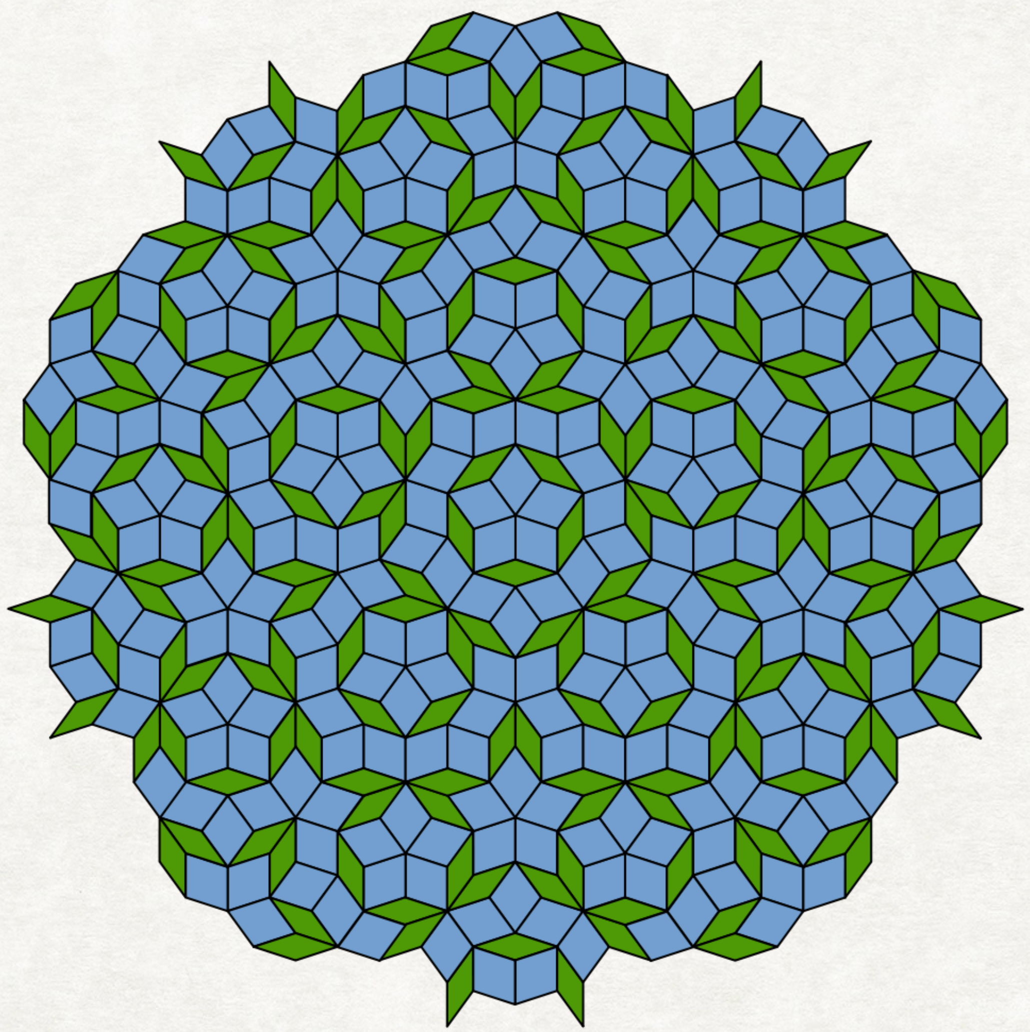
CHARLIE MAIER, SIMONE MEZZASOMA, BENJAMIN MOY, HUNG TAN

LOCALIZATION PROBLEM IN ONE DIMENSION: MAPPING AND ESCAPE

M. KOHMOTO, L. KADANOFF, C. TANG

MOTIVATION

- Quasi-Crystal vs. Periodic Crystal: ordered but not periodic



Penrose tiling: an example of a 2D quasicrystal*

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- Not solvable with analytical tools used in periodic systems
- Allow us to study transition between localized and extended states

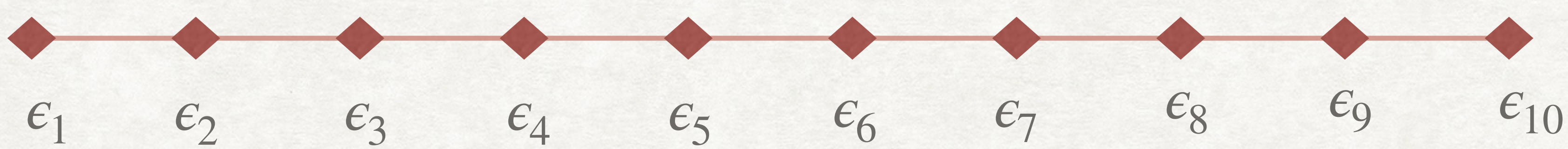
1-D QUASI-CRYSTAL

- Tight Binding Model

- ψ(x) $\rightarrow \psi_n$

1-D QUASI-CRYSTAL

- Tight Binding Model
- $\psi(\mathrm{x}) \to \psi_n$
- $\psi_{n-1} + \psi_{n+1} + (\epsilon_n - E)\psi_n = 0$ (Schrodinger Equation)

