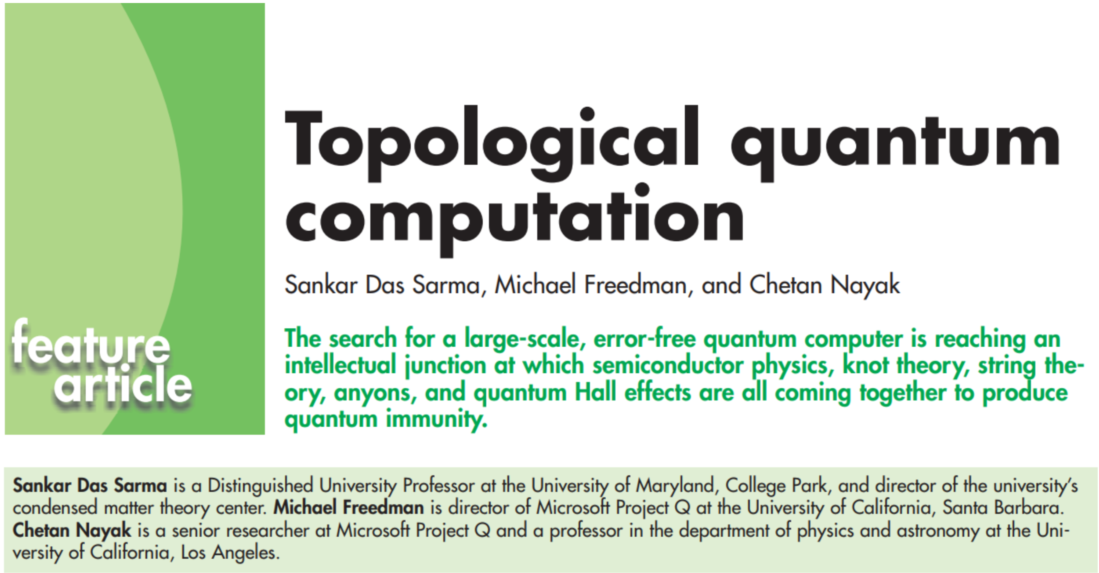
Topological Quantum Computation

596 Journal Club Presentation



Topological quantum computation

Sankar Das Sarma, Michael Freedman, and Chetan Nayak

feature article

The search for a large-scale, error-free quantum computer is reaching an intellectual junction at which semiconductor physics, knot theory, string theory, anyons, and quantum Hall effects are all coming together to produce quantum immunity.

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Physics Today **59**, 7, 32 (2006)

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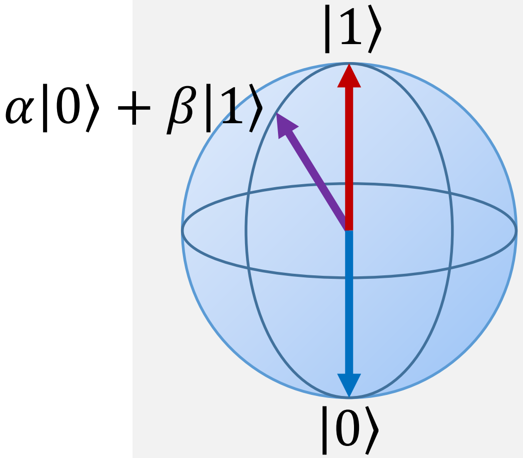
Quantum Computation Oversimplified: How, Why and Why not yet

How

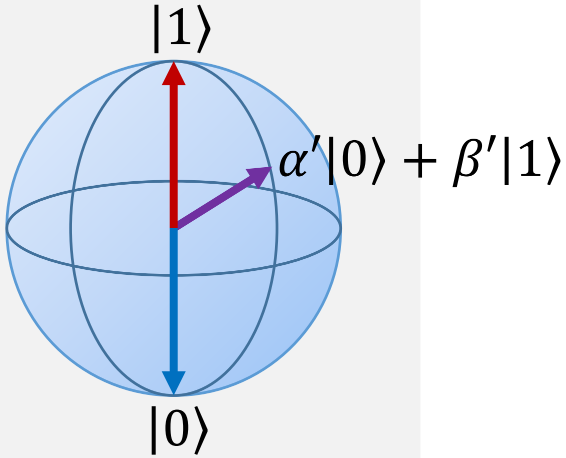


Initialization

"Single Qubit Gate"

$$|\psi(t)\rangle = e^{-\imath Ht}|\psi(0)\rangle$$

Manipulation



Measurement

Why

Promises:

