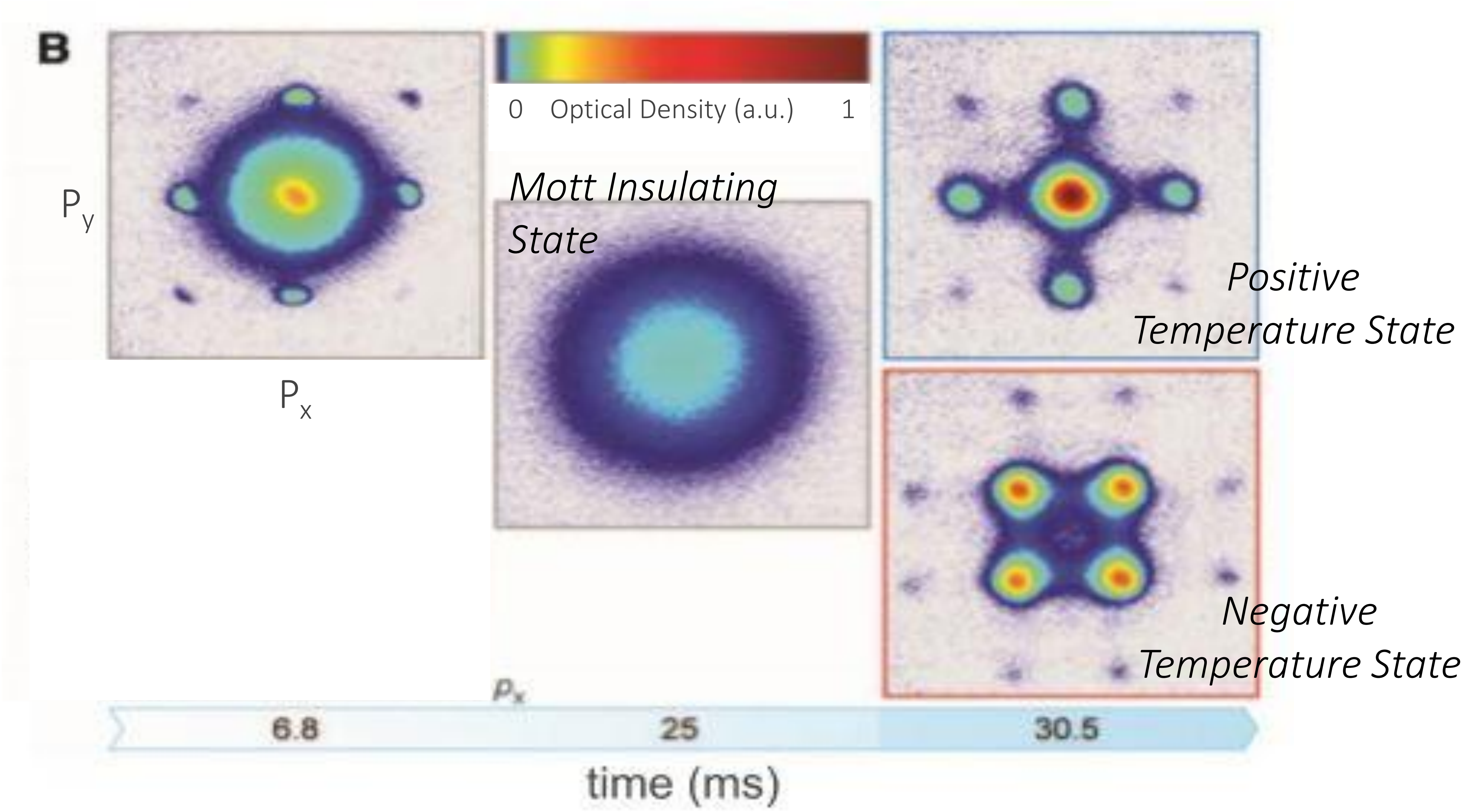
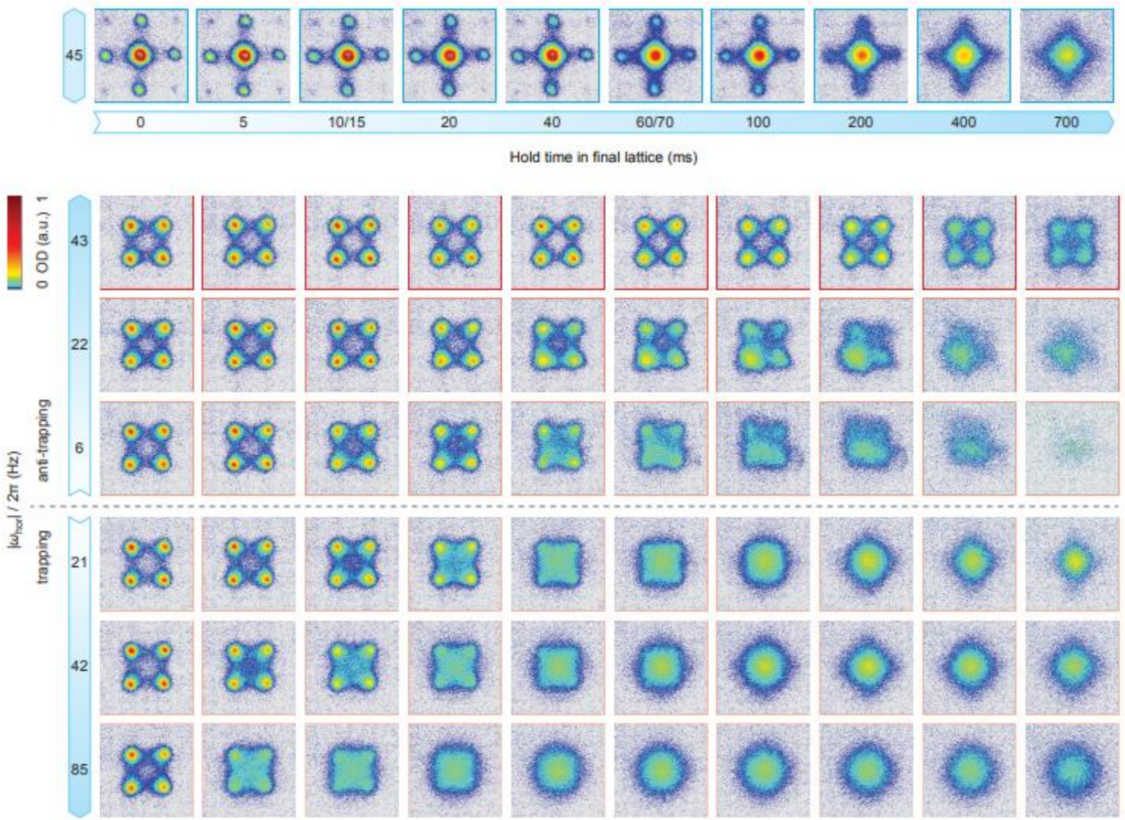
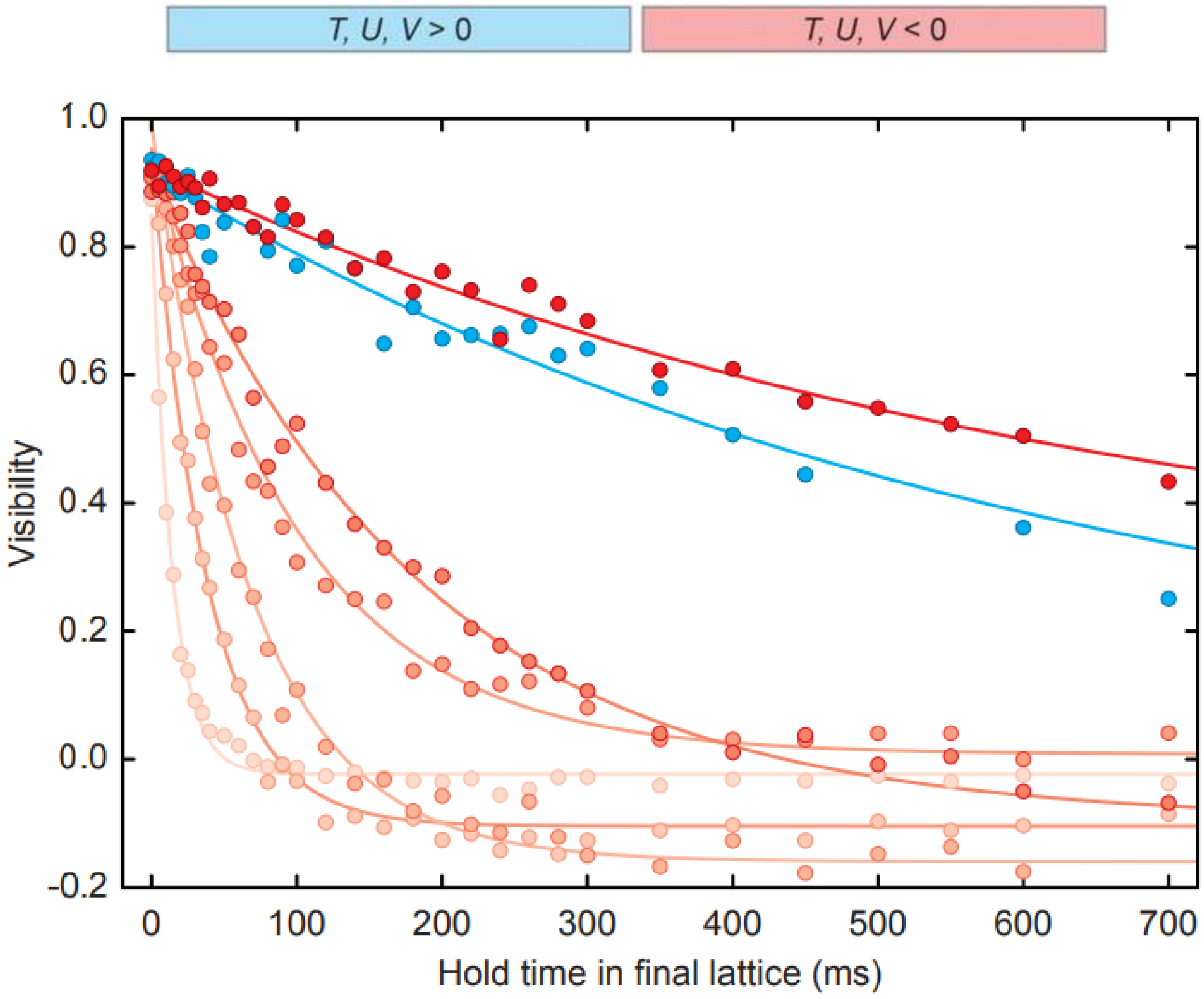


Coherence of Positive vs. Negative Temperature States



Coherence of Positive vs. Negative Temperature States



Our Overview of the Paper

- Produced a Hamiltonian from the Bose-Hubbard model which described a negative temperature state
- Used time of flight measurements to detect a negative temperature state in a Bose-Einstein condensate
- Observed negative temperature states with stability greater than the positive temperature state

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- Produced a Hamiltonian from the Bose-Hubbard model which described a negative temperature state
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Our takeaway?

Critical Analysis

Weaknesses

- Not very accessible to non-experts
- Jargon heavy
- Based experiment on another paper, but don't talk about the explicit details
- Didn’t mention whether they tried many trials for the positive temperature state

Strengths

- Provided supplemental material that was useful

Citation Analysis

- Number of citations: 208
- Number of citations for most cited paper: 396
- Scopus Impact Score: 3.42

- The article has been cited by well-regarded papers on:
  - fundamental thermodynamics,
  - non-equilibrium statistical mechanics and
  - topological condensed matter.