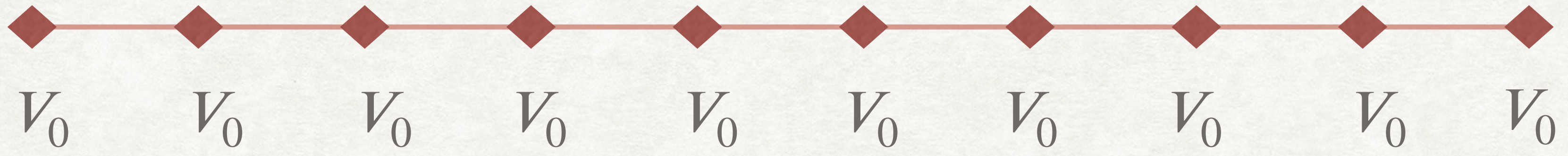


1-D QUASI-CRYSTAL

POTENTIAL

$\epsilon_n \in \{V_0, V_1\}$

$S_0 = \{V_0\}$



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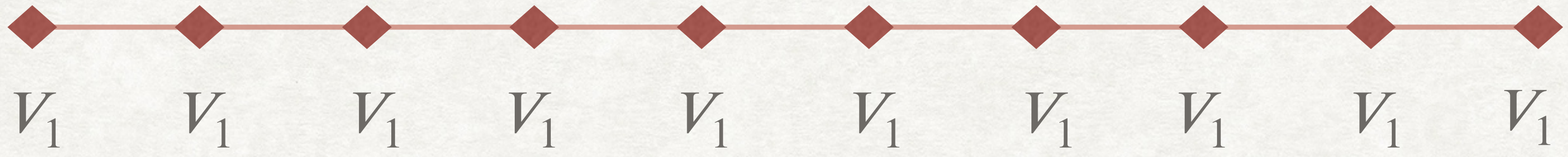

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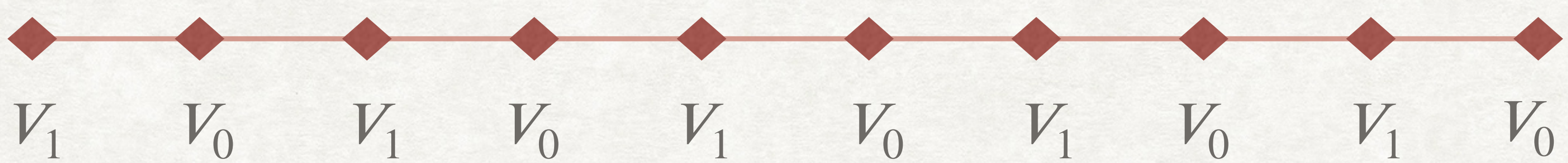

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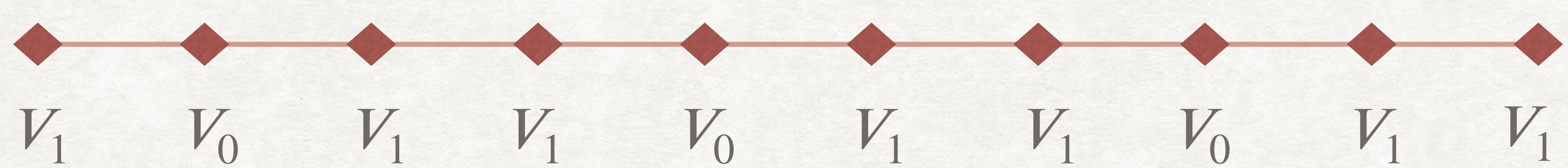

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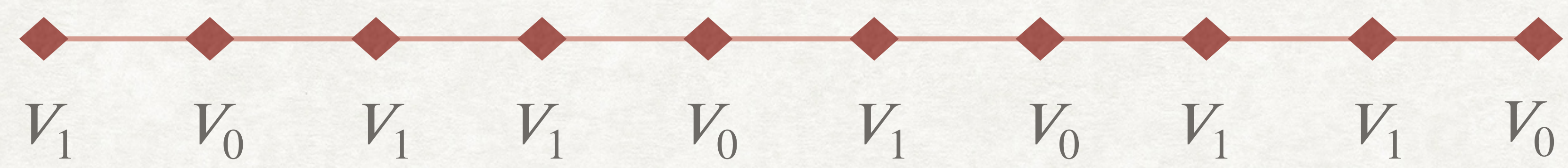