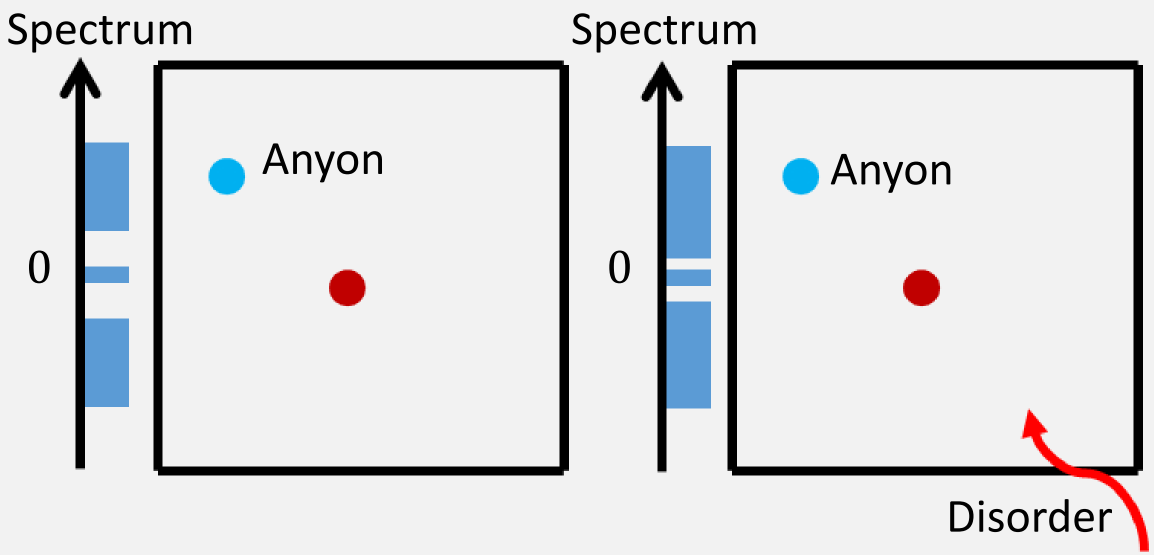
Factorization (Shor, 1994), database searching (Grover, 1996)…

Why not yet

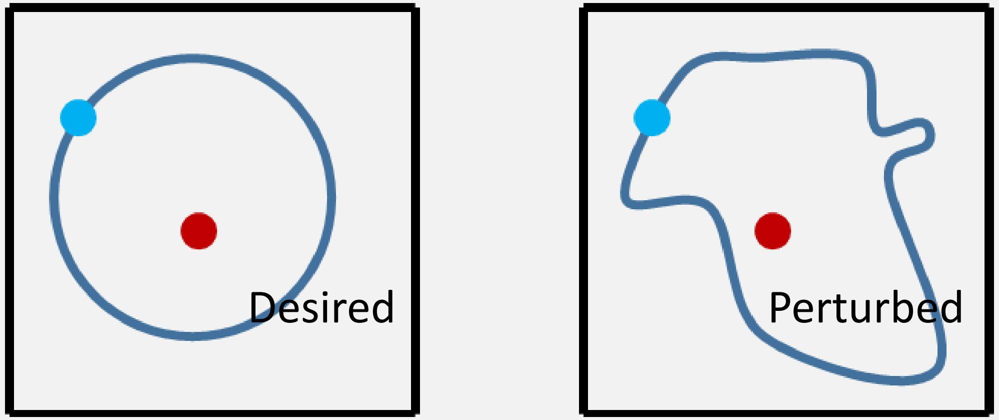
Error sources:

- Decoherence: System interacts with environment
- Manipulation: $H = H_0 + H_{\mathrm{disorder}}$

Topological Quantum Computation Addresses Difficulties in Conventional Quantum Computation