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$$\cdots$$

$$S_l = \{S_{l-1}, S_{l-2}\} \qquad F_l = F_{l-1} + F_{l-2}$$

1-D QUASI-CRYSTAL

TRACE MAP

$$\psi_{n-1} + \psi_{n+1} + (\epsilon_n - E)\psi_n = 0$$

$$T_n \begin{pmatrix} \psi_n \\ \psi_{n-1} \end{pmatrix} = \begin{pmatrix} \psi_{n+1} \\ \psi_n \end{pmatrix}$$

$$T_n = \begin{pmatrix} E - \epsilon_n & -1 \\ 1 & 0 \end{pmatrix}$$

$$M_l = T_{F_l - 1} T_{F_l - 2} \cdots T_0$$

$$M_{l+1} = M_{l-1} M_l$$

$$x_{l+1} = 2 x_l \, x_{l-1} - x_{l-2}$$

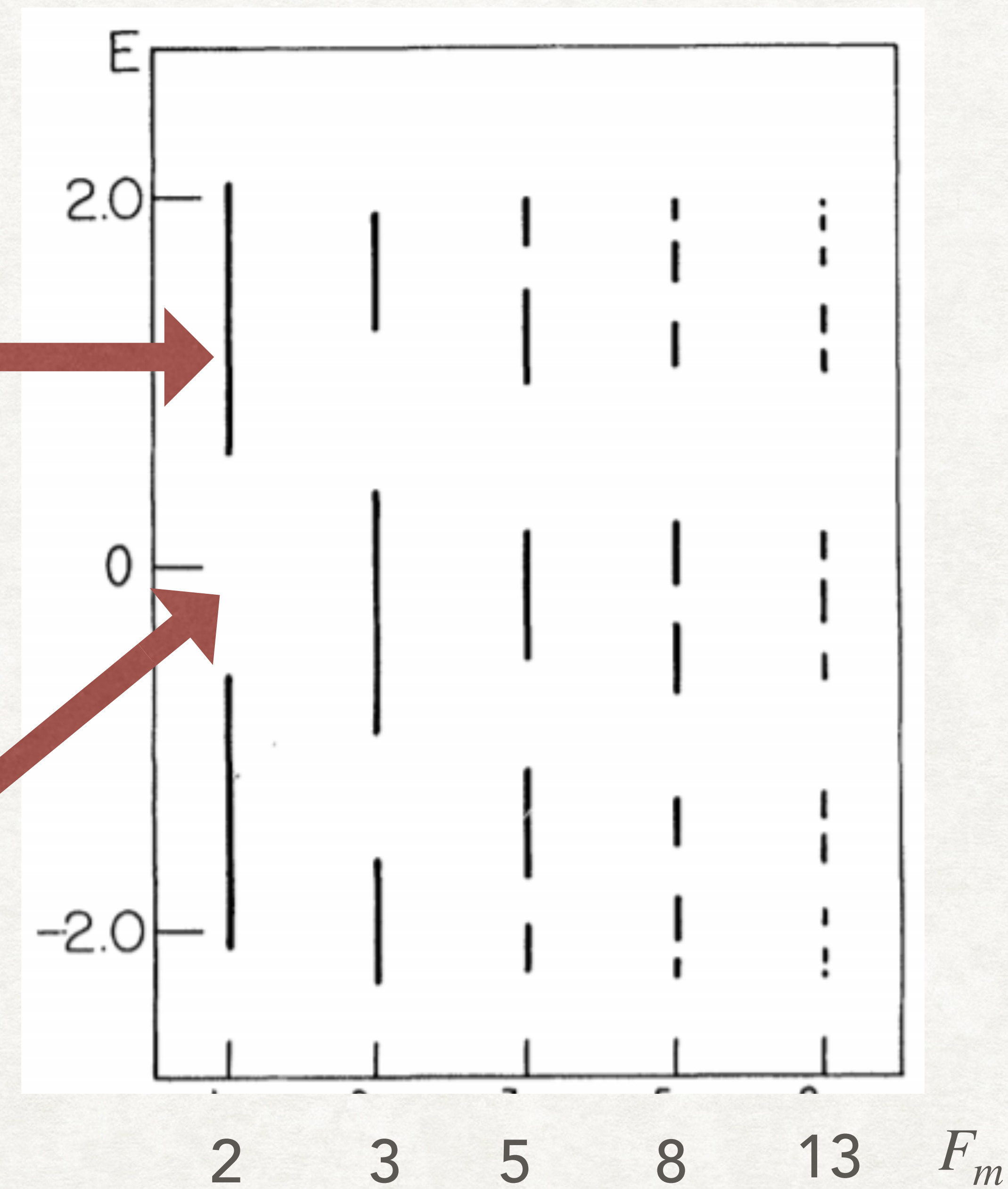
$$x_l = \frac{1}{2} \mathrm{Tr}\, M_l$$

1-D QUASI-CRYSTAL

ENERGY BANDS

$\left|x_m(E)\right| \leq 1$: Allowed energy
Non-escaping

$\left|x_m(E)\right| > 1$: Forbidden energy
Escaping



Increasing period

1-D QUASI-CRYSTAL

ENERGY BANDS

Cantor-like set

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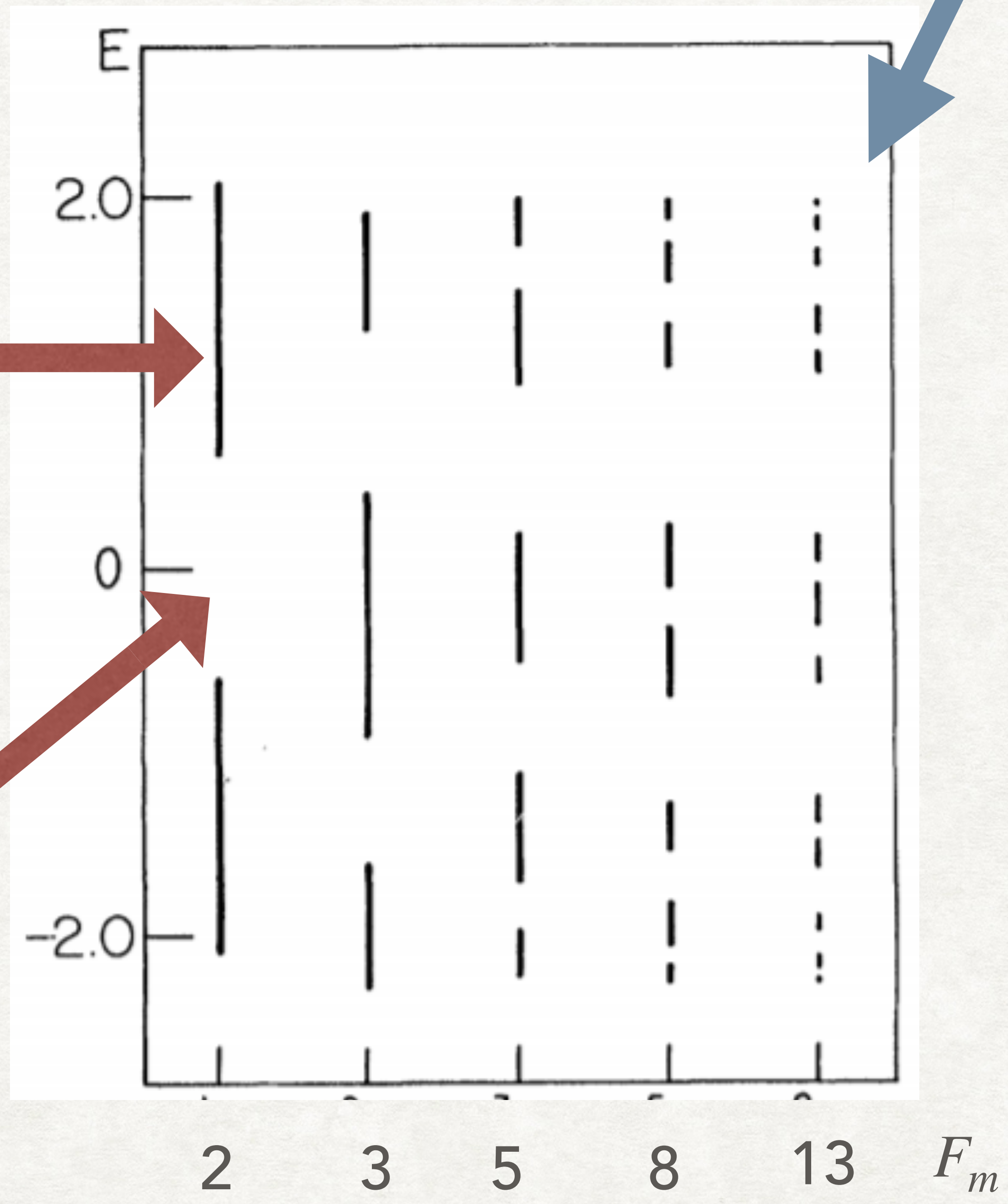

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- Periodic potential: solvable
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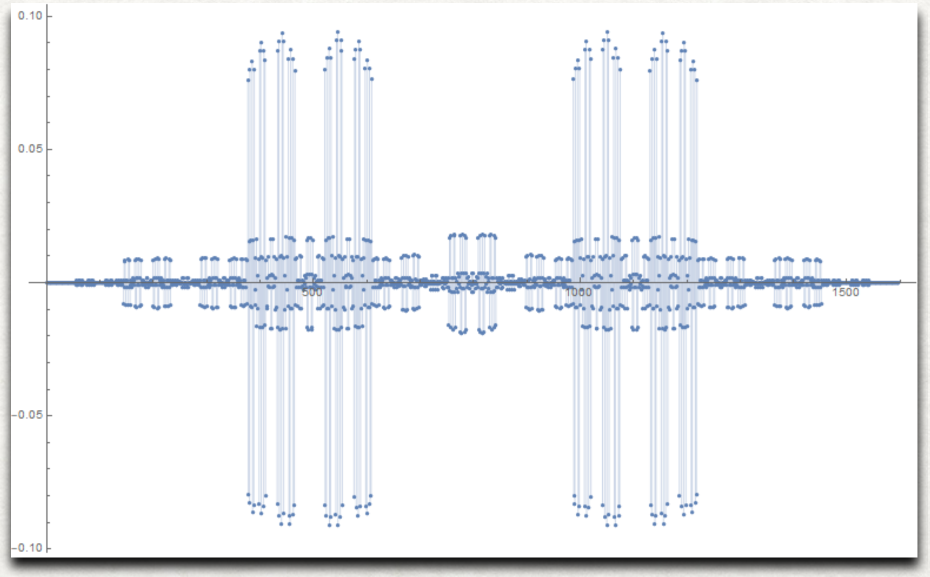
- Quasi-crystal: not directly solvable
  - Reach solution in the infinite period (incommensurate) limit
- In the incommensurate limit:
  - Energy band width shrinks exponentially
  - Allowed energies become a Cantor-like set
  - States reach a compromise between localized and extended

Generalization of extended states



ANALYSIS OF RESULTS

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- Methods very general
    - Applicable to many fields
- Terse explanations; could be more accessible
    - Never mentions “tight-binding model”
    - Elaborate further upon allowed/forbidden energies & escape
- Introduction and conclusion are dated
    - Motivation is very cursory
    - Conclusion is non-existent

PREVIOUS (AND CONCURRENT) RESEARCH

- Conceptually motivated by previous mathematical research
- Methods incrementally built on previous physics research
    - Mapping to trace recursion
    - Stronger analysis of energy band widths