Materials hosting Anyons have
topological order, *resistant* to disorder

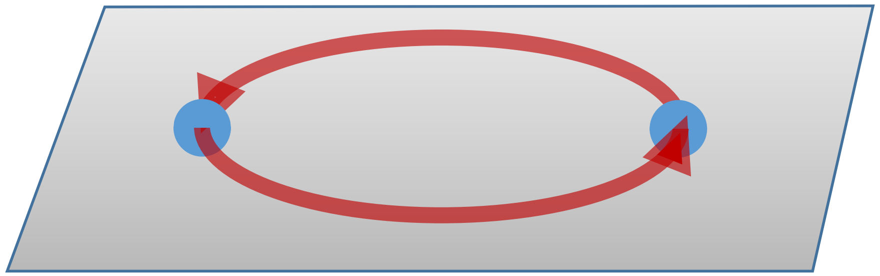


Manipulations are topological,
resistant to deformations

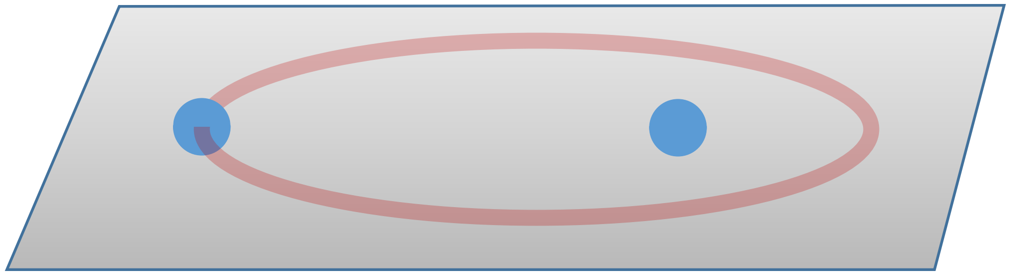
Decoherence: System interacts with environment