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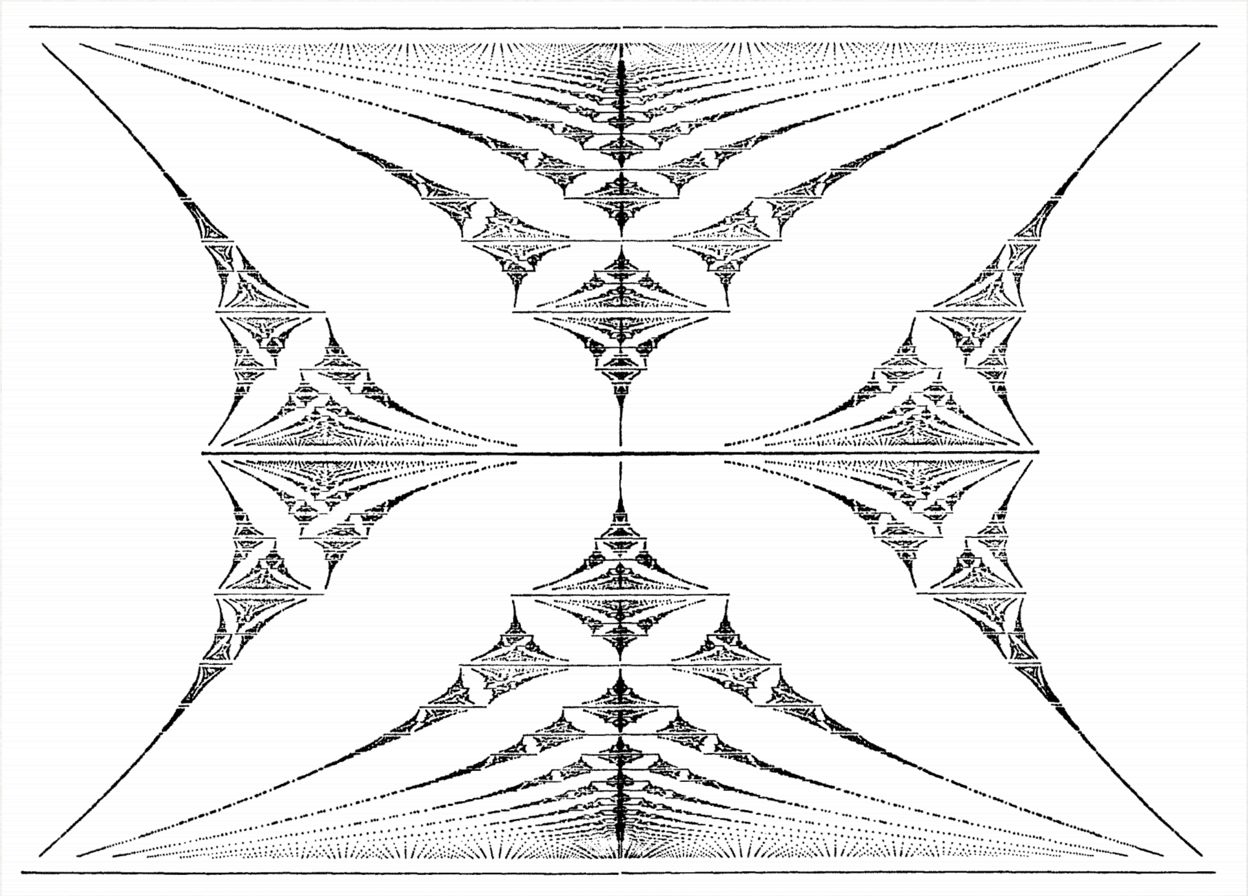
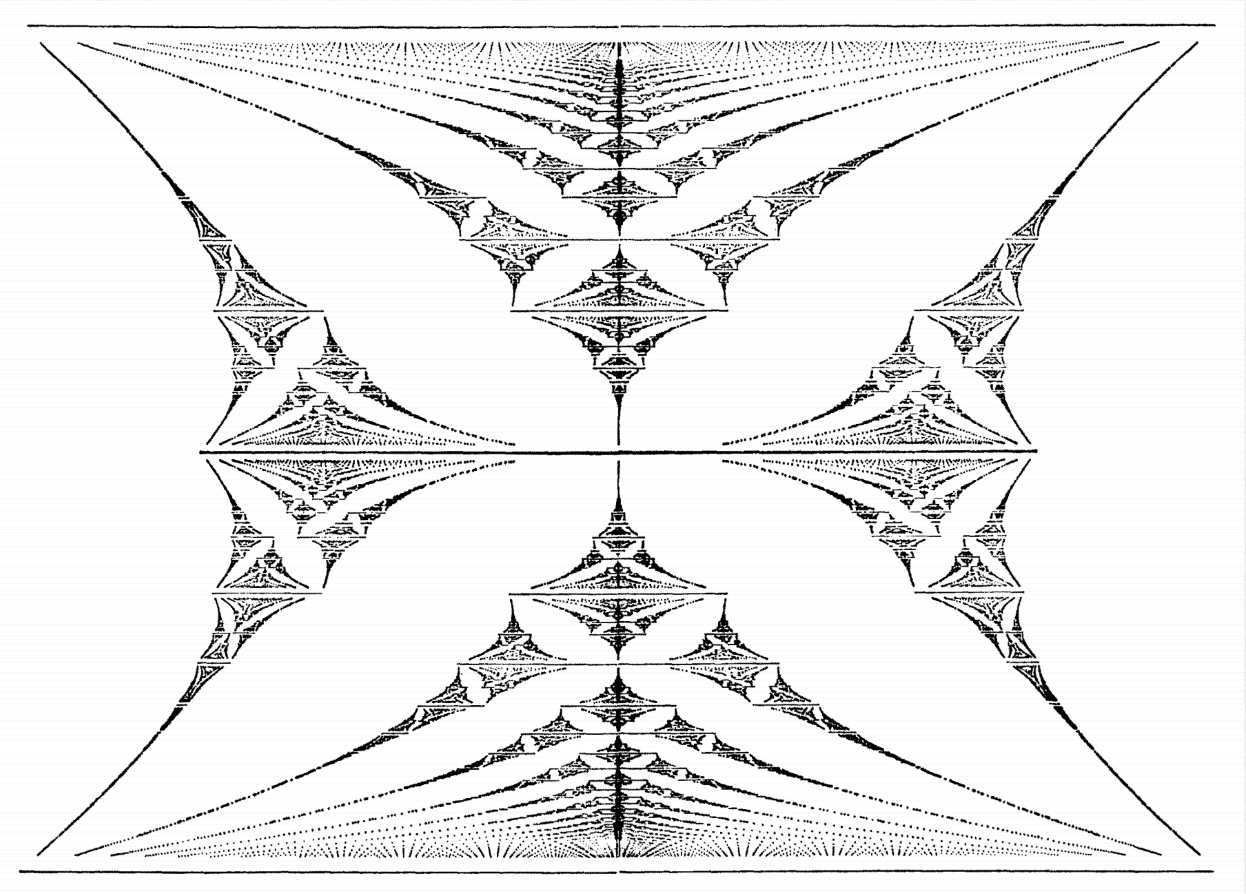


Magnetic potentials on lattices can produce similar Cantor spectra. Hofstadter, (1976).