Manipulation: $H = H_0 + H_{\text{disorder}}$

Equivalence of Exchanging Twice and Winding Once for Identical Particles

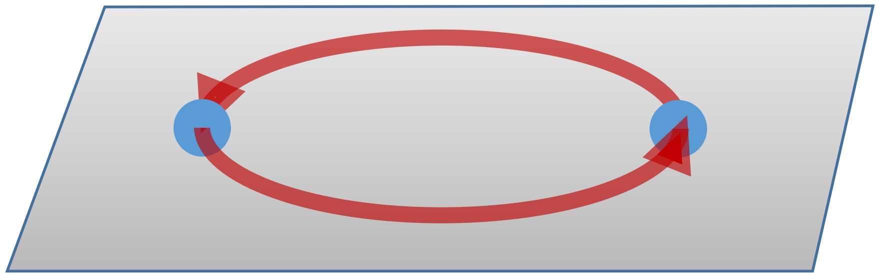


Exchange of two identical particles

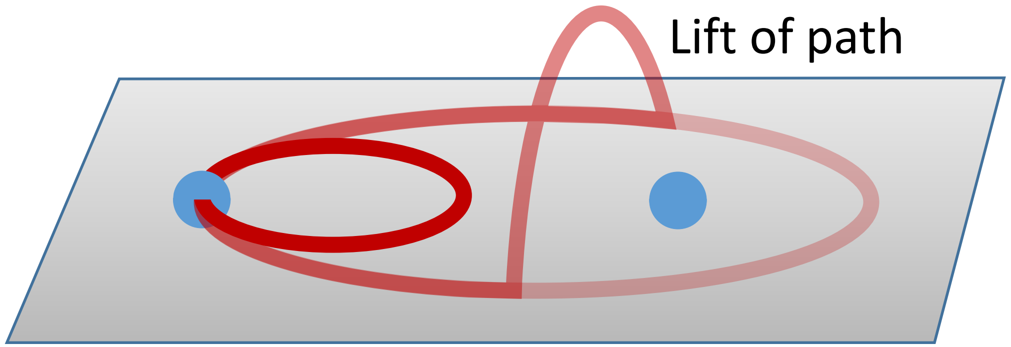


Winding of one identical particles around another

Winding is Topologically Trivial in 3D, not Necessarily 2D



Exchange of two identical particles



Winding of one identical particles around another

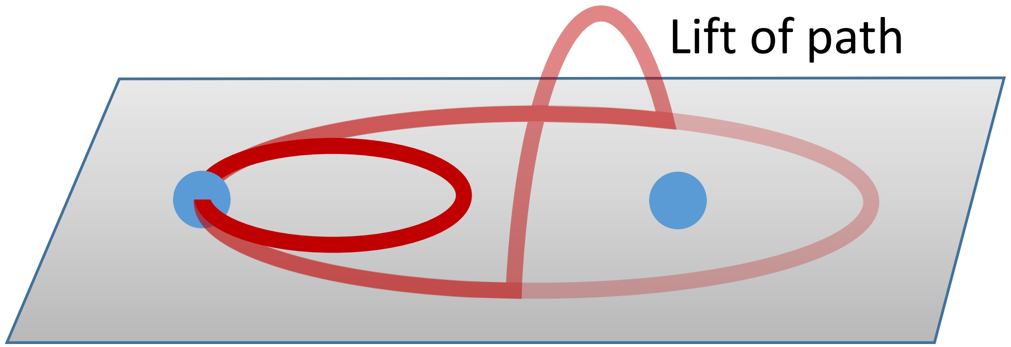
Identical particles in 3D

$$T|\psi\rangle = \lambda|\psi\rangle$$

$$T^2|\psi\rangle = \lambda^2|\psi\rangle = |\psi\rangle$$

$$\lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

Bosons/Fermions



Winding of one identical particles around another behaves differently in 2D and 3D

Building Blocks for Topological Quantum Computation: Anyons

Identical particles in 3D

$$T|\psi\rangle = \lambda|\psi\rangle$$

$$T^2|\psi\rangle = \lambda^2|\psi\rangle = |\psi\rangle$$

$$\lambda^2 = 1 \Rightarrow \lambda = \pm 1$$

Fermions/Bosons

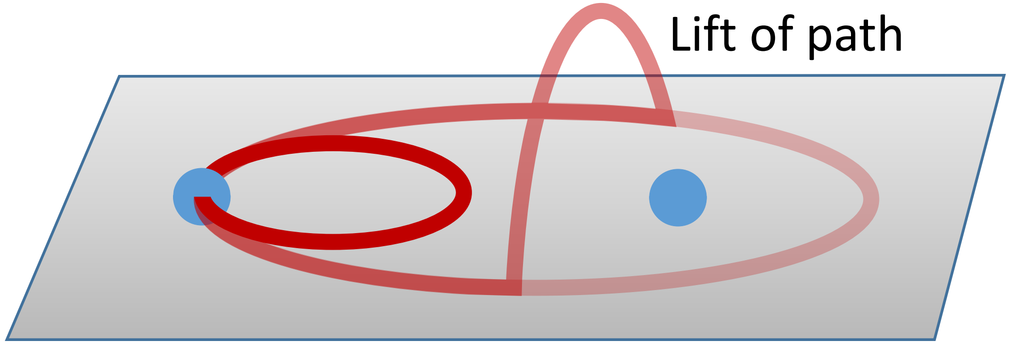
Identical particles in 2D

$$T|\psi\rangle = \lambda|\psi\rangle$$

$$T^2|\psi\rangle = \lambda^2|\psi\rangle \neq |\psi\rangle$$

$$\lambda^n = 1 \Rightarrow \lambda = e^{\frac{\iota\pi}{n}}$$

Anyons

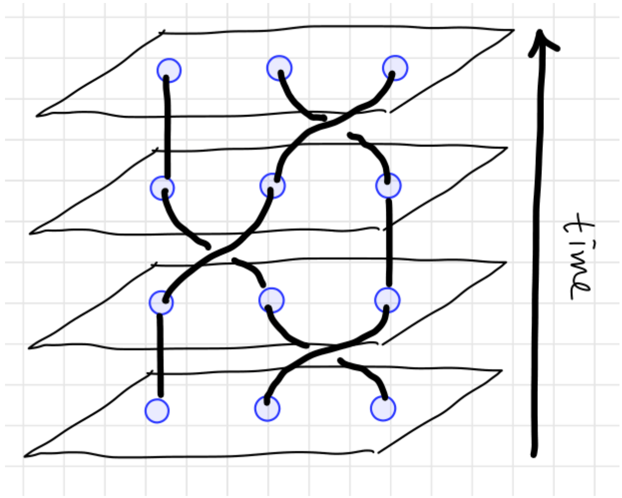


Winding of one identical particles around another behaves differently in 2D and 3D

Open Questions in Using Anyons to Build Topological Quantum Computers

- Identifying systems that can host anyons
  - Toy model, *ab initio*, experimental
  - Stable to observe, manipulate
  - Right types of anyons

- Invent a scheme to perform quantum gates
  - The most promising way: by consecutive exchanging (**braiding**) anyons
  - How to build a **universal** quantum computer with the gates



(Braiding) exchanging positions of three particles on 2

Critique: No Known Experimental System for Topological Quantum Computing

Issue: No known system with correct non-abelian statistics.