PREVIOUS (AND CONCURRENT) RESEARCH

- Conceptually motivated by previous mathematical research
- Methods incrementally built on previous physics research
    - Mapping to trace recursion
    - Stronger analysis of energy band widths
- Published in parallel with very similar paper
    - Overlap in problem & results
    - Different methods



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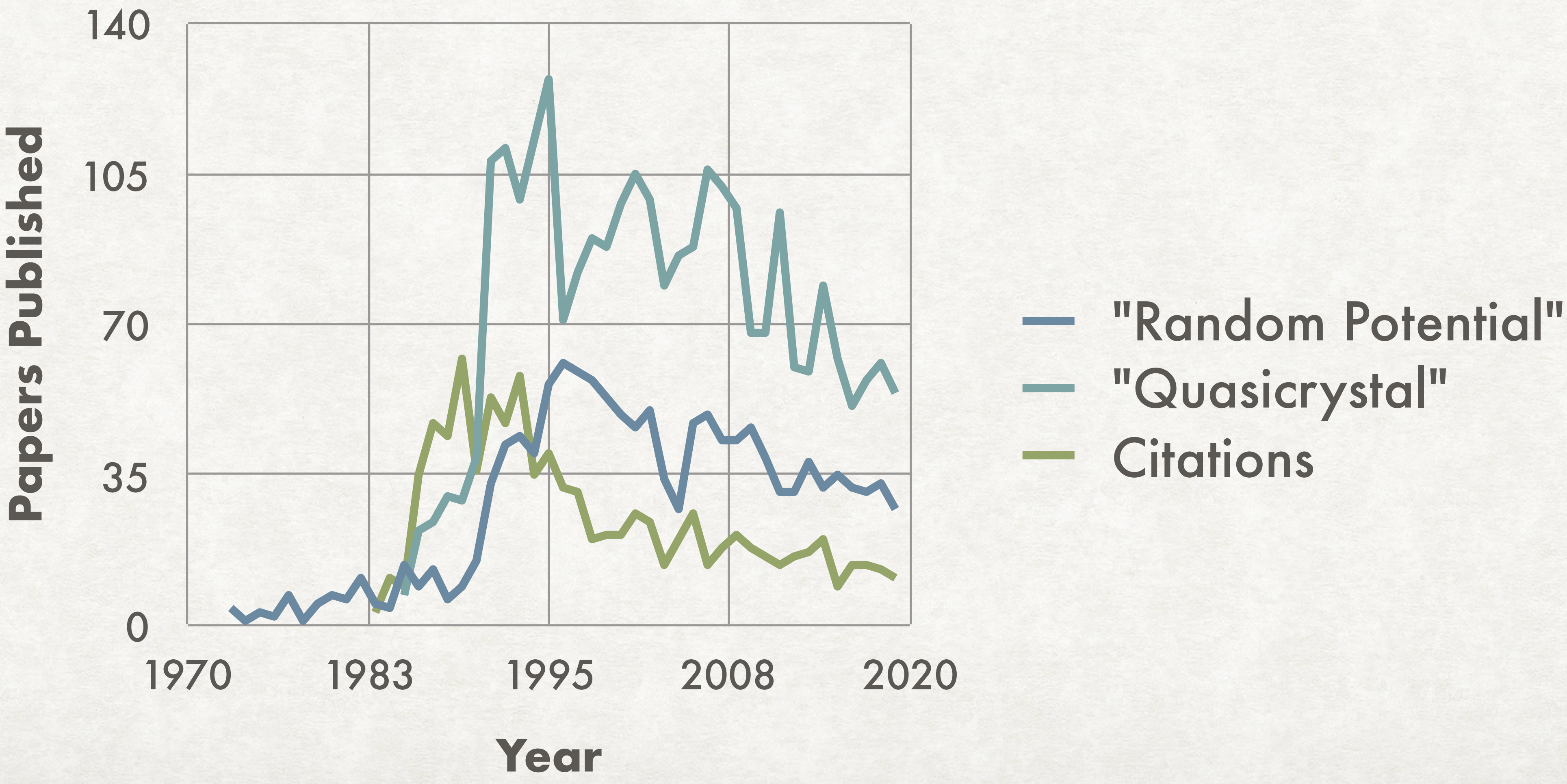
IMPACT ON THE FIELD