Admitted in the outlook section of paper.

Critique: Topological Quantum Computers have a Long Road Ahead

R 25 PHYSICAL REVIEW LETTERS

Demonstration of a Fundamental Quantum Logic Gate

C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland

letters to nature

Coherent control of macroscopic quantum states in a single-Cooper-pair box

Y. Nakamura*, Yu. A. Pashkin† & J. S. Tsai*

First implementation of ion trap logic gate: **1995**.

Current progress: high fidelity gates, a lot of progress, but still a long ways to go until large scale computer![1]

First implementation of superconducting qubit: **1999**

Current progress: same story as trapped ions: high fidelity gates, but there's still a long way to go.[2]

Take away: Once a platform has been experimentally demonstrated, there is a long way towards a good implementation of that platform.

1:Bruzewicz, C. D., Chiaverini, J., Mcconnell, R., & Sage, J. M. (2019). Trapped-ion quantum computing: Progress and challenges. Applied Physics Reviews, 6(2), 021314.

2:Huang, H., Wu, D., Fan, D. *et al.* Superconducting quantum computing: a review. *Sci. China Inf. Sci.* **63,** 180501 (2020).

History of Topological Quantum Computation

- Theoretics of 2+1-dimensional models first worked out by Witten* in 1989

- Kitaev+ applied these models to topological quantum computation
  - Worked out how to make quantum gates with anyons in 2003 paper
  - Sparked broader interest in the field

- Fractional quantum Hall effect remains leading candidate for use in quantum computation

*Witten, E. (1989). Quantum field theory and the jones polynomial. *Communications in Mathematical Physics*, *121*(3), 351-399. doi:10.1007/BF01217730

+Kitaev, A. Y. (2003). Fault-tolerant quantum computation by anyons. *Annals of Physics, 303*(1), 2-30. doi:10.1016/S0003-4916(02)00018-0

Alternate Approaches

- Our paper focused on the fractional quantum Hall effect, but this approach has issues

- It's difficult to realize
  - Even harder to realize for universal gates

- Doing actual manipulation is difficult as well

- A lot of research into potential alternate strategies for topological systems

Hybrid Systems*

- Start with stabler non-universal fractional quantum Hall
- Prepare (noisy) arbitrary initial states
- Then use topological methods on these states
- Pro: System is simpler, universal, and still robust
- Con: Need to deal with noise from input states

*Bravyi, S. (2006). Universal quantum computation with the v=5/2 fractional quantum hall state. *Physical Review A - Atomic, Molecular, and Optical Physics, 73*(4) doi:10.1103/PhysRevA.73.042313

Topological Features in Other Systems

- Can find anyons and topological physics in non-Hall effect systems
- Certain holonomic ion traps have some anyonic features
- More recently (2014) some researchers [including the authors of our paper] have investigated combinations of various "simpler" materials[+]
  - Combinations of standard quantum Hall states/superconductors can realize anyons
- Pro: Can use systems of well-understood materials to obtain desired behavior
- Con: Theoretical and experimental complexity grows quickly the more complicated the structures become

*Pachos, J. (2002). Topological features in ion-trap holonomic computation. *Physical Review A - Atomic, Molecular, and Optical Physics, 66*(4), 423181-423186. doi:10.1103/PhysRevA.66.042318
+Mong, R. S. K et al. Universal topological quantum computation from a superconductor-abelian quantum hall heterostructure. *Physical Review X, 4*(1) doi:10.1103/PhysRevX.4.011036

The Jones Polynomial, Quantum Hall effect, and Kitaev's Quantum Computation review led to this paper

These are the papers that have the highest network *degree* - the number of references that also refer to these papers.

The maths leading to this paper was worked out by *Witten*.

This was first applied to condensed matter by *Moore*.

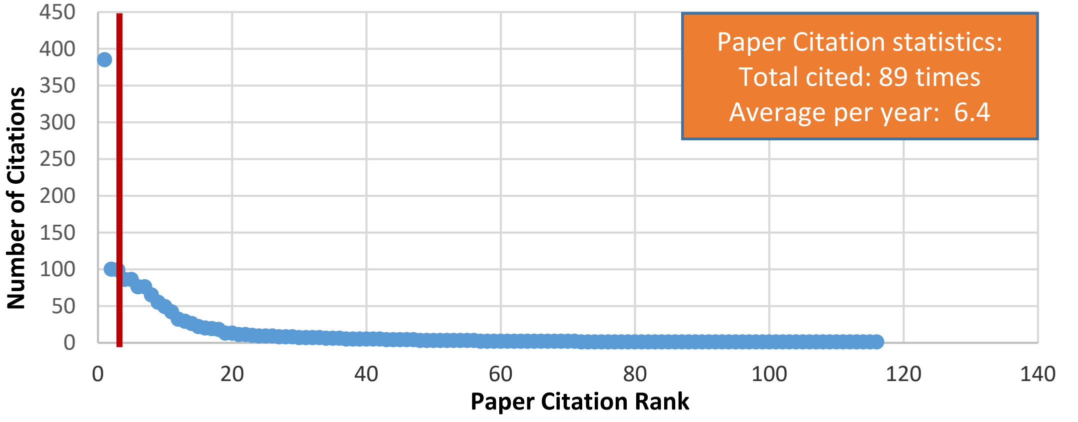
And *Kitaev* worked out how to put the two together in quantum computation.

Title	First Author	Year	Cit	Deg	Ref
Quantum field theory and the Jones polynomial	Witten	1989	5658	4	39
Nonabelions in the fractional quantum Hall effect	Moore	1991	1977	4	32
Fault tolerant quantum computation by anyons	Kitaev	2003	4822	2	23
Observation of an even-denominator quantum number in the fractional quantum Hall effect	Willett	1987	778	2	0
2n-quasihole states realize 2n−1-dimensional spinor braiding statistics in paired quantum Hall states	Nayak	1996	350	2	18
A Modular Functor Which is Universal for Quantum Computation	Freedman	2002	491	1	25
Topologically protected qubits from a possible non-Abelian fractional quantum Hall state.	Sarma	2005	731	1	1
Proposed experiments to probe the non-Abelian nu = 5/2 quantum hall state.	Stern	2006	242	1	0
Electron correlation in the second Landau level: a competition between many nearly degenerate quantum phases.	Xia	2004	362	1	35
A Chern-Simons effective field theory for the Pfaffian quantum Hall state	Fradkin	1998	268	1	13
A polynomial invariant for knots via von Neumann algebras	Jones	1985	2047	0	18
Controlling spin exchange interactions of ultracold atoms in optical lattices.	Duan	2003	1253	0	2
Detecting Non-Abelian Statistics in the nu=5/2 Fractional Quantum Hall State	Bonderson	2006	243	0	32
Braid topologies for quantum computation.	Bonesteel	2005	163	0	3

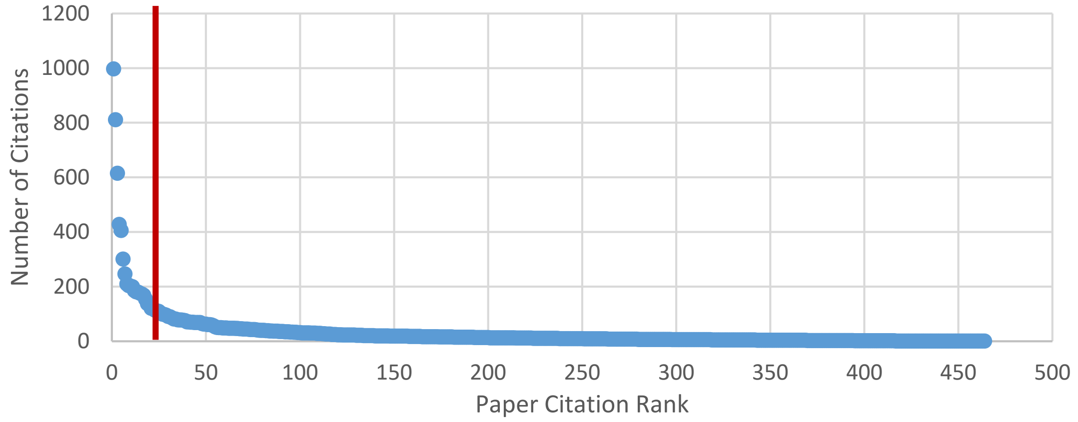
This was one of the top-ranking review papers at the start of explosion of interest in topological quantum computation

"Topological quantum computation" was in the top 5% of papers in Physics Today and of condensed matter physics on the same topic…