- Foundational work in analyzing quasi-periodic potentials
    - Stepping stone for the authors for future work

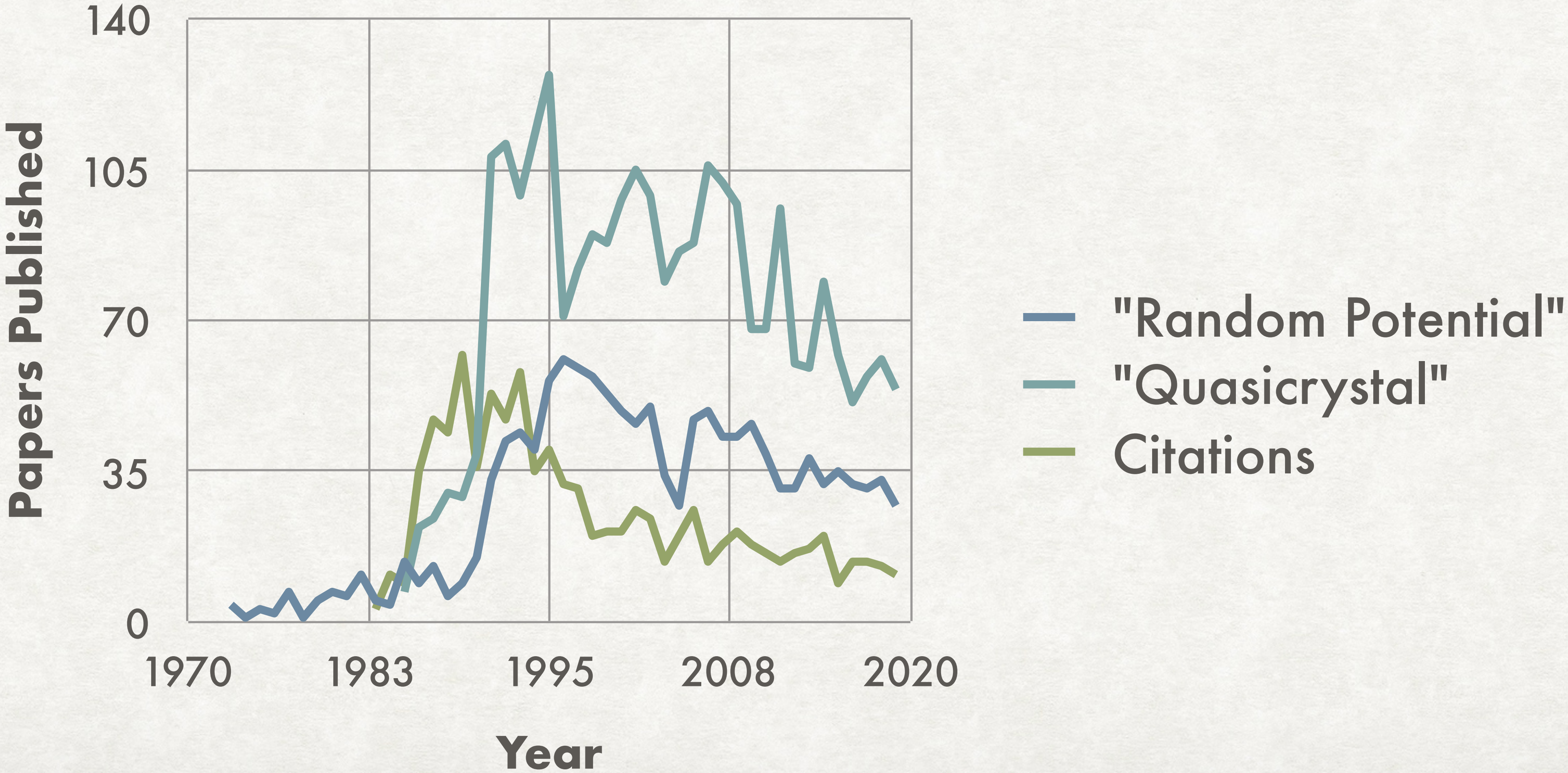
EVOLUTION IN FIELD AND TECHNIQUES

- Published at the beginning of research on quasi-periodic systems



EVOLUTION IN FIELD AND TECHNIQUES

- Published at the beginning of research on quasi-periodic systems
- Shortly proceeded by larger interest in computational physics
    - Further development/availability
    - Changed how people analyzed these systems



RECAP

- Authors have developed useful methods
- They use these methods to solve for the energies in a specific case
- Can also be applied to other physical realizations
- Not very accessible to non-experts
- Foundational work but dated because of advent of computation