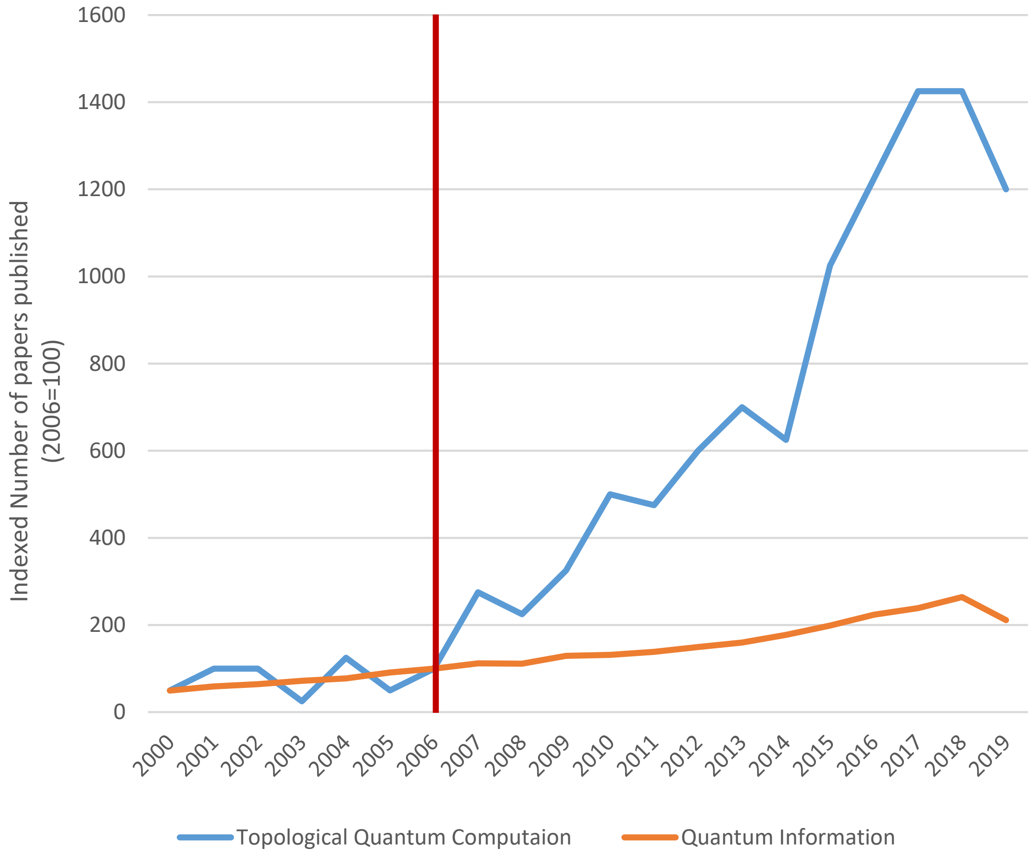
Citation distribution for papers with more than one citation published in Physics Today in 2006



Citation distribution for condensed matter papers on topological quantum computation published in 2006



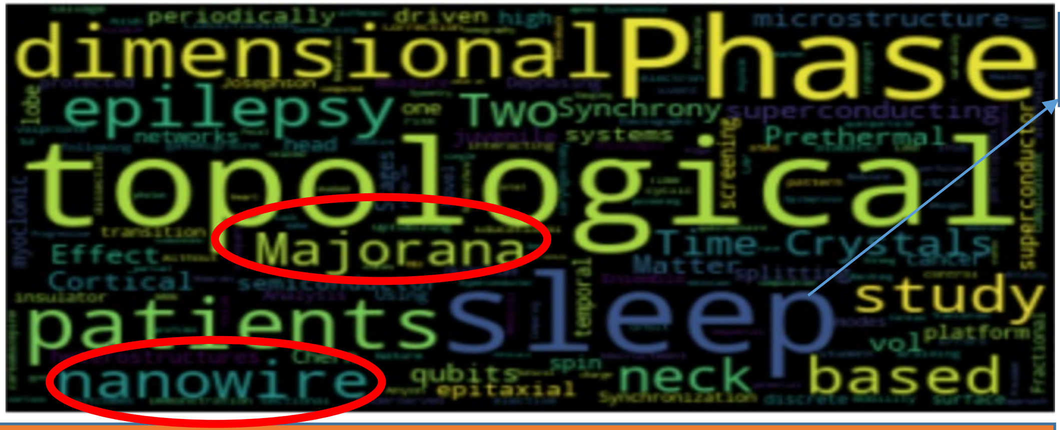
…And came at the start of an explosion in interest in topological quantum computation



Just like the rest of the field, the lead author's initial interest in Hall states has focused in on Majorana states

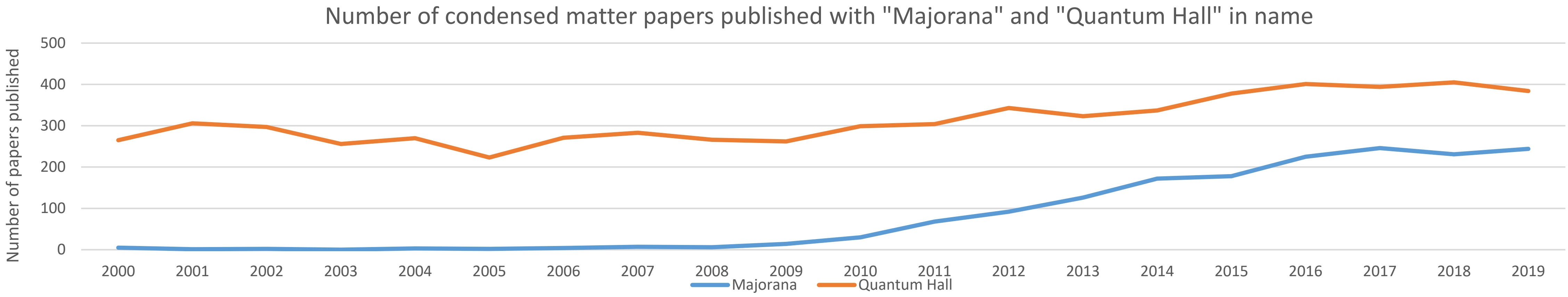
Word cloud of paper titles from 2006-2010 – Nayak was focused on quantum hall states

Nayak then focused on Majorana modes as the most promising way to realise a quantum computer



Learnt from experience?

Which largely mirrors the rest of the field



Authors find two main issues facing Topological Quantum Computation

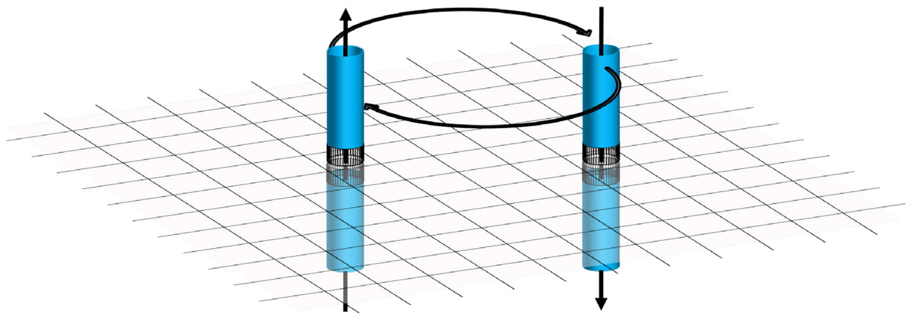
1. Identifying a suitable system with non-abelian statistics

- **Known:** fractional quantum Hall states (e.g. $\nu = 1/3$) that support abelian anyonic excitations exist in nature—*but not sufficient*

- **Need:** system with non-abelian statistics—the $\nu = 5/2$ state may satisfy this condition

2. Finding a way to perform the necessary braiding operations

- Toy model exists for performing topological quantum computation, but not yet experimentally realized



Our outlook & conclusions about the paper

- Even if a system satisfying the described criteria is identified, it is still a long road until a fully realizable implementation is possible
- The paper doesn't address other systems or give detailed explanation topological quantum computation
- Suggest intro reading: Lahtinen & Pachos "A Short Introduction to Topological Quantum Computation" (2017)
- Our outlook: for aspiring scientists, the field is intellectually exciting despite experimental challenges


Danke
Merci
Cảm ơn bạn
Teşekkür ederiz
Grazie
谢谢
Obrigado
謝謝
Gracias
Thank you
شكرا
Terima kasih
धन्यवाद
ధన్యవాదాలు
Спасибо
شكرا
நன்றி
धन्यवाद
ありがとう
ขอบคุณ
감사합니다